

## UNICYCLIC REFLEXIVE GRAPHS WITH SEVEN LOADED VERTICES OF THE CYCLE\*

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### Abstract

A simple graph is reflexive if the second largest eigenvalue of its  $(0, 1)$ -adjacency matrix does not exceed 2. A vertex of the cycle of unicyclic simple graph is said to be loaded if its degree is greater than 2. In this paper we establish that the length of the cycle of unicyclic reflexive graph with seven loaded vertices is at most 10 and find all such graphs with the length of the cycle 10, 9 and 8.

## 1 Introduction

For a graph  $G$ , having  $(0, 1)$ -adjacency matrix  $A = A(G)$ , the roots of its *characteristic polynomial*  $P_G(\lambda) = \det(\lambda I - A)$  are defined as the *eigenvalues* of  $G$ , making up its *spectrum*. If  $G$  is a simple graph (non-oriented and without loops or multiple edges),  $A$  is symmetric and the eigenvalues are all real numbers, in which case we assume their non-increasing order:  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ . In a connected graph  $\lambda_1 > \lambda_2$  holds and the spectrum of a disconnected graph is the union of the spectra of its components. Spectra of a graph  $G$  and an induced subgraph  $H$  of  $G$  are related by the *interlacing theorem*:

*Let  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$  be the eigenvalues of a graph  $G$  and  $\mu_1 \geq \mu_2 \geq \dots \geq \mu_m$  eigenvalues of its induced subgraph  $H$ . Then the inequalities  $\lambda_{n-m+i} \leq \mu_i \leq \lambda_i$  hold.*

Thus,  $\lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \dots$ . Also, if  $G$  is connected,  $\lambda_1 > \mu_1$  holds.

All graphs in this paper are simple. By saying *subgraph*, we always mean *induced subgraph*. A graph  $G$  is *reflexive* if  $\lambda_2(G) \leq 2$ . Reflexive graphs correspond to some sets of vectors in the Lorentz space  $\mathbb{R}^{p,1}$  and have some applications in construction and classification of reflection groups [5]. Among reflexive graphs that have been investigated so far are trees, various classes of cacti (graphs whose all cycles are edge-disjoint) and, recently, some specified classes of bicyclic and unicyclic graphs (results in [4], [6], [7] and some other papers).

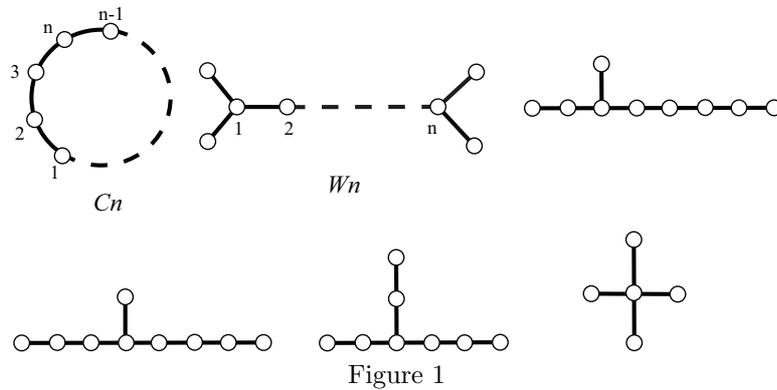
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According to the interlacing theorem, the property of being a reflexive graph is a hereditary one, i. e. all subgraphs of such a graph preserve this property. Therefore it is usual to present reflexive graphs via sets of maximal (connected) graphs.

A connected graph  $G$  is a maximal reflexive graph inside a given class  $\mathcal{C}$  if  $G$  is reflexive and any extension  $G + v$  by a new vertex  $v$  that belongs to  $\mathcal{C}$  has  $\lambda_2 > 2$ .

In this paper we deal with unicyclic reflexive graphs. We will say that a vertex  $v$  of the cycle of such a graph is loaded if  $\text{deg}(v) > 2$ . After some necessary preliminary results in Section 2, we prove in Section 3 that the cycle of unicyclic reflexive graph of a length  $l > 10$  cannot have more than 6 loaded vertices. In Section 4 we find all maximal unicyclic reflexive graphs for  $l = 8, 9, 10$  with 7 loaded vertices of the cycle. These are main results of master thesis [4].

Generally, the terminology concerning theory of graph spectra in this paper follows [2].

## 2 Preliminaries

We will first describe the set of connected graphs whose largest eigenvalue (the index) equals 2.

**Lemma 1.** ([10]) *The index of a graph  $G$  satisfies  $\lambda_1(G) \leq 2$  ( $\lambda_1(G) < 2$ ) if and only if each component of  $G$  is a subgraph (a proper subgraph) of some of the graphs displayed in Fig. 1, all of which have  $\lambda_1 = 2$ .*

These graphs are known as Smith graphs. The label  $W_n$  is usual for the second one of these graphs (which than has diameter  $n + 1$ ). Of course, the last Smith graph is  $W_1$ , yet sometimes it is convenient to treat it separately.

Suitable relations between the values of characteristic polynomials  $P_G(\lambda)$  and  $P_H(\lambda)$  of a graph  $G$  and some of its subgraphs  $H$  are given by the following facts.

**Lemma 2.** ([9]) *Given a graph  $G$ , let  $\mathcal{C}(v)$  ( $\mathcal{C}(uv)$ ) denote the set of all cycles containing a vertex  $v$  and an edge  $uv$  of  $G$ , respectively. Then*

$$\begin{aligned}
 (i) \quad & P_G(\lambda) = \lambda P_{G-v}(\lambda) - \sum_{u \in \text{Adj}(v)} P_{G-v-u}(\lambda) - 2 \sum_{C \in \mathcal{C}(v)} P_{G-V(C)}(\lambda), \\
 (ii) \quad & P_G(\lambda) = P_{G-uv}(\lambda) - P_{G-v-u}(\lambda) - 2 \sum_{C \in \mathcal{C}(uv)} P_{G-V(C)}(\lambda),
 \end{aligned}$$



Figure 2

where  $Adj(v)$  denotes the set of neighbours of  $v$ , while  $G - V(C)$  is the graph obtained from  $G$  by removing the vertices belonging to the cycle  $C$ .

**Corollary 1.** Let  $G$  be a graph obtained by joining a vertex  $v_1$  of a graph  $G_1$  to a vertex  $v_2$  of a graph  $G_2$  by an edge. Let  $G'_1(G'_2)$  be the subgraph of  $G_1(G_2)$  obtained by deleting the vertex  $v_1(v_2)$  from  $G_1$  (resp.  $G_2$ ). Then

$$P_G(\lambda) = P_{G_1}(\lambda)P_{G_2}(\lambda) - P_{G'_1}(\lambda)P_{G'_2}(\lambda).$$

**Corollary 2.** Let  $G$  be a graph with a pendant edge  $v_1v_2$ ,  $v_1$  being of degree 1. Then

$$P_G(\lambda) = \lambda P_{G_1}(\lambda) - P_{G_2}(\lambda),$$

where  $G_1(G_2)$  is the graph obtained from  $G$  (resp.  $G_1$ ) by deleting vertex  $v_1$  (resp.  $v_2$ ).

It will be useful in the coming investigations to have values  $P_G(2)$  of some small graphs.

**Lemma 3 [8].** Let  $P_n$  and  $Z_n$  be the graphs depicted in Fig. 2. Then

1.  $P_{P_n}(2) = n + 2$ ;
2.  $P_{Z_n}(2) = 4$ .

**Lemma 4.**  $P'_{Z_n}(2) = 2(n^2 + 2n + 2)$ .

**Proof.** We shall apply the principal of mathematical induction. For  $n = 1, 2$

$$P_{Z_1}(\lambda) = \lambda^3 - 2\lambda \Rightarrow P'_{Z_1}(\lambda) = 3\lambda^2 - 2 \Rightarrow P'_{Z_1}(2) = 10 = 2(1^2 + 2 \cdot 1 + 2),$$

$$P_{Z_2}(\lambda) = \lambda^4 - 3\lambda^2 \Rightarrow P'_{Z_2}(\lambda) = 4\lambda^3 - 6\lambda \Rightarrow P'_{Z_2}(2) = 20 = 2(2^2 + 2 \cdot 2 + 2).$$

Suppose, now, that the statement holds for every integer  $n, n \geq 3$ . We shall prove that the statement then holds for the integer  $n + 1$ . Corollary 2 gives

$$P_{Z_{n+1}}(\lambda) = \lambda P_{Z_n}(\lambda) - P_{Z_{n-1}}(\lambda),$$

which implies

$$P'_{Z_{n+1}}(\lambda) = P_{Z_n}(\lambda) + \lambda P'_{Z_n}(\lambda) - P'_{Z_{n-1}}(\lambda) =$$

$$= P_{Z_n}(2) + 2P'_{Z_n}(2) - P'_{Z_{n-1}}(2) =$$

$$= 4 + 2 \cdot 2(n^2 + 2n + 2) - 2((n - 1)^2 + 2(n - 1) + 2) =$$

$$= 2(n^2 + 4n + 5) = 2((n + 1)^2 + 2(n + 1) + 2). \quad \square$$

In what follows the next general theorem is to play the crucial role.

**Theorem RS ([8]).** Let  $G$  be a graph with a cut-vertex  $u$ .

(i) If at least two components of  $G - u$  are supergraphs of Smith graphs, and if at least one of them is a proper supergraph, then  $\lambda_2(G) > 2$ .

(ii) If at least two components of  $G - u$  are Smith graphs, and the rest are subgraphs of Smith graphs, then  $\lambda_2(G) = 2$ .

(iii) *If at most one component of  $G-u$  is a Smith graph, and the rest are proper subgraphs of Smith graphs, then  $\lambda_2(G) < 2$ .*

In a lot of cases this theorem is sufficient to answer the question whether a given graph is reflexive or not. If it cannot give the answer, we will say that such a graph is RS-indefinite.

If a reflexive graph is unicyclic, its cycle has a limited number of loaded vertices.

**Theorem 1.** ([6]) The cycle of unicyclic reflexive graph of length  $l > 8$  cannot have more than 7 loaded vertices.

If  $l = 8$ , there are six maximal unicyclic reflexive graphs with 8 loaded vertices, which were found in [6].

### 3 The length of the cycle with seven loaded vertices

The next bound is given by the following theorem.

**Theorem 2.** The cycle of unicyclic reflexive graph of length  $l > 10$  cannot have more than 6 loaded vertices.

**Proof.** Suppose  $l > 10$  and let the cycle have (at least) 7 loaded vertices.

Case 1: Suppose there are two vertices of the cycle,  $u$  and  $v$ , whose removal gives rise to two components such that one of them contains 4 loaded vertices of the cycle, while the other one contains 3 loaded vertices. Suppose besides that  $u$  or  $v$  has a non-loaded neighbour on the cycle. Then by removing e. g.  $u$  we get a subgraph for which Theorem RS, applied to the cut-vertex  $v$ , gives  $\lambda_2 > 2$ , since we get two proper supergraphs of  $W_n$ .

If such a neighbour on the cycle does not exist, there must be non-loaded vertices between the 3 loaded vertices of one component or between the 4 loaded vertices of another one. This case gives rise to the three possibilities displayed in Fig. 3 (a), (b) and (c). By removing vertices  $s$  and  $t$  in the cases (a) and (c) and  $s$  and  $u$  in the case (b) we always obtain two supergraphs (at least one being proper) of  $W_n$ , and according to Theorem RS  $\lambda_2 > 2$  holds.

Case 2: If the described vertices  $u$  and  $v$  do not exist, the cycle must have 7, 6 or 5 consecutive loaded vertices.

If there are 7 and if we remove the non-loaded vertex of the cycle adjacent to the first of the seven loaded vertices, the remaining tree has  $\lambda_2 > 2$ .

In case of 6 consecutive loaded vertices suppose that there are at least 3 consecutive non-loaded vertices between the first of the 6 loaded vertices and the remaining seventh loaded vertex (Fig. 3(d)). Then by removing the indicated vertices  $s$  and  $t$  we get a proper supergraph of  $W_2$  and a proper supergraph of  $W_n$  and Theorem RS gives  $\lambda_2 > 2$ . Otherwise (if there are no 3 consecutive non-loaded vertices, which actually means  $l = 11$ ), we find  $\lambda_2 > 2$  by direct checking.

If there are only 5 consecutive loaded vertices of the cycle, suppose that the first non-loaded neighbour  $s$  of this sequence has also a non-loaded neighbour (Fig. 3(e)). Then by removing vertices  $s$  and  $t$  we have a proper supergraph of  $W_2$  and a

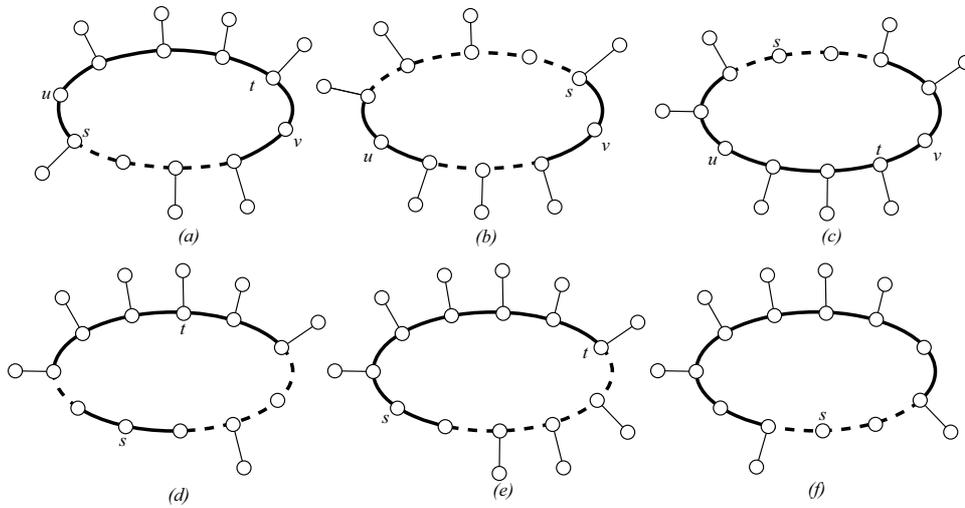


Figure 3

supergraph of  $W_n$ , implying  $\lambda_2 > 2$ . The remaining possibility is displayed in Fig. 3(f), where the removal of the vertex  $s$  gives a tree with  $\lambda_2 > 2$ , which completes the proof.  $\square$

## 4 Maximal graphs

We will now find all maximal unicyclic reflexive graphs with seven loaded vertices if the length of the cycle is  $l = 8, 9, 10$ .

### 4.1 $l = 10$

Among all ways of loading the 10 vertices of the cycle, the condition  $\lambda_2 \leq 2$  admits only that one displayed in Fig. 4. The maximal graphs can easily be found by direct calculating the spectra.

**Theorem 3.** If  $l = 10$ , the only maximal unicyclic reflexive graphs are the two graphs of Fig. 4 (both having  $\lambda_2 = 2$ ).

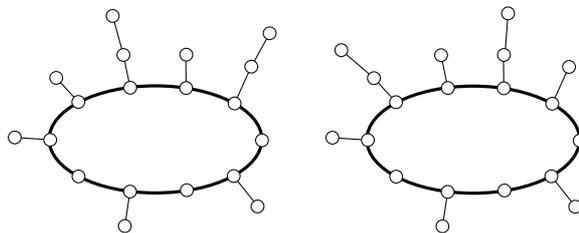


Figure 4

4.2  $l = 9$

Let now the length of the cycle be 9. There are two different ways to load seven vertices of such a cycle and to keep its reflexivity:

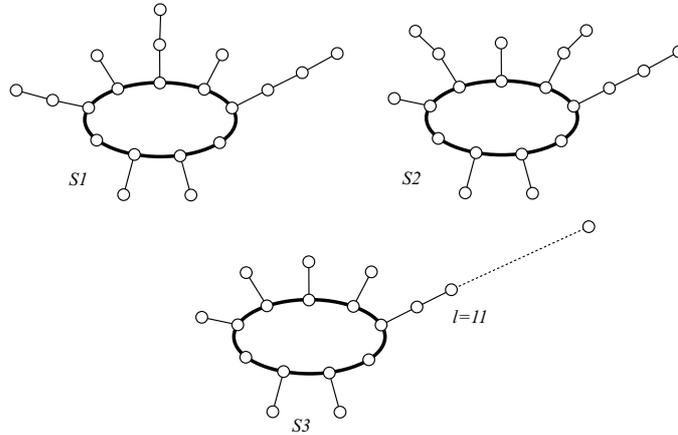


Figure 5

Case 1: Loaded vertices are grouped in two sets of consecutive vertices of the form  $5 + 2$ . Such a graph can be extended only by paths of different lengths, and the three maximal graphs are found by directly calculating the spectra, and are given in Figure 5.

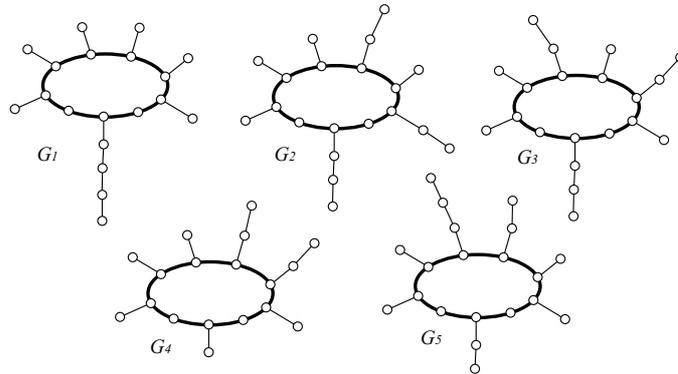


Figure 6

Case 2: There are six consecutive loaded vertices of the cycle and one loaded vertex with both non-loaded neighbours (the form  $6 + 1$ ). This graph can also be extended by paths of different lengths, and the five maximal graphs of that form are also found by directly calculating the spectra, and are given in Figure 6.

But, besides, two middle vertices in the set of 6 consecutive loaded vertices can be loaded by graphs  $Z_i, i \in \mathbb{N}$  (see Figure 2(b)).

We shall prove that all graphs of the form of Figure 7(a) have  $\lambda_2 = \lambda_3 = 2$  and are maximal reflexive graphs in the observed family.

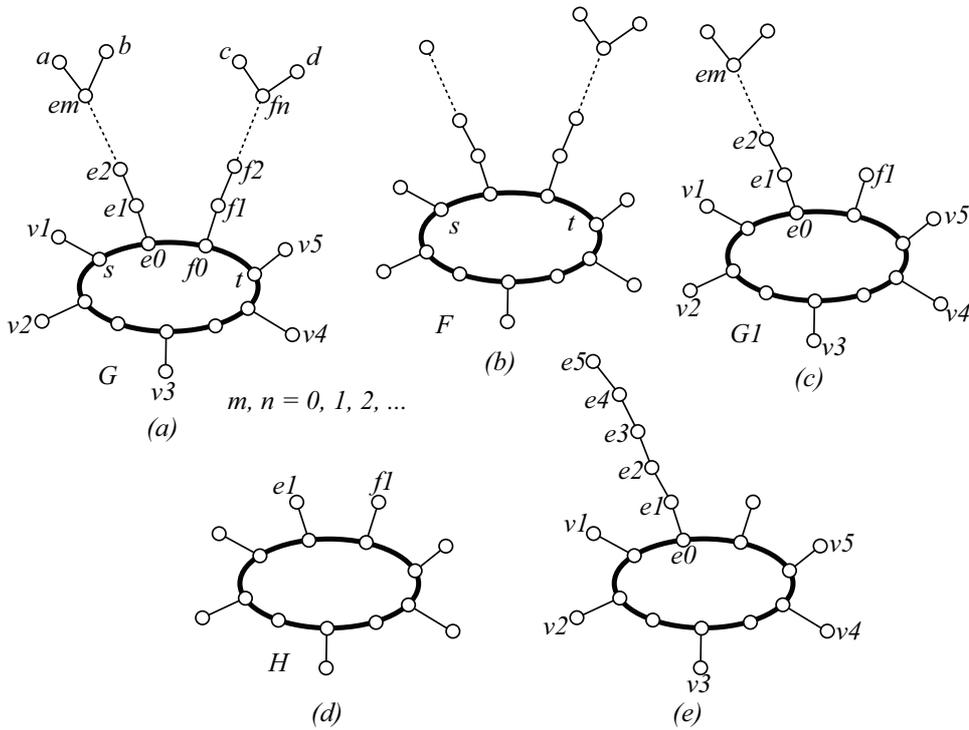


Figure 7

To do that, consider graph  $F$  given in Fig. 7(b) (which is a subgraph of graph  $G$  given in Fig. 7(a)). If we remove vertex  $s$  of  $F$  (thus getting subgraph  $F'$ ) and apply Theorem RS to (vertex  $t$  of)  $F'$ , we see that  $\lambda_2(F') < 2$  and now the interlacing theorem gives  $\lambda_3(F) < 2$  and  $\lambda_4(G) < 2$ .

Consider now graph  $G_1$  given in Figure 7(c). Applying Corollary 1 to the edge  $e_1e_2$  we have:

$$P_{G_1}(2) = P_H(2)P_{Z_{m-1}}(2) - P_{H-e_1}(2)P_{Z_{m-2}}(2),$$

( $H$  being the graph of Fig.7(d)).

Direct calculation of  $P_H(2)$  and  $P_{H-e_1}(2)$  and the application of Lemma 3 ( $P_{Z_n}(2) = 4$ ) gives  $P_{G_1}(2) = 0$ . Now, by repeated application of Corollary 1 we find:  $P_G(\lambda) = P_{G_1}(\lambda)P_{Z_{n-1}}(\lambda) - P_{G_1-f_1}(\lambda)P_{Z_{n-2}}(\lambda)$ .

To calculate  $P_{G_1-f_1}(\lambda)$ , we have to apply again Corollary 1:  $P_{G_1-f_1}(\lambda) = P_{H-f_1}(\lambda)P_{Z_{m-1}}(\lambda) - P_{H-e_1-f_1}(\lambda)P_{Z_{m-2}}(\lambda)$ .

Now, we can easily calculate  $P_G(2)$  and  $P'_G(2)$  by direct calculating the values  $P_{H-f_1}(2)$ ,  $P_{H-e_1-f_1}(2)$ ,  $P'_{H-f_1}(2)$  and  $P'_{H-e_1-f_1}(2)$  and by using the results of Lemma 3 and Lemma 4, and we see that  $P_G(2) = P'_G(2) = 0$  holds.

Consider, now graphs shown in Figure 7(c) and 7(e). Any extension of the graph of Figure 7(e) by a pendant edge at any vertex  $v_i$ ,  $i = 1, 2, 3, 4, 5$ , has  $\lambda_2 > 2$ , implying that any such an extension is impossible for its supergraph of Fig. 7(c) (for  $m \geq 4$ ). If  $m = 0, 1, 2, 3$  for the graph in Fig. 7(c) - direct calculating the

spectra leads us to the same conclusion - any extension by a pendant edge at any vertex  $v_i$ ,  $i = 1, 2, 3, 4, 5$ , has  $\lambda_2 > 2$ .

This means that graph of Fig. 7(a) cannot be extended at any vertex  $v_i$ . It also cannot be extended at any vertex  $a, b, c, d, e_i, f_i$ , because the removal of the vertices  $s$  and  $t$  and the application of the Theorem RS gives  $\lambda_2 > 2$  in such an extension. Thus, graph given in Fig. 7(a) is a maximal reflexive graph in the observed family of graphs.

We have come to the following conclusion:

**Theorem 4.** If  $l = 9$ , the only maximal unicyclic reflexive graphs with 7 loaded vertices of the cycle, all having  $\lambda_2 = 2$ , are the graphs given in Fig. 5, Fig. 6 and Fig. 7(a).

### 4.3 $l = 8$

Let now the length of the cycle be 8. There is only one way to load 7 vertices of such a cycle - all loaded vertices are adjacent.

It turns out that such a graph can be extended:

1. by paths of different lengths,
2. by graphs  $Z_n$ ,  $n \in \mathbb{N}$ ,
3. by trees given in Fig. 8 ( $l_1 \in \mathbb{N} \cup \{0\}, l_2, l_3 \in \mathbb{N}$ ).

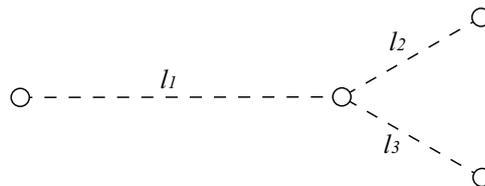


Figure 8

In the first case there are 46 maximal reflexive graphs, and they are all found by directly calculating the spectra. Those graphs are given in Table 1. Columns of the table represent the 8 vertices of the graphs, and the number in each field marks the length of the path that the corresponding vertex is loaded with. Graphs marked with an asterisk have  $\lambda_2 < 2$ .

No.	V1	V2	V3	V4	V5	V6	V7	V8
1	1	2	1	2	3	1	1	0
2	1	2	2	1	2	1	1	0
3	1	2	1	2	1	1	1	0
4	1	2	3	4	1	1	1	0
5	2	2	1	1	1	1	1	0
6	2	2	1	2	1	1	2	0
7	2	2	1	1	1	1	1	0
8	2	2	1	2	1	1	6	0
9	2	2	1	1	2	1	5	0
10	2	2	1	7	2	1	4	0
11	2	2	2	1	1	1	3	0
12	2	2	1	1	2	1	3	0
13	2	2	1	1	4	1	2	0
14	2	2	1	1	4	1	1	0
15	2	2	5	1	2	1	1	0
16	2	2	4	1	7	1	1	0
17	2	2	3	3	2	1	1	0
18	3	2	2	1	1	1	1	0
19	3	3	1	2	2	1	1	0
20	3	3	1	1	1	1	5	0
21	3	3	1	1	18	1	3	0
22	3	3	1	1	50	1	2	0
23	3	3	1	1	2	2	1	0
24	3	3	1	1	2	2	1	0
25	3	3	2	1	2	1	1	0
26	4	4	1	7	1	1	4	0
27	4	4	4	7	1	1	1	0
28	4	4	13	6	1	1	1	0
29	4	4	22	5	1	1	1	0
30	4	4	31	4	1	1	1	0
31	4	4	40	3	1	1	1	0
32	4	4	49	2	1	1	1	0
33	4	4	58	1	1	1	1	0
34	5	5	1	3	1	1	2	0
35	5	5	8	3	1	1	1	0
36	5	5	17	2	1	1	1	0
37	5	5	26	1	1	1	1	0
38	6	6	6	2	1	1	1	0
39	6	7	15	1	1	1	1	0
40	7	7	1	2	1	1	1	0
41	8	8	10	1	1	1	1	0
42	9	9	6	1	1	1	1	0
43	10	10	4	3	1	1	1	0
44	11	11	3	1	1	1	1	0
45	11	11	2	1	1	1	1	0
46	12	12	1	1	1	1	1	0

Table 1

In the second case, the procedure similar to the one described in previous section, leads us to the 10 families of maximal reflexive graphs shown in Fig. 9.

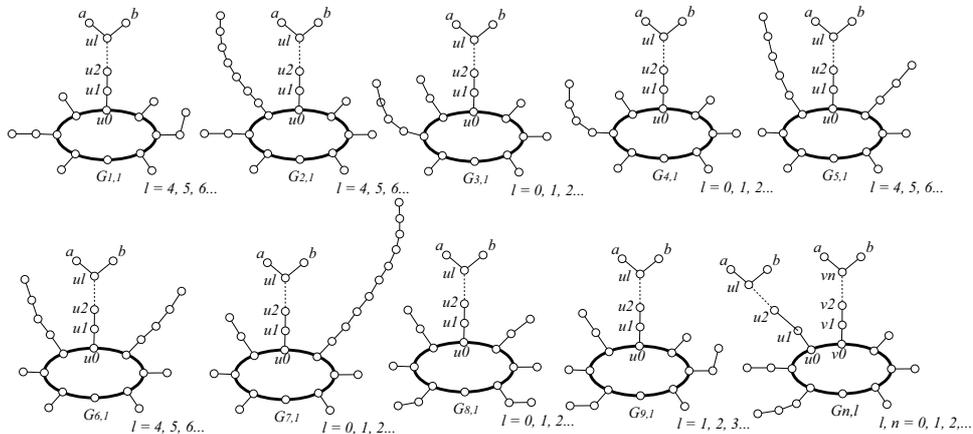


Figure 9

Graphs  $G_{i,l}$ ,  $i = 1, 2, 5, 6$ ,  $l = 0, 1, 2, 3$ , and graph  $G_{9,0}$  are not maximal. They can be extended at one or both of the vertices  $a$  and  $b$ . This observation leads us

to the third case - vertices of the cycle are loaded not only by paths, or graphs  $Z_n$ , but one vertex is loaded by graph given in Fig. 8.

By direct calculating the spectra we can see that there are 44 new maximal reflexive graphs. They are given in Table 2.

To understand the meaning of Tab. 2, consider, first, Fig. 10.

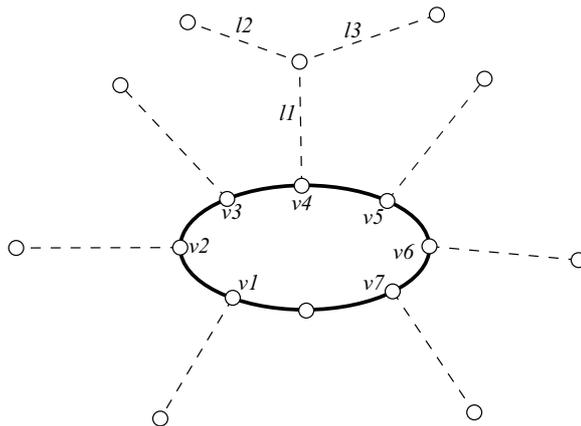


Figure 10

The columns  $v_i$ ,  $i = 1, 2, 3, 5, 6, 7$ , of the table represent six vertices of the graph that are loaded by paths of different lengths, the columns  $l_1, l_2, l_3$  represent the lengths of the paths of the graph given in Fig. 8, vertex  $v_4$  being loaded by graph of Fig. 8.

No.	V1	V2	V3	$l_1$	$l_2$	$l_3$	V5	V6	V7
1	1	1	1	0	1	13	1	1	1
2	1	1	2	0	1	10	1	1	1
3	1	1	2	0	1	9	2	1	1
4	1	1	3	0	1	8	2	1	1
5	1	1	4	0	1	7	4	1	1
6	1	1	5	0	1	6	2	1	1
7	1	2	7	0	1	5	1	1	1
8	1	2	1	0	1	5	1	2	1
9	1	1	6	0	1	5	2	1	1
10	1	1	5	0	1	5	3	1	1
11	1	1	10	0	1	4	1	1	1
12	1	1	7	0	1	4	2	1	1
13	1	1	16	0	1	3	1	1	1
14	1	1	8	0	1	3	2	1	1
15	1	1	1	0	2	3	1	1	1
16	1	1	34	0	1	2	1	1	1
17	1	1	9	0	1	2	2	1	1
18	1	1	1	0	1	2	1	3	1
19	1	1	2	0	1	2	1	2	1
20	1	2	1	0	2	2	1	2	1
21	1	2	7	0	2	2	1	1	1
22	1	1	6	0	2	2	2	1	1
23	1	2	5	0	2	2	3	1	1
24	1	1	4	0	2	2	4	1	1
25	1	1	1	1	1	4	1	1	1
26	1	2	1	1	1	3	1	2	1
27	1	2	7	1	1	3	1	1	1
28	1	1	6	1	1	3	2	1	1
29	1	1	5	1	1	3	3	1	1
30	1	1	4	1	1	3	4	1	1
31	1	1	25	1	1	2	1	1	1
32	1	3	1	1	1	2	1	1	1
33	1	1	8	1	1	2	2	1	1
34	1	1	16	2	1	2	1	1	1
35	1	1	8	2	1	2	2	1	1
36	1	1	5	2	1	2	3	1	1
37	1	1	4	2	1	2	4	1	1
38	1	2	7	2	1	2	1	1	1
39	1	2	1	2	1	2	1	2	1
40	1	2	7	3	1	2	1	1	1
41	1	1	6	3	1	2	2	1	1
42	1	1	5	3	1	2	3	1	1
43	1	1	4	3	1	2	4	1	1
44	1	2	1	3	1	2	1	2	1

Table 2

Now we can formulate the next theorem:

**Theorem 5.** If  $l = 8$ , the only maximal unicyclic reflexive graphs with 7 loaded vertices of the cycle are the graphs given in Table 1, Figure 9, and Table 2.

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## References

- [1] V. Brankov, D. Cvetković, S. Simić, D. Stevanović: *Simultaneous editing and multilabelling of graphs in system newGRAPH* Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. 17 (2006), 112-121.
- [2] D. Cvetković, M. Doob, H. Sachs: *Spectra of Graphs-Theory and Application*. Deutscher Verlag der Wissenschaften-Academic Press, Berlin-New York, 1980; second edition 1982; third edition, Johann Ambrosius Barth Verlag, Heidelberg-Leipzig, 1995.

- [3] D. Cvetković, L. Kraus, S. S. Simić: *Discussing graph theory with a computer, Implementation of algorithms*. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat. Fiz., No. **716**-No. **734** (1981), 100-104.
- [4] T. Koledin: *Reflexive graphs with a small number of cycles* (Serbian). Master thesis, Univ. Beograd, School of Electrical Engineering, 2007.
- [5] A. Neumaier, J. J. Seidel: *Discrete hyperbolic geometry*. *Combinatorica*, **3** (1983), 219-237.
- [6] Z. Radosavljević: *On unicyclic reflexive graphs*. *Applicable Analysis and Discrete Mathematics*, Vol. **1**, No. **1** (2007), 228-240.
- [7] Z. Radosavljević, B. Mihailović, M. Rašajski: *On bicyclic reflexive graphs*. *Discrete Math.* **308** (2008), 715-725.
- [8] Z. Radosavljević, S. Simić: *Which bicyclic graphs are reflexive?*. Univ. Beograd, Publ. Elektrotehn. Fak., Ser. Mat., **7** (1996), 90-104.
- [9] A. J. Schwenk: *Computing the characteristic polynomial of a graph*. In: *Graphs and Combinatorics* (Lecture Notes in Math. 406, ed. R. Bari, F. Harary) Springer-Verlag, Berlin-Heidelberg-New York, 1974, 153-172.
- [10] J. H. Smith: *Some properties of the spectrum of a graph*. In: *Combinatorial Structures and Their Applications* (Ed. R. Guy, H. Hanani, N. Sauer, J. Schonheim ) Gordon and Breach, Science Publ., Inc., New York-London-Paris 1970, 403-406.

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