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COMMON FIXED POINT THEOREMS FOR FAMILIES OF MAPS IN COMPLETE $\mathcal{L}$-FUZZY METRIC SPACES *

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Abstract

In this paper, we prove some common fixed point theorems for any even number of compatible mappings in complete $\mathcal{L}$-fuzzy metric spaces. Our main results extend and generalize some known results in fuzzy metric spaces, intuitionistic metric spaces and $\mathcal{L}$-fuzzy metric spaces.

1 Introduction

As a generalization of fuzzy sets introduced by Zadeh [1], Atanassov [2] introduced the idea of intuitionistic fuzzy set. Since then, various concepts of fuzzy metric and intuitionistic fuzzy metric spaces were considered in [3-6]. Recently, Saadati et al. [7] introduced the concept of $\mathcal{L}$-fuzzy metric spaces which is a generalization of fuzzy metric spaces [8] and intuitionistic fuzzy metric spaces [9-10].

Fixed point and common fixed point properties for mappings defined on fuzzy metric spaces, intuitionistic fuzzy metric spaces, and $\mathcal{L}$-fuzzy metric spaces have been studied by many authors[11-15]. Most of the properties which provide the existence of fixed points and common fixed points are of linear contractive type conditions. On the other hand, there are many generalizations [16-20] of commutativity for functions defined on spaces. Sessa [18] defined a generalization of commutativity, which is called weak commutativity. Further Jungck [21] introduced more generalized commutativity, so called compatibility. Mishra et al. [22] obtained common fixed point theorems for compatible maps on fuzzy metric spaces. Recently, Abu-Donia et al.[23] introduced the concept of compatible mappings of type (α) in intuitionistic fuzzy metric spaces, which is equivalent to the concept of compatible

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mappings under some conditions and proved common fixed point theorems for five mappings satisfying some conditions in intuitionistic fuzzy metric spaces. Alaca, et al.[24] introduced the concept of compatible mappings type (I) and (II) and proved common fixed point theorems for four mappings satisfying some conditions in intuitionistic fuzzy metric spaces. Adibi et al.[11] introduced the concept of compatible mappings and proved common fixed point theorems for four mappings satisfying some conditions in $\mathcal{L}$-fuzzy metric spaces.

In this paper, we shall prove a common fixed point theorem for any even number of compatible mappings in $\mathcal{L}$-fuzzy metric spaces. In Section 2, we recall the concept of $\mathcal{L}$-fuzzy metric spaces and some definitions on compatible and weakly compatible mappings in $\mathcal{L}$-fuzzy metric spaces and their relations. In Section 3, we prove the common fixed point theorem for any even number of compatible mappings in $\mathcal{L}$-fuzzy metric spaces. Our results generalize and extend many known results in fuzzy metric spaces, intuitionistic metric spaces and $\mathcal{L}$-fuzzy metric spaces.

2 Preliminaries

Throughout this paper, the letter N will denote the set of positive integers. In this section, some definitions and preliminary results are given which will be used in this paper.

Definition 2.1 [13] Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice, and U a non-empty set called a universe. An $\mathcal{L}$-fuzzy set $\mathcal{A}$ on U is defined as a mapping $\mathcal{A} : U \to L$. For each u in U, $\mathcal{A}(u)$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Lemma 2.1 [25-26] Consider the set L^* and the operation $\leq_{L^*}$ defined by:

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2 \ \text{ and } \ x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$ for all $(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is a complete lattice.

Classically, a triangular norm T on $([0,1], \leq)$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1, x) = x$, for all $x \in [0,1]$. These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (L, \leq_L)$. Define first $0_{\mathcal{L}} = \inf L$ and $1_{\mathcal{L}} = \sup L$.

Definition 2.2 [7] A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathcal{T} : L^2 \to L$ satisfying the following conditions:

(a) $(\forall x \in L), (\mathcal{T}(x, 1_{\mathcal{L}}) = x)$ (boundary condition);

(b) $(\forall (x,y) \in L^2), (\mathcal{T}(x,y) = \mathcal{T}(y,x))$(commutativity);

(c) $(\forall (x,y,z) \in L^3), (\mathcal{T}(x, \mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y),z))$ (associativity);

(d) $(\forall (x,x',y,y') \in L^4), x \leq_L x'$ and $y \leq_L y' \Rightarrow (\mathcal{T}(x,y) \leq_L \mathcal{T}(x',y'))$ (monotonicity).

A t-norm $\mathcal{T}$ on $\mathcal{L}$ is said to be continuous if for any $x, y \in \mathcal{L}$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y we have

$$\lim_{n\to\infty} \mathcal{T}(x_n, y_n) = \mathcal{T}(x,y).$$

For example, $\mathcal{T}(x,y) = \min(x,y)$ and $\mathcal{T}(x,y) = xy$ are two continuous t-norms on $[0,1]$. A t-norm can also be defined recursively as an $(n+1)$-ary operation ($n \in N$) by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n(x_1, x_2, \cdots, x_{n+1}) = \mathcal{T}(\mathcal{T}^{n-1}(x_1, x_2, \cdots, x_n), x_n)$$

for $n \geq 2$ and $x_i \in L, 1 \leq i \leq n+1$.

Definition 2.3 [7] A negation on $\mathcal{L}$ is any decreasing mapping $\mathcal{N} : \mathcal{L} \to \mathcal{L}$ satisfying $\mathcal{N}(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and $\mathcal{N}(1_{\mathcal{L}}) = 0_{\mathcal{L}}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in \mathcal{L}$, then $\mathcal{N}$ is called an involutive negation. In this paper, the involutive negation is fixed.

Definition 2.4 [7] The 3-tuple $(X, \mathcal{M}, \mathcal{T})$ is said to be an $\mathcal{L}$-fuzzy metric space if X is an arbitrary (non-empty) set, $\mathcal{T}$ is a continuous t-norm on $\mathcal{L}$ and $\mathcal{M}$ is an $\mathcal{L}$-fuzzy sets on $X^2 \times (0, +\infty)$ satisfying the following conditions: for all $x, y, z \in X, s, t > 0$,

(a) $\mathcal{M}(x,y,t) >_L 0_{\mathcal{L}}$;

(b) $\mathcal{M}(x,y,t) = 1_{\mathcal{L}}$ for all $t > 0$ if and only if $x = y$;

(c) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$;

(d) $\mathcal{T}(\mathcal{M}(x,y,t), \mathcal{M}(y,z,s)) \leq_L \mathcal{M}(x,z,t+s)$ for all $x,y,z \in X, s,t > 0$;

(e) $\mathcal{M}(x,y,\cdot) : (0,+\infty) \to L$ is continuous and $\lim_{t\to+\infty} \mathcal{M}(x,y,t) = 1_{\mathcal{L}}$.

In this case $\mathcal{M}$ is called an $\mathcal{L}$-fuzzy metric. If $\mathcal{M} = \mathcal{M}_{M,N}$ is an intuitionistic fuzzy set then the 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be an intuitionistic fuzzy metric space.

Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. For $t \in (0, +\infty)$, we define the open ball $B(x,r,t)$ with center $x \in X$ and a fixed radius $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, as

$$B(x,r,t) = \{y \in X : \mathcal{M}(x,y,t) >_L \mathcal{N}(r)\}.$$

A subset $A \subseteq X$ is called open if for each $x \in A$, there exist $t > 0$ and $r \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ such that $B(x,r,t) \subseteq A$. Let $\tau_{\mathcal{M}}$ denote the family of all open subsets of X. Then $\tau_{\mathcal{M}}$ is called the topology induced by the $\mathcal{L}$-fuzzy metric $\mathcal{M}$.

Example 2.1 [27] Let (X, d) be a metric space. Denote $\mathcal{T}(a,b) = (a_1 b_1, \min(a_2 + b_2, 1))$, for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in L^* and let M and N be fuzzy sets on $X^2 \times (0, +\infty)$ be defined as follows:

$$\mathcal{M}_{M,N}(x,y,t) = (M(x,y,t), N(x,y,t)) = (\frac{t}{t + d(x,y)}, \frac{d(x,y)}{t + d(x,y)}).$$

Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is an intuitionistic fuzzy metric space.

Lemma 2.2 [8] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. Then, $\mathcal{M}(x,y,t)$ is nondecreasing with respect to t, for all x, y in X.

Definition 2.5 [7] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\varepsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $t > 0$, there exist $n_0 \in N$ such that for all $m \geq n \geq n_0 (n \geq m \geq n_0)$,

$$\mathcal{M}(x_n, x_m, t) >_L \mathcal{N}(\varepsilon).$$

(b) A sequence $\{x_n\}$ in X is said to be converged to x in X (denote by $x_n \to x$) if $\mathcal{M}(x_n, x, t) = \mathcal{M}(x, x_n, t) \to 1_{\mathcal{L}}$ whenever $n \to \infty$ for each $t > 0$.

An $\mathcal{L}$-fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent. It is called compact if every sequence contains a convergent subsequence.

Henceforth, we assume that $\mathcal{T}$ is a continuous t-norm on the lattice $\mathcal{L}$ such that for every $\mu \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}$, there is a $\lambda \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}$ such that

$$\mathcal{T}^{n-1}(\mathcal{N}(\lambda), \mathcal{N}(\lambda), \cdots, \mathcal{N}(\lambda)) >_L \mathcal{N}(\mu).$$

For more information see [7].

Lemma 2.3 [7, 28] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. If we define $E_{\lambda,\mathcal{M}} : X^2 \to R^+ \cup \{0\}$ by

$$E_{\lambda,\mathcal{M}} = \inf\{t > 0 : \mathcal{M}(x, y, t) >_L \mathcal{N}(\lambda)\}$$

for each $\lambda \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}$ and any $x, y \in X$. Then we have

(a) For any $\mu \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}$, there exists $\lambda \in L \setminus \{0_\mathcal{L}, 1_\mathcal{L}\}$ such that

$$E_{\mu,\mathcal{M}}(x_1, x_n) \leq E_{\lambda,\mathcal{M}}(x_1, x_2) + E_{\lambda,\mathcal{M}}(x_2, x_3) + \cdots + E_{\lambda,\mathcal{M}}(x_{n-1}, x_n)$$

for any $x_1, x_2, \cdots, x_n \in X$.

(b) The sequence $\{x_n\}$ is convergent to x with respect to $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if $E_{\lambda,\mathcal{M}}(x_n, x) \to 0$. Also the sequence $\{x_n\}$ is Cauchy with respect to $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if it is Cauchy with $E_{\lambda,\mathcal{M}}$.

Definition 2.6 [28] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. $\mathcal{M}$ is said to be continuous on $X^2 \times (0, +\infty)$ if

$$\lim_{n\to\infty} \mathcal{M}(x_n, y_n, t_n) = \mathcal{M}(x, y, t).$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, +\infty)$ converges to a point $(x, y, t) \in X^2 \times (0, +\infty)$ i.e. $\lim_{n\to\infty} \mathcal{M}(x_n, x, t) = \lim_{n\to\infty} \mathcal{M}(y_n, y, t) = 1_\mathcal{L}$ and $\lim_{n\to\infty} \mathcal{M}(x, y, t_n) = \mathcal{M}(x, y, t)$.

Lemma 2.4 [28] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is a continuous function on $X^2 \times (0, +\infty)$.

Definition 2.7 [11] Let S and T be two mappings from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself and $\{x_n\}$ be a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$. Then the mapping S and T are said to be

(a) weakly commuting if $\mathcal{M}(STx, TSx, t) \geq_L \mathcal{M}(Sx, Tx, t)$ for all $x \in X$ and $t > 0$,

(b) compatible if $\lim_{n\to\infty} \mathcal{M}(STx_n, TSx_n, t) = 1_\mathcal{L}$ for all $t > 0$,

(c) compatible of type (A) if $\lim_{n\to\infty} \mathcal{M}(STx_n, TTx_n, t) = 1_\mathcal{L}$, and

$$\lim_{n\to\infty} \mathcal{M}(TSx_n, SSx_n, t) = 1_\mathcal{L}$$

for all $t > 0$,

(d) compatible of type (P) if $\lim_{n\to\infty} \mathcal{M}(SSx_n, TTx_n, t) = 1_\mathcal{L}$ for all $t > 0$.

Definition 2.8 [11] Let S and T be mappings from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself. The maps S and T are said to be weakly compatible if they commute at their coincidence points, i.e. if $Sp = Tp$ for some $p \in X$, then $STp = TSp$.

Proposition 2.1 [11] Self mappings S and T of an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ are compatible then they are weakly compatible.

Proposition 2.2 [11] Let S and T be two mappings from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself. If S and T are compatible then they are compatible of type (A).

Proposition 2.3 [11] Let S and T be compatible mappings of type (A) from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself. If one of S and T is continuous, then S and T are compatible.

Proposition 2.4 [11] Let S and T be two mappings from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself. Then S and T are compatible if and only if they are compatible of type (P).

Proposition 2.5 [11] Let S and T be compatible mappings of type (A) from an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ into itself. If one of S and T is continuous, then S and T are compatible of type (P).

Lemma 2.5 [11] Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that

$$\mathcal{M}(y_n, y_{n+1}, kt) \geq_L \mathcal{M}(y_{n-1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, \cdots$, then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.9 [29] We say that the $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ has the property (C), if it satisfies the following condition:

$$\mathcal{M}(x, y, t) = C, \quad \text{for all} \quad t > 0 \quad \text{implies} \quad C = 1_{\mathcal{L}}.$$

Lemma 2.6 Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space, witch has the property (C). If for all $x, y \in X, t > 0$ and for a number $k \in (0, 1)$

$$\mathcal{M}(x, y, kt) \geq_L \mathcal{M}(x, y, t),$$

then $x = y$.

Proof Since $k \in (0, 1)$, it follows that $\mathcal{M}(x, y, kt) \leq_L \mathcal{M}(x, y, t)$ for all $x, y \in X, t > 0$ by Lemma 2.2. By condition $\mathcal{M}(x, y, kt) \geq_L \mathcal{M}(x, y, t)$, we have $\mathcal{M}(x, y, t) = C$ for all $t > 0$. Since $(X, \mathcal{M}, \mathcal{T})$ has the property (C), it follows that $C = 1_{\mathcal{L}}$, i.e., $x = y$.

3 Main results

Firstly, we prove a common fixed point theorem for any even number of compatible mappings in a complete $\mathcal{L}$-fuzzy metric space in this section.

Theorem 3.1 Let $(X, \mathcal{M}, \mathcal{T})$ be a complete $\mathcal{L}$-fuzzy metric space with $\mathcal{T}(t, t) \geq_{\mathcal{L}} t$ for all $t \in L$, which has the property (C). Let $P_1, P_2, \cdots, P_{2n}, Q_0$ and Q_1 be mappings from X into itself that satisfy conditions:

(1) $Q_0(X) \subset P_1P_3\cdots P_{2n-1}(X), Q_1(X) \subset P_2P_4\cdots P_{2n}(X)$;

(2)$P_2(P_4\cdots P_{2n}) = (P_4\cdots P_{2n})P_2$,

$P_2P_4(P_6\cdots P_{2n}) = (P_6\cdots P_{2n})P_2P_4$,

$\vdots$

$P_2\cdots P_{2n-2}(P_{2n}) = (P_{2n})P_2\cdots P_{2n-2}$,

$Q_0(P_4\cdots P_{2n}) = (P_4\cdots P2n)Q_0$,

$Q_0(P_6\cdots P_{2n}) = (P_6\cdots P_{2n})Q_0$,

$\vdots$

$Q_0P_{2n} = P_{2n}Q_0$,

$P_1(P_3\cdots P_{2n-1}) = (P_3\cdots P_{2n-1})P_1$,

$P_1P_3(P_5\cdots P_{2n-1}) = (P_5\cdots P_{2n-1})P_1P_3$,

$\vdots$

$P_1\cdots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1\cdots P_{2n-3}$,

$Q_1(P_3\cdots P_{2n-1}) = (P_3\cdots P2n-1)Q_1$,

$Q_1(P_5\cdots P_{2n-1}) = (P_5\cdots P_{2n-1})Q_1$,

$\vdots$

$Q_1P_{2n-1} = P_{2n-1}Q_1$;

(3) Either $P_2\cdots P_{2n}$ or Q_0 is continuous;

(4) $(Q_0, P_2\cdots P_{2n})$ is compatible and $(Q_1, P_1\cdots P_{2n-1})$ is weakly compatible;

(5) There exist a number $k \in (0,1)$ such that

$\mathcal{M}(Q_0x, Q_1y, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P_2P_4\cdots P_{2n}x, Q_0x, t), \mathcal{T}(\mathcal{M}(P_1P_3\cdots P_{2n-1}y, Q_1y, t), \mathcal{T}(\mathcal{M}(P_2P_4\cdots P_{2n}x, P_1P_3\cdots P_{2n-1}y, t), \mathcal{T}(\mathcal{M}(P_1P_3\cdots P_{2n-1}y, Q_0x, \beta t), \mathcal{M}(P_2P_4\cdots P_{2n}x, Q_1y, (2-\beta)t)))))$

for all $x, y \in X, \beta \in (0,2)$ and $t > 0$. Then $P_1, P_2, \cdots, P_{2n}, Q_0$ and Q_1 have a unique common fixed point in X.

Proof Let x_0 be an arbitrary point in X. From the condition (1) there exist $x_1, x_2 \in X$ such that

$Q_0x_0 = P_1P_3\cdots P_{2n-1}x_1 = y_0$ and $Q_1x_1 = P_2P_4\cdots P_{2n}x_2 = y_1$.

Inductively we can construct sequences x_n and y_n in X:

$Q_0x_{2k} = P_1P_3\cdots P_{2n-1}x_{2k+1} = y_{2k}$ and $Q_1x_{2k+1} = P_2P_4\cdots P_{2n}x_{2k+2} = y_{2k+1}$ for $k = 0, 1, \cdots$.

By the condition (5), for all $t > 0$ and $\beta = 1-q$ with $q \in (0,1)$, we have

$\mathcal{M}(y_{2k}, y_{2k+1}, kt)$
$= \mathcal{M}(Q_0x_{2k}, Q_1x_{2k+1}, kt) \geq_L \mathcal{T}(\mathcal{M}(P_2P_4\cdots P_{2n}x_{2k}, Q_0x_{2k}, t),$
$\mathcal{T}(\mathcal{M}(P_1P_3\cdots P_{2n-1}x_{2k+1}, Q_1x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P_2P_4\cdots P_{2n}x_{2k}, P_1P_3\cdots P_{2n-1}x_{2k+1}, t), \mathcal{T}(\mathcal{M}(P_1P_3\cdots P_{2n-1}x_{2k+1}, Q_0x_{2k}, \beta t),$
$\mathcal{M}(P_2P_4\cdots P_{2n}x_{2k}, Q_1x_{2k+1}, (2-\beta)t))))))$
$= \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k}, (1-q)t),$
$\mathcal{M}(y_{2k-1}, y_{2k+1}, (1+q)t)))))$
$= \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t),$
$\mathcal{T}(1_{\mathcal{L}}, \mathcal{M}(y_{2k-1}, y_{2k+1}, (1+q)t)))))$
$\geq_L \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t),$
$\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, qt)))))$

$= \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k-1}, y_{2k}, t)),$
$\mathcal{M}(y_{2k}, y_{2k+1}, qt))))$
$\geq_L \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, qt))))$
$= \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{M}(y_{2k-1}, y_{2k}, t)), \mathcal{M}(y_{2k}, y_{2k+1}, qt)))$
$= \mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{M}(y_{2k-1}, y_{2k}, t)), \mathcal{M}(y_{2k}, y_{2k+1}, qt)))$
$= \mathcal{T}(\mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, t)), \mathcal{M}(y_{2k}, y_{2k+1}, qt)))$
$\geq_L \mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, t)), \mathcal{M}(y_{2k}, y_{2k+1}, qt)).$

As t-norm $\mathcal{T}$ is continuous, letting $q \to 1$ we get:

$\mathcal{M}(y_{2k}, y_{2k+1}, kt) \geq_L \mathcal{T}(\mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, t)), \mathcal{M}(y_{2k}, y_{2k+1}, t))$
$= \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{M}(y_{2k}, y_{2k+1}, t)))$
$\geq_L \mathcal{T}(\mathcal{M}(y_{2k-1}, y_{2k}, t), \mathcal{M}(y_{2k}, y_{2k+1}, t)).$

Similarly, $\mathcal{M}(y_{2k+1}, y_{2k+2}, kt) \geq_L \mathcal{T}(\mathcal{M}(y_{2k}, y_{2k+1}, t), \mathcal{M}(y_{2k+1}, y_{2k+2}, t))$. Therefore, for all m even or odd we have:

$$\mathcal{M}(y_m, y_{m+1}, kt) \geq_L \mathcal{T}(\mathcal{M}(y_{m-1}, y_m, t), \mathcal{M}(y_m, y_{m+1}, t)).$$

Consequently, it follows that, for positive integers m, p

$$\mathcal{M}(y_m, y_{m+1}, kt) \geq_L \mathcal{T}(\mathcal{M}(y_{m-1}, y_m, t), \mathcal{M}(y_m, y_{m+1}, t/k^p)).$$

By noting that $\mathcal{M}(y_m, y_{m+1}, t/k^p) \to 1_{\mathcal{L}}$, we have, for $m = 1, 2, \cdots$

$$\mathcal{M}(y_m, y_{m+1}, kt) \geq_L \mathcal{M}(y_{m-1}, y_m, t).$$

By Lemma2.5, $\{y_m\}$ is a Cauchy sequence in X. Since X is complete, $\{y_m\}$ converges to a point $z \in X$. Also, for its subsequences we have

$$Q_1 x_{2k+1} \to z \text{ and } P_1 P_3 \cdots P_{2n-1} x_{2k+1} \to z,$$

$$Q_0 x_{2k} \to z \text{ and } P_2 P_4 \cdots P_{2n} x_{2k} \to z.$$

Case 1. $P_2 P_4 \cdots P_{2n}$ is continuous.

Define $P'_1 = P_2 P_4 \cdots P_{2n}$. Since P'_1 is continuous, $P'_1 \circ P'_1 x_{2k} \to P'_1 z$ and $P'_1 Q_0 x_{2k} \to P'_1 z$. Also, as (Q_0, P'_1) is compatible, this implies that $Q_0 P'_1 x_{2k} \to P'_1 z$.

(a) Putting $x = P_2 P_4 \cdots P_{2n} x_{2k} = P'_1 x_{2k}, y = x_{2k+1}$, and $P'_2 = P_1 P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$\mathcal{M}(Q_0 P'_1 x_{2k}, Q_1 x_{2k+1}, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P'_1 P'_1 x_{2k}, Q_0 P'_1 x_{2k}, t), \mathcal{T}(\mathcal{M}(P'_2 x_{2k+1}, Q_1 x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P'_1 P'_1 x_{2k}, P'_2 x_{2k+1}, t), \mathcal{T}(\mathcal{M}(P'_1 P'_1 x_{2k}, Q_1 x_{2k+1}, t), \mathcal{M}(P'_2 x_{2k+1}, Q_0 P'_1 x_{2k}, t)))))$

which implies that as $k \to \infty$

$\mathcal{M}(P'_1 z, z, kt) \geq_L \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(P'_1 z, z, t), \mathcal{T}(\mathcal{M}(z, P'_1 z, t), M(P'_1 z, z, t))))) \geq_L$
$\mathcal{M}(P'_1 z, z, t).$

Therefore, by Lemma 2.6, we have $P'_1 z = z$ i.e., $P_2 P_4 \cdots P_{2n} z = z$.

(b) Putting $x = z, y = x_{2k+1}, P'_1 = P_2 P_4 \cdots P_{2n}$, and $P'_2 = P_1 P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$\mathcal{M}(Q_0 z, Q_1 x_{2k+1}, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P'_1 z, Q_0 z, t), \mathcal{T}(\mathcal{M}(P'_2 x_{2k+1}, Q_1 x_{2k+1}, t), \mathcal{T}(\mathcal{M}(P'_1 z, P'_2 x_{2k+1}, t),$

$\mathcal{T}(\mathcal{M}(P_1'z, Q_1x_{2k+1}, t), \mathcal{M}(P_2'x_{2k+1}, Q_0z, t)))))$
which implies that as $k \to \infty$
$\mathcal{M}(Q_0z, z, kt) \geq_L \mathcal{T}(\mathcal{M}(z, Q_0z, t), \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(1_{\mathcal{L}}, \mathcal{M}(z, Q_0z, t))))) \geq_L \mathcal{M}(Q_0z, z, t)$.

Therefore, by Lemma 2.6, we have $Q_0z = z$. Hence, $Q_0z = P_2P_4 \cdots P_{2n}z = z$.

(c) Putting $x = P_4P_6 \cdots P_{2n}z, y = x_{2k+1}, P_1' = P_2P_4 \cdots P_{2n}$, and $P_2' = P_1P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), and using the condition $P_2(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})P_2$ and $Q_0(P_4 \cdots P_{2n}) = (P_4 \cdots P2n)Q_0$ in condition (2), we have
$\mathcal{M}(Q_0P_4P_6 \cdots P_{2n}z, Q_1x_{2k+1}, kt)$
$\geq_L \mathcal{T}(\mathcal{M}(P_1'P_4P_6 \cdots P_{2n}z, Q_0P_4P_6 \cdots P_{2n}z, t), \mathcal{T}(\mathcal{M}(P_2'x_{2k+1}, Q_1x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P_1'P_4P_6 \cdots P_{2n}z, P_2'x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P_1'P_4P_6 \cdots P_{2n}z, Q_1x_{2k+1}, t), \mathcal{M}(P_2'x_{2k+1}, Q_0P_4P_6 \cdots P_{2n}z, t)))))$
which implies that as $k \to \infty$

$$\begin{aligned}\mathcal{M}(P_4P_6 \cdots P_{2n}z, z, kt) &\geq_L \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(P_4P_6 \cdots P_{2n}z, z, t), \\ &\quad \mathcal{T}(\mathcal{M}(P_4P_6 \cdots P_{2n}z, z, t), \mathcal{M}(z, P_4P_6 \cdots P_{2n}z, t))))) \\ &\geq_{\mathcal{L}} \mathcal{M}(P_4P_6 \cdots P_{2n}z, z, t).\end{aligned}$$

Therefore, by Lemma 2.6, we have $P_4P_6 \cdots P_{2n}z = z$. Hence, $P_2(P_4P_6 \cdots P_{2n})z = P_2z$ and so $P_2z = P_2P_4P_6 \cdots P_{2n}z = z$.

Continuing this procedure, we obtain

$$Q_0z = P_2z = P_4z = \cdots = P_{2n}z = z.$$

(d) As $Q_0(X) \subset P_1P_3 \cdots P_{2n-1}(X)$, there exist $y \in X$ such that $z = Q_0z = P_1P_3 \cdots P_{2n-1}y$. Putting $x = x_{2k}, P_1' = P_2P_4 \cdots P_{2n}$, and $P_2' = P_1P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have
$\mathcal{M}(Q_0x_{2k}, Q_1y, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_0x_{2k}, t), \mathcal{T}(\mathcal{M}(P_2'y, Q_1y, t), \mathcal{T}(\mathcal{M}(P_1'x_{2k}, P_2'y, t), \mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_1y, t),$
$\mathcal{M}(P_2'y, Q_0x_{2k}, t)))))$
which implies that as $k \to \infty$
$\mathcal{M}(z, Q_1y, kt) \geq_L$
$\mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(z, Q_1y, t), \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(z, Q_1y, t), 1_{\mathcal{L}})))) \geq_L \mathcal{M}(z, Q_1y, t)$.

Therefore, by Lemma 2.6, we have $Q_1y = z$. Hence, $P_1P_3 \cdots P_{2n-1}y = Q_1y = z$. As $(Q_1, P_1P_3 \cdots P_{2n-1})$ is weakly compatible, we have

$$P_1P_3 \cdots P_{2n-1}Q_1y = Q_1P_1P_3 \cdots P_{2n-1}y.$$

Thus $P_1P_3 \cdots P_{2n-1}z = Q_1z$.

(e) Putting $x = x_{2k}, y = z, P_1' = P_2P_4 \cdots P_{2n}$, and $P_2' = P_1P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have
$\mathcal{M}(Q_0x_{2k}, Q_1z, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_0x_{2k}, t), \mathcal{T}(\mathcal{M}(P_2'z, Q_1z, t), \mathcal{T}(\mathcal{M}(P_1'x_{2k}, P_2'z, t),$
$\mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_1z, t), \mathcal{M}(P_2'z, Q_0x_{2k}, t)))))$
which implies that as $k \to \infty$
$\mathcal{M}(z, Q_1z, kt) \geq_L$
$\mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(z, Q_1z, t), \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(z, Q_1z, t), 1_{\mathcal{L}})))) \geq_L \mathcal{M}(z, Q_1z, t)$.

Therefore, by Lemma 2.6, we have $Q_1 z = z$. Hence, $P_1P_3\cdots P_{2n-1}z = Q_1z = z$.

(f) Putting $x = x_{2k}, y = P_3\cdots P_{2n-1}z, P_1' = P_2P_4\cdots P_{2n}$, and $P_2' = P_1P_3\cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$\mathcal{M}(Q_0x_{2k}, Q_1P_3\cdots P_{2n-1}z, kt) \geq_L$
$\mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_0x_{2k}, t), \mathcal{T}(\mathcal{M}(P_2'P_3\cdots P_{2n-1}z, Q_1P_3\cdots P_{2n-1}z, t),$
$\mathcal{T}(\mathcal{M}(P_1'x_{2k}, P_2'P_3\cdots P_{2n-1}z, t), \mathcal{T}(\mathcal{M}(P_1'x_{2k}, Q_1P_3\cdots P_{2n-1}z, t),$
$\mathcal{M}(P_2'P_3\cdots P_{2n-1}z, Q_0x_{2k}, t)))))$

which implies that as $k \to \infty$

$\mathcal{M}(z, P_3\cdots P_{2n-1}z, kt) \geq_L$
$\mathcal{T}(1_\mathcal{L}, \mathcal{T}(1_\mathcal{L}, \mathcal{T}(\mathcal{M}(z, P_3\cdots P_{2n-1}z, t), \mathcal{T}(\mathcal{M}(z, P_3\cdots P_{2n-1}z, t),$
$\mathcal{M}(P_3\cdots P_{2n-1}z, z, t))))) \geq_L \mathcal{M}(z, P_3\cdots P_{2n-1}z, t)$.

Therefore, by Lemma 2.6, we have $P_3\cdots P_{2n-1}z = z$. Hence, $P_1 z = z$. Continuing this procedure, we have

$$Q_1z = P_1z = P_3z = \cdots = P_{2n-1}z.$$

Thus we have prove

$$Q_0z = Q_1z = P_1z = P_2z = P_3z = \cdots = P_{2n-1}z = P_{2n}z = z.$$

Case 2. Q_0 is continuous.

Since Q_0 is continuous, $Q_0^2x_{2k} \to Q_0z$ and $Q_0(P_2P_4\cdots P_{2n})x_{2k} \to Q_0z$. As $(Q_0, P_2P_4\cdots P_{2n})$ is compatible, we have $(P_2P_4\cdots P_{2n})Q_0x_{2k} \to Q_0z$.

(g) Putting $x = Q_0x_{2k}, y = x_{2k+1}, P_1' = P_2P_4\cdots P_{2n}$, and $P_2' = P_1P_3\cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$\mathcal{M}(Q_0Q_0x_{2k}, Q_1x_{2k+1}, kt)$
$\geq_L \mathcal{T}(\mathcal{M}(P_1'Q_0x_{2k}, Q_0Q_0x_{2k}, t), \mathcal{T}(\mathcal{M}(P_2'x_{2k+1}, Q_1x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P_1'Q_0x_{2k}, P_2'x_{2k+1}, t),$
$\mathcal{T}(\mathcal{M}(P_1'Q_0x_{2k}, Q_1x_{2k+1}, t), \mathcal{M}(P_2'x_{2k+1}, Q_0Q_0x_{2k}, t)))))$

which implies that as $k \to \infty$

$\mathcal{M}(Q_0z, z, kt) \geq_L \mathcal{T}(1_\mathcal{L}, \mathcal{T}(1_\mathcal{L}, \mathcal{T}(\mathcal{M}(Q_0z, z, t), \mathcal{T}(\mathcal{M}(Q_0z, z, t), \mathcal{M}(Q_0z, z, t))))) \geq_L$
$\mathcal{M}(z, Q_0z, t)$.

Therefore, by Lemma 2.6, we have $Q_0z = z$. Now using step (d), (e) and (f) and continuing step (f) gives us

$$Q_1z = P_1z = P_3z = \cdots = P_{2n-1}z = z.$$

(h) As $Q_1(X) \subset P_2P_4\cdots P_{2n}(X)$, there exist $w \in X$ such that $z = Q_1z = P_2P_4\cdots P_{2n}w$. Putting $x = w, y = x_{2k+1}, P_1' = P_2P_4\cdots P_{2n}$, and $P_2' = P_1P_3\cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$\mathcal{M}(Q_0w, Q_1x_{2k+1}, kt)$
$\geq_L \mathcal{T}(\mathcal{M}(P_1'w, Q_0w, t), \mathcal{T}(\mathcal{M}(P_2'x_{2k+1}, Q_1x_{2k+1}, t), \mathcal{T}(\mathcal{M}(P_2'x_{2k+1}, Q_0w, t),$
$\mathcal{T}(\mathcal{M}(P_1'w, Q_1x_{2k+1}, t), \mathcal{M}(P_1'w, P_2'x_{2k+1}, t)))))$

which implies that as $k \to \infty$

$\mathcal{M}(Q_0w, z, kt) \geq_L \mathcal{T}(\mathcal{M}(z, Q_0w, t), \mathcal{T}(1_\mathcal{L}, \mathcal{T}(1_\mathcal{L}, \mathcal{T}(1_\mathcal{L}, \mathcal{M}(z, Q_0w, t)))))$
$\geq_L \mathcal{M}(z, Q_0w, t)$.

Therefore, by Lemma 2.6, we have $Q_0w = z$. Hence, $Q_0w = z = P_2P_4 \cdots P_{2n}w$. As $(Q_0, P_2P_4 \cdots P_{2n})$ is weakly compatible, we have

$$P_2P_4 \cdots P_{2n}Q_0w = Q_0P_2P_4 \cdots P_{2n}w.$$

Thus $P_2P_4 \cdots P_{2n}z = Q_0z = z$.

Similarly to in step (c) it can be shown that $P_2z = P_4z = \cdots = P_{2n}z = Q_0z = z$. Thus we have proved that

$$Q_0z = Q_1z = P_1z = P_2z = P_3z = \cdots = P_{2n}z = z.$$

Proof of uniqueness Let z' be another common fixed point of the aforementioned mappings, then $Q_0z' = Q_1z' = P_1z' = P_2z' = P_3z' = \cdots = P_{2n}z' = z'$. Putting $x = z, y = z', P_1' = P_2P_4 \cdots P_{2n}$ and $P_2' = P_1P_3 \cdots P_{2n-1}$ with $\beta = 1$ in condition (5), we have

$$\begin{aligned}&\mathcal{M}(Q_0z, Q_1z', kt)\\ &\quad \geq_L \mathcal{T}(\mathcal{M}(P_1'z, Q_0z, t), \mathcal{T}(\mathcal{M}(P_2'z', Q_1z', t), \mathcal{T}(\mathcal{M}(P_1'z, P_2'z', t),\\ &\quad \mathcal{T}(\mathcal{M}(P_1'z, Q_1z', t), \mathcal{M}(P_2'z', Q_0z, t)))))\end{aligned}$$

which implies that as $k \to \infty$

$$\mathcal{M}(z, z', kt) \geq_L \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(1_{\mathcal{L}}, \mathcal{T}(\mathcal{M}(z', z, t), \mathcal{T}(\mathcal{M}(z, z', t), \mathcal{M}(z, z', t))))) \geq_L \mathcal{M}(z, z', t).$$

Therefore, by Lemma 2.6, we have $z = z'$, and this shows z is a unique common fixed point of mappings.

Now we shall prove a common fixed point theorem, which is a slight generalization of Theorem 3.1.

Theorem 3.2 Let $(X, \mathcal{M}, \mathcal{T})$ be a complete $\mathcal{L}$-fuzzy metric space with $\mathcal{T}(t, t) \geq_{\mathcal{L}} t$ for all $t \in L$, which has the property (C). Let $\{T_\mu\}_{\mu \in J}$ and $\{P_i\}_{i=1}^{2n}$ be two families of self-mappings of X. Suppose that there exists a fixed $\nu \in J$ such that:

(1) $T_\mu(X) \subset P_2P_4 \cdots P_{2n}(X)$, for each $\mu \in J$ and $T_\nu(X) \subset P_1P_3 \cdots P_{2n-1}(X)$ for some $\nu \in J$;

(2) $P_2(P_4 \cdots P_{2n}) = (P_4 \cdots P_{2n})P_2$,
$P_2P_4(P_6 \cdots P_{2n}) = (P_6 \cdots P_{2n})P_2P_4$,
$\vdots$
$P_2 \cdots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \cdots P_{2n-2}$,
$T_\nu(P_4 \cdots P_{2n}) = (P_4 \cdots P2n)T_\nu$,
$T_\nu(P_6 \cdots P_{2n}) = (P_6 \cdots P_{2n})T_\nu$,
$\vdots$
$T_\nu P_{2n} = P_{2n}T_\nu$,
$P_1(P_3 \cdots P_{2n-1}) = (P_3 \cdots P_{2n-1})P_1$,
$P_1P_3(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})P_1P_3$,
$\vdots$
$P_1 \cdots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \cdots P_{2n-3}$,
$T_\mu(P_3 \cdots P_{2n-1}) = (P_3 \cdots P2n - 1)T_\mu$,

$T_\mu(P_5 \cdots P_{2n-1}) = (P_5 \cdots P_{2n-1})T_\mu,$

$\vdots$

$T_\mu P_{2n-1} = P_{2n-1}T_\mu;$

(3) Either $P_2 \cdots P_{2n}$ or T_ν is continuous;

(4) $(T_\nu, P_2 \cdots P_{2n})$ is compatible and $(T_\mu, P_1 \cdots P_{2n-1})$ is weakly compatible;

(5)There exist a $k \in (0,1)$ such that

$\mathcal{M}(T_\nu x, T_\mu y, kt) \geq_L \mathcal{T}(\mathcal{M}(P_2P_4 \cdots P_{2n}x, T_\nu x, t), \mathcal{T}(\mathcal{M}(P_1P_3 \cdots P_{2n-1}y, T_\mu y, t),$
$\mathcal{T}(\mathcal{M}(P_2P_4 \cdots P_{2n}x, P_1P_3 \cdots P_{2n-1}y, t), \mathcal{T}(\mathcal{M}(P_1P_3 \cdots P_{2n-1}y, T_\nu x, \beta t),$
$\mathcal{M}(P_2P_4 \cdots P_{2n}x, T_\mu y, (2-\beta)t)))))$

for all $x, y \in X, \beta \in (0,2)$ and $t > 0$. Then P_i and T_μ have a unique common fixed point in X.

Proof Let T_{μ_0} be a fixed element in $\{T_\mu\}_{\mu \in J}$. By Theorem 3.1 with $Q_0 = T_\nu$ and $Q_1 = T_{\mu_0}$, it follows that there exists some $z \in X$ such that

$$T_\nu z = T_{\mu_0}z = P_2P_4 \cdots P_{2n}z = P_1P_3 \cdots P_{2n-1}z = z$$

Let $\mu \in J$ be arbitrary and $\beta = 1$. Then from condition (5),

$$\mathcal{M}(T_\nu z, T_\mu z, kt) \geq_L \mathcal{T}(\mathcal{M}(P_2P_4 \cdots P_{2n}z, T_\nu z, t), \mathcal{T}(\mathcal{M}(P_1P_3 \cdots P_{2n-1}z, T_\mu z, t), \mathcal{T}(\mathcal{M}(P_2P_4 \cdots P_{2n}z, P_1P_3 \cdots P_{2n-1}z, t), \mathcal{T}(\mathcal{M}(P_1P_3 \cdots P_{2n-1}z, T_\nu z, t), \mathcal{M}(P_2P_4 \cdots P_{2n}z, T_\mu z, t)))))$$

and hence, $\mathcal{M}(z, T_\mu z, kt) \geq_L \mathcal{T}(1_\mathcal{L}, \mathcal{T}(\mathcal{M}(z, T_\mu z, t), \mathcal{T}(1_\mathcal{L}, \mathcal{T}(1_\mathcal{L}, \mathcal{M}(z, T_\mu z, t))))) \geq_L \mathcal{M}(z, T_\mu z, t)$.

Therefore, by Lemma 2.6, we have $T_\mu z = z$ for each $\mu \in J$. Since condition (5) implies the uniqueness of the common fixed point, Theorem 3.2 is proved.

Remark 3.1 Observe that Theorem 3.1 and 3.2 generalize the Theorem 2.11 of Adibi et al.[11] and the Theorem 2.1 and 2.2 of Saadati et. al.[28] in many aspects.

If we put $Q_0 = A, Q_1 = B, P_2P_4 \cdots P_{2n} = S$ and $P_1P_3 \cdots P_{2n-1} = T$ in Theorem 3.1. We have the following:

Corollary 3.1 Let A, S, B and T be self-mappings on a complete $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ with $\mathcal{T}(t,t) \geq_\mathcal{L} t$ for all $t \in L$, which has the property (C), satisfying:

(1) $A(X) \subseteq T(X), B(X) \subseteq S(X)$;

(2) Either A or S is continuous;

(3) (A, S) is compatible and (B, T) is weakly compatible;

(4) There exists $k \in (0,1)$ such that

$$\mathcal{M}(Ax, By, kt) \geq_L \mathcal{T}(\mathcal{M}(Sx, Ax, t), \mathcal{T}(\mathcal{M}(Ty, By, t), \mathcal{T}(\mathcal{M}(Sx, Ty, t), \mathcal{T}(\mathcal{M}(Ty, Ax, \beta t), \mathcal{M}(Sx, By, (2-\beta)t)))))$$

for all $x, y \in X, \beta \in (0,2)$ and $t > 0$. Then A, S, B and T have a unique common fixed point in X.

Remark 3.2 Above Corollary improves the result of Adibi. et al.[11], Theorem 2.11 in the sense that the requirement of weak compatibility is more general than of compatibility or compatible of type(P). Also the number of required continuities of maps has been reduced to one only in our results.

If we put $Q_0 = L, Q_1 = R, P_2P_4 \cdots P_{2n} = ST$ and $P_1P_3 \cdots P_{2n-1} = AB$ in Theorem 3.1. We have the following:

Corollary 3.2 Let A, B, S, T, L and R be self-mappings on a complete $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ with $\mathcal{T}(t,t) \geq_{\mathcal{L}} t$ for all $t \in L$, which has the property (C), satisfying:

(1) $L(X) \subseteq AB(X), R(X) \subseteq ST(X)$;

(2) $AB = BA, ST = TS, LT = TL, RB = BR$

(3) Either L or ST is continuous;

(4) (L, ST) is compatible and (R, AB) is weakly compatible;

(5) There exists $k \in (0,1)$ such that

$$\mathcal{M}(Lx, Ry, kt) \geq_L \mathcal{T}(\mathcal{M}(STx, Lx, t), \mathcal{T}(\mathcal{M}(ABy, Ry, t), \mathcal{T}(\mathcal{M}(STx, ABy, t), \mathcal{T}(\mathcal{M}(ABy, Lx, \beta t), \mathcal{M}(STx, Ry, (2-\beta)t)))))$$

for all $x, y \in X, \beta \in (0,2)$ and $t > 0$. Then A, B, S, T, L and R have a unique common fixed point in X.

Remark 3.3 Our results generalize and extend many known results in fuzzy metric spaces and intuitionistic metric spaces (see[15, 20, 23-24]).

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