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Preservations of *so*-metrizable spaces

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Abstract. A space is called an *so*-metrizable space if it is a regular space with a σ-locally finite sequentially open network. This paper proves that *so*-metrizable spaces are preserved under perfect mappings and under closed sequence-covering mappings, which give an affirmative answer to a question on preservations of *so*-metrizable spaces under some closed mappings. Also, we prove that the closed image of an *so*-metrizable space is an *so*-metrizable space if it is a topological group.

1. Introduction

Sequentially open networks were introduced and investigated by S. Lin in [18], where sequentially open network was written by *so*-network for short. A space is called to be an *so*-metrizable space if it has a σ-locally finite so-network, which was discussed in [8, 19]. We have the following remark for some important generalized metric spaces including *so*-metrizable spaces.

Remark 1.1. The following implications hold.

$$\begin{array}{ccccc} \textit{metrizable space} & \longrightarrow & \text{g}-\textit{metrizable space} & & \\ \downarrow & & \downarrow & & \\ \text{so}-\textit{metrizable space} & \longrightarrow & \text{sn}-\textit{metrizable space} & \longrightarrow & \aleph-\text{space} \end{array}$$

For these generalized metric spaces, an interesting topic is to investigate their preservation under closed mappings, and the following results are known.

Proposition 1.2. *The following statements hold.*

(1) Metrizable spaces and $\aleph$-spaces are preserved under perfect mappings [16, 23].

(2) g-metrizable spaces and sn-metrizable spaces are not preserved under perfect mappings [10, 17].

(3) g-metrizable spaces and sn-metrizable spaces are preserved under closed finite-to-one mappings [10, 17].

(4) Metrizable spaces, g-metrizable spaces, sn-metrizable spaces and $\aleph$-spaces are preserved under closed sequence-covering mappings [20, 22, 24].

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Take Proposition 1.2 into account, the following question arise naturally.

Question 1.3. *What closed mappings preserve so-metrizable spaces?*
In particular, one can ask:
(1) Are so-metrizable spaces preserved under perfect mappings?
(2) Are so-metrizable spaces preserved under closed sequence-covering mappings?

Related to the above question 1.3, X. Ge proved that clopen mappings preserve *so*-metrizable spaces [8]. However, he did not know even whether closed finite-to-one mappings preserve *so*-metrizable spaces [8].

In this paper, we give an affirmative answer for Question 1.3. Especially, we obtain that closed finite-to-one mappings preserve *so*-metrizable spaces. Also, we prove that the closed image of an *so*-metrizable space is an *so*-metrizable space if it is a topological group.

Throughout this paper, all spaces are assumed to be regular T_1, and all mappings are continuous and onto. $\mathbb{N}$ and ω denote the set of all natural numbers and the first infinite ordinal, respectively. The sequence $\{x_n : n \in \mathbb{N}\}$ is abbreviated to $\{x_n\}$. Let P be a subset of a space X and $\{x_n\}$ be a sequence in X. $\{x_n\}$ converging to x is eventually in P if $\{x_n : n > k\} \cup \{x\} \subset P$ for some $k \in \mathbb{N}$; $\{x_n\}$ is frequently in P if $\{x_{n_k}\}$ is eventually in P for some subsequence $\{x_{n_k}\}$ of $\{x_n\}$. Let $f : X \longrightarrow Y$ be a mapping and $A \subset X$. $f|_A$ denotes the restriction of f on restriction A, i.e., for each $x \in A$, $f|_A(x) = f(x)$.

2. Preliminaries

Definition 2.1. ([7]) Let X be a space.

(1) Let $x \in P \subset X$. P is called a sequential neighborhood of x in X if whenever $\{x_n\}$ is a sequence converging to x, then $\{x_n\}$ is eventually in P.

(2) Let $P \subset X$. P is called a sequentially open subset of X if P is a sequential neighborhood of x in X for each $x \in P$. F is called a sequentially closed subset of X if $X - F$ is sequentially open of X.

(3) X is called a sequential space if each sequentially open subset of X is open in X.

(4) X is called a Fréchet space if for each $P \subset X$ and for each $x \in \overline{P}$, there exists a sequence $\{x_n\}$ in P converging to the point x.

Remark 2.2. The following are well known.

(1) P is a sequential neighborhood of x in X if and only if each sequence $\{x_n\}$ converging to x is frequently in P.

(2) The intersection of finitely many sequentially open subsets of X is a sequentially open subset of X.

Definition 2.3. Let $\mathscr{P} = \bigcup\{\mathscr{P}_x : x \in X\}$ be a cover of a space X, where each $x \in \bigcap \mathscr{P}_x$.

(1) $\mathscr{P}$ is called a network of X [2], if whenever $x \in U$ with U open in X there is $P \in \mathscr{P}_x$ such that $x \in P \subset U$, where $\mathscr{P}_x$ is called a network at x in X.

(2) $\mathscr{P}$ is called a *cs*-network of X [12], if for every convergent sequence S converging to a point $x \in U$ with U open in X, S is eventually in $P \subset U$ for some $P \in \mathscr{P}_x$, where $\mathscr{P}_x$ is called a *cs*-network at x in X.

(3) $\mathscr{P}$ is called a *k*-network of X [28], if for every compact subset $K \subset U$ with U open in X, there is a finite $\mathscr{F} \subset \mathscr{P}$ such that $K \subset \bigcup \mathscr{F} \subset U$.

Definition 2.4. Let $\mathscr{P} = \bigcup\{\mathscr{P}_x : x \in X\}$ be a cover of a space X. Assume that $\mathscr{P}$ satisfies the following (a) and (b) for each $x \in X$.

(a) $\mathscr{P}_x$ is a network at x in X.

(b) If $P_1, P_2 \in \mathscr{P}_x$, then there is $P \in \mathscr{P}_x$ such that $P \subset P_1 \cap P_2$.

(1) $\mathscr{P}$ is called an *so*-network of X [18] if every element of $\mathscr{P}_x$ is a sequentially open subset for each $x \in X$, where $\mathscr{P}_x$ is called an *so*-network at x in X.

(2) $\mathscr{P}$ is called an *sn*-network of X [18] if every element of $\mathscr{P}_x$ is a sequential neighborhood of x for each $x \in X$, where $\mathscr{P}_x$ is called an *sn*-network at x in X.

(3) $\mathscr{P}$ is called a weak base of X [3] if whenever $G \subset X$ and for each $x \in G$ there is $P \in \mathscr{P}_x$ such that $P \subset G$, then G is open in X, where $\mathscr{P}_x$ is called a weak neighborhood base at x in X.

Remark 2.5. The following implications hold.

$$\begin{array}{ccccc} \textit{base} & \longrightarrow & \textit{weak base} & & \\ \downarrow & & \downarrow & & \\ \text{so}-\textit{network} & \longrightarrow & \text{sn}-\textit{network} & \longrightarrow & \text{cs}-\textit{network} \end{array}$$

Definition 2.6. ([19]) Let X be a space. X is called sof-countable (resp. snf-countable, csf-countable, gf-countable) if for each $x \in X$, there is an *so*-network (resp. *sn*-network, *cs*-network, weak base) $\mathscr{P}_x$ at x in X such that $\mathscr{P}_x$ is countable.

Remark 2.7. The following implications hold.

$$\begin{array}{ccccc} \textit{first countable} & \longrightarrow & \text{gf}-\textit{countable} & & \\ \downarrow & & \downarrow & & \\ \text{sof}-\textit{countable} & \longrightarrow & \text{snf}-\textit{countable} & \longrightarrow & \text{csf}-\textit{countable} \end{array}$$

Definition 2.8. ([5]) Let $\mathscr{P}$ be a collection of subsets of a space X.

(1) $\mathscr{P}$ is called locally finite if whenever $x \in X$ there is an open neighborhood U of x such that $\{P \in \mathscr{P} : U \bigcap P \neq \emptyset\}$ is finite.

(2) $\mathscr{P}$ is called σ-locally finite if $\mathscr{P} = \bigcup\{\mathscr{P}_n : n \in \mathbb{N}\}$ and for each $n \in \mathbb{N}$, $\mathscr{P}_n$ is locally finite.

Definition 2.9. Let X be a space.

(1) X is called an *so*-metrizable space [8, 19] if X has a σ-locally finite *so*-network.

(2) X is called an *sn*-metrizable space [10, 19] if X has a σ-locally finite *sn*-network.

(3) X is called an $\aleph$-space [13] if X has a σ-locally finite k-network (equivalent, *cs*-network).

(4) X is called a g-metrizable space [30] if X has a σ-locally finite weak base.

Remark 2.10. ([10]) For a sequential space, the following hold.

(1) weak base $\Longleftrightarrow$ *sn*-network;

(2) base $\Longleftrightarrow$ *so*-network;

(3) gf-countable $\Longleftrightarrow$ snf-countable;

(4) first countable $\Longleftrightarrow$ sof-countable.

Here, "sequential spaces" can not be weakened to "k-spaces".

Remark 2.11. For a k-space, the following hold.

(1) g-metrizable $\Longleftrightarrow$ *sn*-metrizable [19];

(2) metrizable $\Longleftrightarrow$ *so*-metrizable.

Remark 2.12. ([10]) For a space, the following hold.

First countable $\Longleftrightarrow$ Fréchet, gf-countable $\Longleftrightarrow$ Fréchet, sof-countable $\Longleftrightarrow$ Fréchet, snf-countable.

Definition 2.13. (1) Let $T = \{1/n : n \in \mathbb{N}\} \cup \{0\}$ be a space with the usual topology, and $\alpha \geq \omega$ be an ordinal number. For each $\beta < \alpha$, let T_β be a copy of T. Then S_α denotes the quotient space obtained from the topological sum $\bigoplus_{\beta<\alpha} T_\beta$ by identifying all the nonisolated points into one point. In particular, S_ω is called a sequential fan [4].

(2) Let $L_0 = \{a_n : n \in \mathbb{N}\}$ be a sequence converging to $b \notin L_0$. For each $n \in \mathbb{N}$, let L_n be a sequence converging to b_n, where $b_n \notin L_n$. Put $T_0 = L_0 \cup \{b\}$ and $T_n = L_n \cup \{b_n\}$ for each $n \in \mathbb{N}$. Let M be the topological sum of $\{T_n : n \geq 0\}$. Then S_2 denotes the quotient space, which is called an Arens space [1], obtained from the topological sum M by identifying a_n with b_n for each $n \in \mathbb{N}$.

Remark 2.14. It is well known that S_2 and S_ω is not first countable. So each first countable space contains no copy of S_2 or S_ω (for example, see [23, Example 1.8.7] and [9, Proposition 3.2]).

Definition 2.15. ([19]) Let (X, τ) be a topological space. Put

$$\sigma_\tau = \{P \subset X : P \textit{ is sequentially open in } X\}.$$

Then σ_τ is a topology on X. The space (X, σ_τ) is called sequential coreflection of X, and denoted by σX.

Remark 2.16. ([19]) X and σX have the same convergent sequences.

Proposition 2.17. ([8, 19]) *The following are equivalent for a space X.*
(1) X is an so-metrizable space.
(2) X is an $\aleph$-space and contains no closed subspace having S_2 or S_ω as its sequential coreflection.
(3) X is an so$f$-countable, sn-metrizable space.

Definition 2.18. ([6, 29]) Let $f : X \longrightarrow Y$ be a mapping.
(1) f is called a closed (resp. an open) mapping if $f(B)$ is closed (resp. open) in Y for every closed (resp. open) subset B in X.
(2) f is called a compact mapping if $f^{-1}(y)$ is a compact subset of X for each $y \in Y$.
(3) f is called a perfect mapping if f is a closed compact mapping.
(4) f is called a sequence-covering mapping if whenever $\{y_n\}$ is a sequence converging to y in Y, there is a sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.

3. Main results

In this section, we give the main theorems of this paper. At first, we prove that perfect mappings preserve *so*-metrizable spaces.

Lemma 3.1. ([14]) *For a space X, if Y is a sequentially closed subspace of a space X and F is a sequentially closed subset of Y, then F is a sequentially closed subset of X.*

Proposition 3.2. *Let Y be a sequentially closed subset of X. Then $\sigma_\tau|Y = \sigma_{\tau|Y}$.*

Proof. Let $U \in \sigma_\tau|Y$. Then there is a sequentially open subset V of X such that $U = V \cap Y$. So U is sequentially open subset of Y with Y as a subspace of X. It follows that $U \in \sigma_{\tau|Y}$. Thus, $\sigma_\tau|Y \subset \sigma_{\tau|Y}$. Conversely, let $F \in (\sigma_{\tau|Y})^c$, i.e., a closed subset in $(Y, \sigma_{\tau|Y})$. Then F is a sequentially closed subset of Y. By Lemma 3.1, F is a sequentially closed subset of X. Thus, F is a closed subset of σX, i.e., $F \in (\sigma_\tau|Y)^c$. Thus, $(\sigma_{\tau|Y})^c \subset (\sigma_\tau|Y)^c$. It follows that $\sigma_{\tau|Y} \subset \sigma_\tau|Y$. So $\sigma_\tau|Y = \sigma_{\tau|Y}$. □

Corollary 3.3. *For a space X and an ordinal number α ($\alpha = 2$ or $\alpha \geq \omega$), X contains a sequentially closed subspace having S_α as its sequential coreflection if and only if σX contains a closed copy of S_α.*

Remark 3.4. In Proposition 3.2, "Y be a sequentially closed subspace of a space X" can be replaced "Y be a sequentially open subspace of a space X".

Recall that a mapping $f : X \longrightarrow Y$ is called sequentially continuous if whenever a sequence $\{x_n\}$ in X converging to $x \in X$, then $\{f(x_n)\}$ is a sequence in Y converging to $f(x) \in Y$.

Lemma 3.5. ([21]) *Let $f : X \longrightarrow Y$ be a mapping, where X is a sequential space. Then f is sequentially continuous if and only if f is continuous.*

We call a space X to have the point-G_δ-property if each point in X is a G_δ-set of X.

Lemma 3.6. ([8]) *Let $f : X \longrightarrow Y$ be a closed mapping and X have the point-G_δ-property. If B is a sequentially closed subset of X, then $f(B)$ is a sequentially closed subset of Y.*

Lemma 3.7. ([23]) *Each compact space with the point-G_δ-property is first countable.*

Proposition 3.8. *Let $f : X \longrightarrow Y$ be a mapping and X have the point-G_δ-property. Put $g = f|_{\sigma X} : \sigma X \longrightarrow \sigma Y$. Then the following hold.*

(1) g is a continuous mapping.

(2) If f is a closed mapping, then g is a closed mapping.

(3) f is a compact mapping if and only if g is a compact mapping.

(4) f is a sequence-covering mapping if and only if g is a sequence-covering mapping.

Proof. (1) Since σX is a sequential space, by Lemma 3.5, we only need to prove $g : \sigma X \longrightarrow \sigma Y$ is sequentially continuous. Note that X and σX (resp. Y and σY) have the same convergent sequences from Remark 2.16. Let $\{x_n\}$ be a convergent sequence in σX. Then $\{x_n\}$ is a convergent sequence in X. Since f is continuous, $\{f(x_n)\}$ is a convergent sequence in Y, hence $\{g(x_n)\}$ is a convergent sequence in σY. This proves that g is sequentially continuous.

(2) Let F is a closed subset of σX. Then F is a sequentially closed subset of X. Since X has the point-G_δ-property and f is a closed mapping, $f(F)$ is a sequentially closed subset of Y by Lemma 3.6. So $g(F) = f(F)$ is a closed subset of σY. This proves that g is a closed mapping.

(3) If part is clear. We only need to prove that only if part. Let $y \in \sigma Y$. Then $f^{-1}(y)$ is a compact subset of X. Since X has the point-G_δ-property, $f^{-1}(y)$ has the point-G_δ-property. So $f^{-1}(y)$ is first countable from Lemma 3.7. By Proposition 3.2, the topology on $f^{-1}(y)$ as a subspace of σX is equivalent to the topology on $f^{-1}(y)$ as a subspace of X. Consequently, $g^{-1}(y) = f^{-1}(y)$ is compact. This proves that g is a compact mapping.

(4) It holds from Remark 2.16. □

Remark 3.9. Lemma 3.8(2) can not be reversed. In fact, let $Y = \beta\mathbb{N}$, and X be the topological space obtained by the set $\{x : x \in Y\}$ endowed discrete topology, and put $f : X \longrightarrow Y$ is the natural mapping. Then X is a metric space. It is clear that f is not a closed mapping. Note that $\sigma\beta\mathbb{N}$ is a discrete space. So $g = f|_{\sigma X} : \sigma X \longrightarrow \sigma Y$ is a closed mapping (also see [20, Example 3.4.7(5)]).

Remark 3.10. It is clear that "X has the point-G_δ-property" for Lemma 3.8(1) and (4) can be omitted from the proof of Lemma 3.8. However, "X has the point-G_δ-property" for Lemma 3.8(2) and (3) can not be omitted. In fact, it can be showed by the following two simple examples.

(1) Let $f : \beta\mathbb{N} \longrightarrow \{x\}$ be the constant mapping. Then f is a compact mapping. Note that $\sigma\beta\mathbb{N}$ is a discrete space. So $g = f|_{\sigma\beta\mathbb{N}} : \sigma\beta\mathbb{N} \longrightarrow \{x\}$ is not a compact mapping.

(2) Let $X = \beta\mathbb{N}$, and $Y = T$ described in Definition 2.13(1). Put $f : X \longrightarrow Y$ as follows: $f(\beta\mathbb{N} - \mathbb{N}) = \{0\}$ and $f(n) = 1/n$ for each $n \in \mathbb{N}$. Then $f : X \longrightarrow Y$ is a perfect mapping, and $g = f|_{\sigma X} : \sigma X \longrightarrow \sigma Y$ is not a closed mapping (also see [20, Example 3.4.7(2)]).

Lemma 3.11. ([27]) *Let $f : X \longrightarrow Y$ be a perfect mapping with X sequential. Then X contains a closed copy of S_2 or S_ω if and only if so does Y.*

Now we give the first main theorem.

Theorem 3.12. *Let $f : X \longrightarrow Y$ be a perfect mapping. If X is an so-metrizable space, then Y is an so-metrizable space.*

Proof. Let X be an *so*-metrizable space. By Proposition 2.17(2), X is an $\aleph$-space and contains no closed subspace having S_2 or S_ω as its sequential coreflection. By Proposition 1.2(1), Y is an $\aleph$-space. If Y contains closed subspace B having S_2 or S_ω as its sequential coreflection, then σB is homeomorphic to S_2 or S_ω. Note that *so*-metrizable spaces are hereditary to subspaces. So $f^{-1}(B)$ is *so*-metrizable, hence $f^{-1}(B)$ is *sof*-countable. Thus $\sigma f^{-1}(B)$ is first countable because $\sigma f^{-1}(B)$ is sequential. On the other hand, by Proposition 3.8, $g = f|_{f^{-1}(B)} : \sigma f^{-1}(B) \longrightarrow \sigma B$ is a perfect mapping. Since σB is homeomorphic to S_2 or S_ω, σB is a paracompact space, then $\sigma f^{-1}(B)$ is a paracompact space by the perfectness of g, thus $\sigma f^{-1}(B)$ is a regular space. By Lemma 3.11, $\sigma f^{-1}(B)$ contains a closed copy of S_2 or S_ω. By Remark 2.14, this contradicts that $\sigma f^{-1}(B)$ is first countable. So Y contains no closed subspace having S_2 or S_ω as its sequential coreflection. By Proposition 2.17(2), Y is an *so*-metrizable space. □

Secondly, we prove that closed sequence-covering mappings preserve *so*-metrizable spaces.

Lemma 3.13. ([22]) *sn-metrizable spaces are preserved by closed sequence-covering mappings.*

Lemma 3.14. ([19]) *The following are equivalent for a space X.*
(1) σX is a first countable space.
(2) X is an sof-countable space.

Now we give the second main theorem.

Theorem 3.15. *Let $f : X \longrightarrow Y$ be a closed sequence-covering mapping. If X is an so-metrizable space, then Y is an so-metrizable space.*

Proof. Let X be an *so*-metrizable space. Then X is an sof-countable, *sn*-metrizable space from Proposition 2.17(3), and Y is an *sn*-metrizable space from Lemma 3.13. In Particular, Y is snf-countable, and hence σY is snf-countable from Remark 2.16. By Proposition 2.17(3), it suffices to prove that Y is sof-countable. Put $g = f|_{\sigma X} : \sigma X \longrightarrow \sigma Y$. By Proposition 3.8(2), g is a closed mapping. Since X is an sof-countable space, σX is a first-countable space from Lemma 3.14, and so σX is a Fréchet space. Note that closed mappings preserve Fréchet spaces. So σY is a Fréchet spaces. By Remark 2.12, σY is first countable. Thus, Y is sof-countable from Lemma 3.14. □

In the end, we give a result for closed images of *so*-metrizable spaces in topological groups. Recall that a family $\mathscr{P}$ of subsets of a space X is called closure-preserving if $\overline{\bigcup \mathscr{P}'} = \bigcup\{\overline{P} : P \in \mathscr{P}'\}$ for each $\mathscr{P}' \subset \mathscr{P}$; is called hereditarily closure-preserving if any family $\{H(P) : P \in \mathscr{P}\}$ is closure-preserving, where each $H(P) \subset P \in \mathscr{P}$. $\mathscr{P} = \bigcup\{\mathscr{P}_n : n \in \mathbb{N}\}$ is called σ-hereditarily closure-preserving if $\mathscr{P}_n$ is hereditarily closure-preserving for each $n \in \mathbb{N}$.

Lemma 3.16. ([15]) *A space X is an $\aleph$-space if and only if X has a σ-hereditarily closure-preserving k-network and contains no (closed) copy of S_{ω_1}.*

Lemma 3.17. ([27]) *Let $f : X \longrightarrow Y$ be a closed mapping with X sequential. If Y contains a closed copy of S_2, then so does X.*

Lemma 3.18. ([26]) *A topological group contains a (closed) copy of S_ω if and only if it contains a (closed) copy of S_2.*

Theorem 3.19. *Let $f : X \longrightarrow Y$ be a closed mapping, where σY is a topological group. If X is an so-metrizable space, then Y is an so-metrizable space.*

Proof. Let X be an *so*-metrizable space. By Proposition 2.17, we only need to prove the following two claims.
Claim 1. Y contains no closed subspace having S_2 or S_ω as its sequential coreflection.
By Corollary 3.3, it suffices to prove that σY contains no closed copy of S_2 or S_ω. Put $g = f|_{\sigma X} : \sigma X \longrightarrow \sigma Y$. Then σX is sequential, and g is a closed mapping from Proposition 3.8(2). Since X is sof-countable, σX is first countable from Lemma 3.14. If σY contains a closed copy of S_2, then σX contains a closed copy of S_2 from Lemma 3.17. This contradicts that σX is first countable. So σY contains no closed copy of S_2. If σY contains a closed copy of S_ω, then σY contains a closed copy of S_2 from Lemma 3.18. This is a contradiction. Consequently, σY contains no closed copy of S_2 or S_ω.
Claim 2. Y is an $\aleph$-space.
By Proposition 2.17, X has a σ-hereditarily closure-preserving k-network. Since closed mappings preserve σ-hereditarily closure-preserving k-networks, Y has a σ-hereditarily closure-preserving k-network. By the proof of the above Claim 1, σY contains no closed copy of S_ω, and hence σY contains no closed copy of S_{ω_1}. Note that the topology on σY is finer than the topology on Y and any convergent sequence in Y is still a convergent sequence in σY. Therefore Y contains no closed copy of S_{ω_1}. By Lemma 3.16, Y is an $\aleph$-space. □

Note that a space is equivalent to its sequential coreflection if this space is sequential. By the above theorem and Remark 2.11(2), we have the following corollary.

Corollary 3.20. *Let $f : X \longrightarrow Y$ be a closed mapping with Y a sequential topological group. If X is an so-metrizable space, then Y is a metrizable space.*

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