Filomat 27:1 (2013), 89–94
DOI 10.2298/FIL1301089D

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

# On $I$-convergence of nets in locally solid Riesz spaces

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**Abstract.** In this paper, following the line of [13] and [6], we introduce the ideas of $I_\tau$-convergence, $I_\tau$-boundedness and $I_\tau$-Cauchy condition of nets in a locally solid Riesz space endowed with a topology $\tau$ and investigate some of its consequences.

## 1. Introduction

The notion of a Riesz space was first introduced by F. Riesz [17] in 1928 and since then it has found several applications in measure theory, operator theory, optimization and also in economics (see [2]). It is well known that a topology on a vector space that makes the operations of addition and scalar multiplication continuous is called a linear topology and a vector space endowed with a linear topology is called a topological vector space. A Riesz space is an ordered vector space which is also a lattice, endowed with a linear topology. Further if it has a base consisting of solid sets at zero, then it is known as a locally solid Riesz space.

The notion of statistical convergence, which is an extension of the idea of usual convergence, was introduced by Fast [7] and Schoenberg [19] and its topological consequences were studied first by Fridy [8] and Šalát [18] (also later by Maddox [15]). Recently Di Maio and Kočinac [16] introduced the concept of statistical convergence in topological spaces and statistical Cauchy condition in uniform spaces and established the topological nature of this convergence (see also [3, 4]). Subsequently, in a very recent development, the idea of statistical convergence of sequences was studied by Albayrak and Pehlivan [1] in locally solid Riesz spaces.

However if one considers the concept of nets instead of sequences (which undoubtedly plays a more important and natural role in general structures like topological spaces, uniform spaces and Riesz spaces) the above approach does not seem to be appropriate because of the absence of any idea of density in arbitrary directed sets. Instead it seems more appropriate to follow the more general approach of [9] where the notion of $I$-convergence of a sequence was introduced by using ideals of the set of positive integers. One can see [5, 6, 10, 12, 13] for more works in this direction where many more references can be found.

In an interesting development, the notion of usual convergence of nets was extended to ideal convergence of nets in [13] where the basic topological nature of this convergence was established (also continued in

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2010 *Mathematics Subject Classification*. Primary 40G15; Secondary 40A35, 46A40
*Keywords*. Ideal, filter, nets, $I_\tau$-convergence, $I_\tau$-boundedness, $I_\tau$-Cauchy condition
Received: 01 March 2012; Revised: 18 April 2012; Accepted: 23 April 2012
Communicated by Ljubiša D.R. Kočinac
The work was done when the first author visited Istanbul Commerce University in 2011. The first author is also thankful to CSIR for granting the project No. 25(0186)/10/EMR-II during the tenure of which this work was done.
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[6]). As a natural consequence, in this paper, we introduce the idea of ideal-$\tau$-convergence of nets in a locally solid Riesz space and study some of its properties by using the mathematical tools of the theory of topological vector spaces. It should be noted that our paper contains all results of [1] as special cases.

## 2. Preliminaries

In this section we recall some of the basic concepts of Riesz spaces and ideal convergence of nets and interested readers can look into [2, 13, 20] for details.

**Definition 2.1.** Let $L$ be a real vector space and let $\leq$ be a partial order on this space. $L$ is said to be an *ordered vector space* if it satisfies the following properties:

(i) If $x, y \in L$ and $y \leq x$, then $y + z \leq x + z$ for each $z \in L$.
(ii) If $x, y \in L$ and $y \leq x$, then $\lambda y \leq \lambda x$ for each $\lambda \geq 0$.
If in addition $L$ is a lattice with respect to the partial ordering, then $L$ is said to be a *Riesz space* (or a *vector lattice*).

For an element $x$ of a Riesz space $L$ the positive part of $x$ is defined by $x^+ = x \bigvee \theta$, the negative part of $x$ by $x^- = (-x) \bigvee \theta$, and the absolute value of $x$ by $|x| = x \bigvee (-x)$, where $\theta$ is the element zero of $L$.

A subset $S$ of a Riesz space $L$ is said to be *solid* if $y \in S$ and $|x| \leq \left|y\right|$ imply $x \in S$.

A topology $\tau$ on a real vector space $L$ that makes the addition and scalar multiplication continuous is said to be a linear topology, that is when the mappings

$$(x, y) \to x + y \quad (\text{from } (L \times L, \tau \times \tau) \to (L, \tau))$$
$$(\lambda, x) \to \lambda x \quad (\text{from } (\mathbb{R} \times L, \sigma \times \tau) \to (L, \tau))$$

are continuous, where $\sigma$ is the usual topology on $\mathbb{R}$. In this case the pair $(L, \tau)$ is called a topological vector space.

Every linear topology $\tau$ on a vector space $L$ has a base $\mathcal{N}$ for the neighborhoods of $\theta$ satisfying the following properties:
a) Each $V \in \mathcal{N}$ is a balanced set, that is $\lambda x \in V$ holds for all $x \in V$ and every $\lambda \in R$ with $|\lambda| \leq 1$.
b)Each $V \in \mathcal{N}$ is an absorbing set, that is for every $x \in L$, there exists a $\lambda > 0$ such that $\lambda x \in V$.
c) For each $V \in \mathcal{N}$ there exists some $W \in \mathcal{N}$ with $W + W \subset V$.

**Definition 2.2.** ([2]) A linear topology $\tau$ on a Riesz space $L$ is said to be *locally solid* if $\tau$ has a base at zero consisting of solid sets. A *locally solid Riesz space* $(L, \tau)$ is a Riesz space $L$ equipped with a locally solid topology $\tau$.

$\mathcal{N}_{sol}$ will stand for a base at zero consisting of solid sets and satisfying the properties (a),(b) and (c) in a locally solid topology.

We now recall the following basic facts from [12] (see also [5, 6]).

A family $I$ of subsets of a non-empty set $X$ is said to be an ideal if (i) $A, B \in I$ implies $A \cup B \in I$, (ii) $A \in I, B \subset A$ imply $B \in I$. $I$ is called non-trivial if $I \neq \left\{\phi\right\}$ and $X \notin I$. $I$ is admissible if it contains all singletons. If $I$ is a proper non-trivial ideal, then the family of sets $F(I) = \{M \subset X : M^c \in I\}$ is a filter on $X$ (where $c$ stands for the complement.) It is called the filter associated with the ideal $I$.

Throughout the paper $(D, \geq)$ will stand for a directed set and $I$ a non-trivial proper ideal of $D$. A net is a mapping from $D$ to $X$ and will be denoted by $\{s_\alpha : \alpha \in D\}$. Let for $\alpha \in D, D_\alpha = \{\beta \in D : \beta \geq \alpha\}$. Then the collection $F_0 = \{A \subset D : A \supset D_\alpha$ for some $\alpha \in D\}$ forms a filter in $D$. Let $I_0 = \{A \subset D : A^c \in F_0\}$. Then $I_0$ is a non-trivial ideal of $D$.

A nontrivial ideal $I$ of $D$ will be called $D$-admissible if $D_\alpha \in F(I)$ $\forall \alpha \in D$.

**Definition 2.3.** A net $\{s_\alpha : \alpha \in D\}$ in a topological space $(X, \tau)$ is said to be *I-convergent* to $x_0 \in X$ if for any open set $U$ containing $x_0$, $\{\alpha \in D : s_\alpha \notin U\} \in I$.

## 3. Ideal topological convergence in locally solid Riesz spaces

We first introduce our main definition.

**Definition 3.1.** Let $(L,\tau)$ be a locally solid Riesz space and $\{s_\alpha : \alpha \in D\}$ be a net in $L$. $\{s_\alpha : \alpha \in D\}$ is said to be *ideal-$\tau$-convergent* (*$I_\tau$-convergent* in short) to $x_0 \in L$ if for any $\tau$-neighborhood $U$ of zero, $\{\alpha \in D : s_\alpha - x_0 \notin U\} \in I$. In this case we write $I_\tau\text{-}\lim s_\alpha = x_0 \left( \text{ or } s_\alpha \to^{I_\tau} x_0 \right)$.

When $I = I_d$ (the ideal of those subsets of $\mathbb{N}$ which have asymptotic density zero, see [7, 8, 16] for detailed definitions) and $D = \mathbb{N}$, the notion of ideal-$\tau$-convergence reduces to statistical-$\tau$-convergence of sequences [1].

**Example 3.2.** In the locally solid Riesz space $(\mathbb{R}^2, \|\cdot\|)$ with the Euclidean norm $\|\cdot\|$ and coordinate ordering choose the neighborhood system $\mathcal{N}_{x_0}$ of any point $x_0 \in \mathbb{R}^2$. It is known that $\mathcal{N}_{x_0}$ is itself a directed set $D$ with respect to inclusion. Take a proper non-trivial ideal $I$ of $D$ which contains $I_0$ properly. Choose $C \in I \setminus I_0$. Let $\{s_U : U \in D\}$ be given by

$$\begin{aligned} s_U &\in U \ \forall\, U \in \mathcal{N}_{x_0} \setminus C \\ s_U &= y_0 \ \forall\, U \in C \end{aligned}$$

where $x_0 \neq y_0$. Then it is easy to observe that $\{s_U : U \in D\}$ cannot converge to $x_0$ usually but it $I_\tau$-converges to $x_0$ as

$$\{U \in D : s_U - x_0 \notin U\} = C \in I$$

for any $\tau$-neighborhood $U$ of zero, which does not contain $y_0 - x_0$ (such neighborhoods exist because of Hausdorffness of $\mathbb{R}^2$).

Note that the above example can be formulated in any Hausdorff locally solid Riesz space $(L,\tau)$ with a point $x_0$ for which $\mathcal{N}_{x_0}$ contains infinitely many members.

**Definition 3.3.** A net $\{s_\alpha : \alpha \in D\}$ is said to be *ideal-$\tau$-bounded* (*$I_\tau$-bounded*) if for any $\tau$-neighborhood $U$ of zero there exists some $\lambda > 0$ such that $\{\alpha \in D : \lambda s_\alpha \notin U\} \in I$.

**Definition 3.4.** A net $\{s_\alpha : \alpha \in D\}$ in a locally solid Riesz space $(L,\tau)$ is said to be *ideal-$\tau$-Cauchy* (*$I_\tau$-Cauchy*) if for every $\tau$-neighborhood $U$ of zero there exists a $\beta \in D$ such that $\left\{\alpha \in D : s_\alpha - s_\beta \notin U\right\} \in I$.

**Theorem 3.5.** *A locally solid Riesz space is Hausdorff if and only if every $I_\tau$-convergent net has a unique limit point for every $D$ admissible ideal $I$.*

The proof readily follows from Theorems 1 and 2 of [13].

As in [13] we can also find the equivalent characterizations of limit points of sets and continuous mappings with respect to $I_\tau$-convergence of nets.

**Theorem 3.6.** *Let $(L,\tau)$ be a locally solid Riesz space and $\{s_\alpha : \alpha \in D\}, \{t_\alpha : \alpha \in D\}$ be two nets in $L$. Then*
*(i)* $I_\tau\text{-}\lim s_\alpha = x_0 \Rightarrow I_\tau\text{-}\lim a s_\alpha = a x_0$ *for each* $a \in R$.
*(ii)* $I_\tau\text{-}\lim s_\alpha = x_0, I_\tau\text{-}\lim t_\alpha = y_0 \Rightarrow I_\tau\text{-}\lim(s_\alpha + t_\alpha) = x_0 + y_0$.

(i) Let $U$ be a $\tau$-neighborhood of zero. Choose $V \in \mathcal{N}_{sol}$ such that $V \subset U$. Since $I\text{-}\lim s_\alpha = x_0$,

$$\{\alpha \in D : s_\alpha - x_0 \in V\} \in F(I).$$

Let $|a| \leq 1$. Since V is balanced, $s_\alpha - x_0 \in V$ implies that $a\,(s_\alpha - x_0) \in V$.
Hence we have

$$\{\alpha \in D : s_\alpha - x_0 \in V\} \subset \{\alpha \in D : a s_\alpha - a x_0 \in V\} \subset \{\alpha \in D : a s_\alpha - a x_0 \in U\}$$

and so

$$\{\alpha \in D : as_\alpha - ax_0 \in U\} \in F(I).$$

Now let $|a| > 1$ and as usual let $[|a|]$ be the smallest integer greater than or equal to $|a|$. There exists a $W \in \mathcal{N}_{sol}$ such that $[|a|]\, W \subset V$. Since $I_\tau\text{-}\lim s_\alpha = x_0$,

$$A = \{\alpha \in D : s_\alpha - x_0 \in W\} \in F(I).$$

Then we have

$$|ax_0 - as_\alpha| = |a|\,|x_0 - s_\alpha| \leq [|a|]\,|x_0 - s_\alpha| \in [|a|]\, W \subset V \subset U$$

for each $\alpha \in A$. Since the set $V$ is solid, we have $as_\alpha - ax_0 \in V$ and so $as_\alpha - ax_0 \in U$ for each $\alpha \in A$. So we get

$$\{\alpha \in D : as_\alpha - ax_0 \in U\} \supset A$$

and so it belongs to $F(I)$. Hence $I_\tau\text{-}\lim as_\alpha = ax_0$ for every $a \in \mathbb{R}$.

(ii) Let $U$ be an arbitrary $\tau$-neighborhood of zero. Choose $V \in \mathcal{N}_{sol}$ such that $V \subset U$. Choose $W \in \mathcal{N}_{sol}$ such that $W + W \subset V$. Since $I_\tau\text{-}\lim s_\alpha = x_0$ and $I_\tau\text{-}\lim t_\alpha = y_0$ so

$$A = \{\alpha \in D : s_\alpha - x_0 \in W\} \in F(I)$$

and

$$B = \{\alpha \in D : t_\alpha - y_0 \in W\} \in F(I).$$

Then $A \cap B \in F(I)$ and clearly

$$(s_\alpha + t_\alpha) - (x_0 + y_0) = (s_\alpha - x_0) + (t_\alpha - y_0) \in W + W \subset V \subset U$$

for each $\alpha \in A \cap B$. Hence we have

$$A \cap B \subset \{\alpha \in D : (s_\alpha + t_\alpha) - (x_0 + y_0) \in U\}$$

and so the set on the right hand side belongs to $F(I)$. Hence $I_\tau\text{-}\lim (s_\alpha + t_\alpha) = x_0 + y_0$.

**Theorem 3.7.** *Let $(L, \tau)$ be a locally solid Riesz space. Let $\{s_\alpha : \alpha \in D\}, \{t_\alpha : \alpha \in D\}, \{v_\alpha : \alpha \in D\}$ be three nets such that $s_\alpha \leq t_\alpha \leq v_\alpha$ for each $\alpha \in D$. If $I_\tau\text{-}\lim s_\alpha = I_\tau\text{-}\lim v_\alpha = x_0$, then $I_\tau\text{-}\lim t_\alpha = x_0$.*

*Proof.* Let $U$ be an arbitrary $\tau$-neighborhood of zero. Choose $V, W \in \mathcal{N}_{sol}$ such that $W + W \subset V \subset U$. Now by our assumption

$$A = \{\alpha \in D : s_\alpha - x_0 \in W\} \in F(I)$$

and

$$B = \{\alpha \in D : v_\alpha - x_0 \in W\} \in F(I).$$

Then $A \cap B \in F(I)$ and for each $\alpha \in A \cap B$

$$s_\alpha - x_0 \leq t_\alpha - x_0 \leq v_\alpha - x_0$$

and so $|t_\alpha - x_0| \leq |s_\alpha - x_0| + |v_\alpha - x_0| \in W + W \subset V$. Since $V$ is solid so

$$t_\alpha - x_0 \in V \subset U.$$

Hence $A \cap B \subset \{\alpha \in D : t_\alpha - x_0 \in U\}$ which implies that $\{\alpha \in D : t_\alpha - x_0 \in U\} \in F(I)$ and this completes the proof of the theorem. □

**Theorem 3.8.** *If a net $\{s_\alpha : \alpha \in D\}$ in a locally solid Riesz space $(L, \tau)$ is $I_\tau$-convergent, then it is $I_\tau$-bounded.*

*Proof.* Let $I_\tau\text{-}\lim s_\alpha = x_0$. Let $U$ be an arbitrary $\tau$-neighborhood of zero. Choose $V, W \in \mathcal{N}_{sol}$ such that $W + W \subset V \subset U$. Now we have

$$C = \{\alpha \in D : s_\alpha - x_0 \notin W\} \in I.$$

Since $W$ is absorbing, there exists a $\lambda > 0$ such that $\lambda x_0 \in W$. We can take $\lambda \leq 1$ since $W$ is solid. Since $W$ is balanced, $s_\alpha - x_0 \in W$ implies that $\lambda (s_\alpha - x_0) \in W$. Then we have

$$\lambda s_\alpha = \lambda (s_\alpha - x_0) + \lambda x_0 \in W + W \subset V \subset U$$

for every $\alpha \in D \setminus C$. Hence

$$\{\alpha \in D : \lambda s_\alpha \notin U\} \in I$$

which shows that $\{s_\alpha : \alpha \in D\}$ is $I_\tau$-bounded. □

**Theorem 3.9.** *If a net $\{s_\alpha : \alpha \in D\}$ in a locally solid Riesz space is $I_\tau$-convergent, then it is $I_\tau$-Cauchy.*

*Proof.* Let $I_\tau\text{-}\lim s_\alpha = x_0$ and let $U$ be an arbitrary $\tau-$neighborhood of zero. Choose $V, W \in \mathcal{N}_{sol}$ such that $W + W \subset V \subset U$. Since $I_\tau\text{-}\lim s_\alpha = x_0$, we have

$$C = \{\alpha \in D : s_\alpha - x_0 \notin W\} \in I.$$

Then for any $\alpha, \beta \in D \setminus C$,

$$s_\alpha - s_\beta = s_\alpha - x_0 + x_0 - s_\beta \in W + W \subset V \subset U.$$

Hence it follows that

$$\left\{\alpha \in D : s_\alpha - s_\beta \notin U\right\} \subset C$$

where $\beta \in D \setminus C$ is fixed. This shows the existence of $\beta \in D$ for which

$$\left\{\alpha \in D : s_\alpha - s_\beta \notin U\right\} \in I.$$

As this holds for each $\tau$-neighborhood $U$ of zero, $\{s_\alpha : \alpha \in D\}$ is $I_\tau$-Cauchy. □

**Theorem 3.10.** *For a net $\{s_\alpha : \alpha \in D\}$ in a locally solid Riesz space $L$, the following are equivalent:*
*(1) $\{s_\alpha : \alpha \in D\}$ is an $I_\tau$-Cauchy net.*
*(2) For every $\tau$-neighborhood $U$ of zero, there exists $A \in I$ such that $\beta, \alpha \notin A$ implies that $s_\beta - s_\alpha \in U$.*
*(3) For every $\tau$-neighborhood $U$ of zero, $\{\beta \in D : E_\beta(U) \notin I\} \in I$ where $E_\beta(U) = \{\alpha \in D : s_\alpha - s_\beta \notin U\}$.*

*Proof.* (1) $\Longrightarrow$ (2) Let $\{s_\alpha : \alpha \in D\}$ be an $I_\tau$-Cauchy net and let $U$ be any $\tau$-neighborhood of zero. Choose $V, W \in \mathcal{N}_{sol}$ such that $W + W \subset V \subset U$. There exists a $\beta \in D$ such that $\{\alpha \in D : s_\alpha - s_\beta \notin W\} \in I$. Then $\{\alpha \in D : s_\alpha - s_\beta \in W\} \in F(I)$. Write $A = \{\alpha \in D : s_\alpha - s_\beta \notin W\}$. Clearly $A \in I$ and $\gamma, \alpha \notin A$ implies that $s_\gamma - s_\beta \in W$ and $s_\alpha - s_\beta \in W$ and hence $s_\gamma - s_\alpha \in W + W \subset V \subset U$.

(2)$\Rightarrow$ (3) Let $U$ be any $\tau$-neighborhood of zero. By (2) there exists an $A \in I$ such that $\nu, \alpha \notin A$ implies $s_\nu - s_\alpha \in U$. We shall show that $\left\{\beta \in D : E_\beta(U) \notin I\right\} \subset A$. Let $\beta \in D$ be such that $E_\beta(U) \notin I$. If possible let $\beta \notin A$. Since $A \in I$ but $E_\beta(U) \notin I$, so $E_\beta(U)$ is not a subset of A. Take $\alpha \in E_\beta(U) \setminus A$. Then $s_\alpha - s_\beta \notin U$ by the definition of $E_\beta(U)$. But $\alpha, \beta \notin A$ implies $s_\alpha - s_\beta \in U$, a contradiction. This proves (3).

(3)$\Rightarrow$ (1) Let $U$ be any $\tau$-neighborhood of zero. By (3) $\left\{\beta \in D : E_\beta(U) \notin I\right\} \in I$. Then $\left\{\beta \in D : E_\beta(U) \in I\right\} \in F(I)$. Since $\phi \notin F(I)$, so $\left\{\beta \in D : E_\beta(U) \in I\right\} \neq \phi$. Choose $\beta_0 \in \left\{\beta \in D : E_\beta(U) \in I\right\}$. Then $\beta_0 \in D$ is such that $E_{\beta_0}(U) = \left\{\alpha \in D : s_\alpha - s_{\beta_0} \notin U\right\} \in I$. This proves (1). □

**Definition 3.11.** A point $y \in L$ is called an *I-cluster point* of a net $\{s_\alpha : \alpha \in D\}$ if for any $\tau$-neighborhood $U$ of zero, $\{\alpha \in D : s_\alpha - y \in U\} \notin I$.

**Theorem 3.12.** *If an $I_\tau$-Cauchy net $\{s_\alpha : \alpha \in D\}$ in a locally solid Riesz space $(L, \tau)$ has an I-cluster point $x_0$, then $\{s_\alpha : \alpha \in D\}$ is $I_\tau$-convergent to $x_0$.*

*Proof.* Let $U$ be any $\tau$-neighborhood of zero. Choose $V, W \in \mathcal{N}_{sol}$ such that $W + W \subset V \subset U$. Let $B = \{\alpha \in D : s_\alpha - x_0 \in W\}$. Since $x_0$ is an $I$-cluster point of $\{s_\alpha : \alpha \in D\}$ so $B \notin I$. Again from the $I_\tau$-Cauchy condition of $\{s_\alpha : \alpha \in D\}$ we can find $A \in I$ such that $\nu, \alpha \notin A$ implies $s_\nu - s_\alpha \in W$ (by Theorem 3.6). Clearly $B \cap A^c \neq \phi$ for otherwise $B \subset A$ and so $B \in I$. Choose $\beta \in B \cap A^c$. Then $s_\beta - x_0 \in W$. Now $\alpha \in A^c$ implies $s_\alpha - s_\beta \in W$ and so

$$s_\alpha - x_0 = s_\alpha - s_\beta + s_\beta - x_0 \in W + W \subset V \subset U.$$

This shows that $A^c \subset \{\alpha \in D : s_\alpha - x_0 \in U\}$. Since $A^c \in F(I)$, $\{\alpha \in D : s_\alpha - x_0 \in U\} \in F(I)$ which implies that $\{s_\alpha : \alpha \in D\}$ is $I_\tau$-convergent to $x_0$. □

## Acknowledgement

We are thankful to the referees for their several valuable suggestions which improved the presentation of the paper.

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