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On the Existence of Global Solutions for a Nonlinear Klein-Gordon Equation

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Abstract. The aim of this work is to study the global existence of solutions for the Cauchy problem of a Klein-Gordon equation with high energy initial data. The proof relies on constructing a new functional, which includes both the initial displacement and the initial velocity: with sign preserving property of the new functional we show the existence of global weak solutions.

1. Introduction

The nonlinear Klein-Gordon equation with quadratic nonlinearity is

$$u_{tt} - u_{xx} + \alpha u - \beta u^2 = 0, \tag{1}$$

where α, and $\beta \neq 0$. Eq. (1) arises in many scientific applications such as solid state physics, nonlinear optics and quantum field theory. The Klein-Gordon equation is the first relativistic equation in quantum mechanics for the wave function of a particle with zero spin. It was proposed as a relativistic generalization of the Schrödinger equation and was investigated in many papers [1, 2, 4–6, 9, 12, 15, 23, 26].

The goal of the present paper is to investigate the existence of global solutions for the Cauchy problem of the Klein-Gordon equation with dissipation

$$u_{tt} - \Delta u + u + u_t = |u|^{p-1} u, \quad x \in R^n, \quad t > 0, \tag{2}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in R^n, \tag{3}$$

where u_0 and u_1 are the initial value functions, $n \geq 2$ and $1 < p < \frac{n+2}{n-2}$ if $n \geq 3$, $1 < p < \infty$ if $n = 2$. Evolution equations with dissipation are studied from various aspects in many papers [3, 13, 16, 17, 19].

In the present paper, we investigate the existence of global solutions by using the potential well method [18]. Sattinger [18] investigated global existence of the initial-boundary value problem of the following nonlinear hyperbolic equation

$$u_{tt} - \nabla^2 u + f(x,u) = 0$$

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in the case of initial energy less than the potential well depth d. Then this result extended to the total energy of the initial data is less than or equal to d [24]. Very recently in a paper of Kutev et al. [8] it was proved that there exist global solutions when the total energy of initial data is greater than d and they established the existence of global weak solutions by constructing a functional which include both the initial displacement and the initial velocity. Because they showed numerically that the initial velocity plays a crucial role in the behaviour of the problem. Problem (2), (3) was already treated in the $E(0) \leq d$ case by Runzhang [25], but the functional $I(u)$ used in their paper fails to prove the $E(0) > d$ case. Although a strongly damped nonlinear Klein-Gordon equation is studied in [26] and a blow up result was given for the high energy intial data, i.e. $E(0) > d$, the global existence was studied for $E(0) \leq d$. In the present paper, we reinvestigate the problem for the case $E(0) \leq d$, where we use a standard functional that include only the initial displacement u_0. Then, we prove that the existence of global solutions for $E(0) > d$ can not be proved via sign invariance of this functional. A new functional which includes both the initial displacement u_0 and initial velocity u_1 will be constructed for the case of high energy initial data. Functionals depending on u_0 and u_1 are introduced for the first time in [8] and then they were successfully applied for proving the global existence to some Boussinesq-type equations in [20–22].

Throughout this paper $H^s = H^s(R^n)$ will denote the L^2 Sobolev space on R^n with norm $\|f\|_{H^s} = \left\|(I-\Delta)^{\frac{s}{2}} f\right\| = \left\|\left(1+k^2\right)^{\frac{s}{2}} \widehat{f}\right\|$, where s is a real number, I is unitary operator. The notation $\|f\|_p$, $\|f\|$ and $\|f\|_\infty$ will be used instead of norms of $L^p(R^n)$, $L^2(R^n)$ and $L^\infty(R^n)$, respectively.

2. Global Existence for $E(0) \leq d$

The present section refers to two points. Firstly, we define a functional which includes only the initial displacement, and prove the existence of global solutions for $E(0) \leq d$ by aid of the sign invariance of this functional. We then show that this functional fails to prove the global existence in the case of $E(0) > d$.

Now, let us define

$$E(t) = E(u(t), u_t(t)) = \frac{1}{2}\left[\|u_t\|^2 + \|\nabla u\|^2 + \|u\|^2\right] - \frac{1}{p+1}\|u\|_{p+1}^{p+1}, \tag{4}$$

$$E(t) + \int_0^t \|u_\tau\|^2 d\tau = E(0)$$

$$J(u) = \frac{1}{2}\left(\|\nabla u\|^2 + \|u\|^2\right) - \frac{1}{p+1}\|u\|_{p+1}^{p+1}, \tag{5}$$

$$I(u) = \left(\|\nabla u\|^2 + \|u\|^2\right) - \|u\|_{p+1}^{p+1}, \tag{6}$$

$$d = \inf_{u \in N} J(u), \tag{7}$$

where $N = \left\{u \in H^1 \mid I(u) = 0, \|u\|_{H^1} \neq 0\right\}$, $E(u(t), u_t(t))$ is the total energy, $J(u)$ is the potential energy and d is the depth of potential well which can exactly be written in terms of the Sobolev constant as

$$d = \frac{p-1}{2(p+1)}\left(S_p^{p+1}\right)^{-2/(p-1)}. \tag{8}$$

Here S_p is the imbedding constant from $H^1(R^n)$ into $L^{p+1}(R^n)$ given by

$$S_p = \sup_{u \in H^1} \frac{\|u\|_{p+1}}{\|u\|_{H^1}}.$$

When $0 < E(0) < d$, by the sign invariance of (6) one can prove the existence of global solutions of (2), (3). Existence of global solutions was proved by such functionals for problem (2), (3) in [25]. It was proved in

[25] that if $I(u) > 0$, then every weak solutions of the problem exist globally, and if $I(u) < 0$, then every weak solutions of the problem blow up in finite time.

For $\sigma > -\frac{p-1}{2}$, define

$$\begin{aligned} I_\sigma(u) &= (1-\sigma)\left(\|\nabla u\|^2 + \|u\|^2\right) - \|u\|_{p+1}^{p+1} \\ &= I(u) - \sigma\left(\|\nabla u\|^2 + \|u\|^2\right). \end{aligned}$$

Then D_σ and N_σ are defined by

$$D_\sigma = \inf_{u\in N_\sigma} J(u), \quad N_\sigma = \left\{u \in H^1 : I_\sigma(u) = 0, \|u\|_{H^1} \neq 0\right\}.$$

Obviously, taking $\sigma = 0$, I_σ corresponds to the functional $I(u)$. Moreover, if $\sigma < -\frac{p-1}{2}$ then $D_\sigma < 0$. In this case for $E(0) = D_\sigma < 0$, all weak solutions of (2), (3) blow-up in a finite time.

For $\sigma \in \left(-\frac{p-1}{2}, 1\right)$, we have the following lemmas.

Lemma 2.1. *Assume that $u \in H^1(R^n)$. If $I_\sigma(u) < 0$, then $\|u\|_{H^1} > \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$. If $I_\sigma(u) = 0$, then $\|u\|_{H^1} \geq \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$ or $\|u\|_{H^1} = 0$.*

Proof. First, since $I_\sigma(u) < 0$, we have $\|u\|_{H^1} \neq 0$. Hence, from

$$(1-\sigma)\|u\|_{H^1}^2 < \|u\|_{p+1}^{p+1} \leq S_p^{p+1}\|u\|_{H^1}^{p+1}, \tag{9}$$

we have $\|u\|_{H^1} > \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$.

If $\|u\|_{H^1} = 0$, then $I_\sigma(u) = 0$. If $I_\sigma(u) = 0$ and $\|u\|_{H^1} \neq 0$, then from

$$(1-\sigma)\|u\|_{H^1}^2 = \|u\|_{p+1}^{p+1} \leq S_p^{p+1}\|u\|_{H^1}^{p+1}$$

it follows that $\|u\|_{H^1} \geq \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$. □

Lemma 2.2. *If $\|u\|_{H^1} < \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$, then $I_\sigma(u) > 0$.*

Proof. By $\|u\|_{H^1} < \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$, we obtain

$$\|u\|_{p+1}^{p+1} \leq S_p^{p+1}\|u\|_{H^1}^{p+1} < (1-\sigma)\|u\|_{H^1}^2$$

from which follows $I_\sigma(u) > 0$. □

Theorem 2.3. *Let D_σ be defined as above. Then for $\sigma > -\frac{p-1}{2}$, we have*

$$D_\sigma = \frac{p-1+2\sigma}{2(p+1)}\left(\frac{|1-\sigma|}{S_p^{p+1}}\right)^{2/(p-1)}.$$

If we write D_σ in terms of d, we obtain

$$D_\sigma = \frac{p-1+2\sigma}{2(p+1)}|1-\sigma|^{2/(p-1)}\frac{2(p+1)}{p-1}d. \tag{10}$$

Proof. If $u \in N_\sigma$, we have by Lemma 2.1 that $\|u\|_{H^1} \geq \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}$. In the proof of Lemma 2.1 the inequality (9) is an equality iff u is a minimizer of the imbedding H^1 into L^{p+1}. Since $\|u\|_{p+1} = S_p \|u\|_{H^1}$ is attained only for $\widetilde{u} = \left(\cosh\left(\frac{p-1}{2}\right)x\right)^{-\frac{2}{p-1}}$ for $n = 1$ [11], and for the ground state solution of (2), (3) for $n > 1$ [14] and it has constant sign, we have

$$\inf_{u\in N_\sigma} \|u\|_{H^1} = \left(\frac{1-\sigma}{S_p^{p+1}}\right)^{1/(p-1)}.$$

Hence from

$$\begin{aligned}\inf_{u\in N_\sigma} J(u) &= \inf_{u\in N_\sigma}\left(\frac{1}{2}\|u\|_{H^1}^2 - \frac{1}{p+1}\|u\|_{p+1}^{p+1}\right)\\ &= \inf_{u\in N_\sigma}\left[\left(\frac{1}{2} - \frac{(1-\sigma)}{p+1}\right)\|u\|_{H^1}^2 + \frac{1}{p+1} I_\sigma(u)\right]\\ &= \left(\frac{1}{2} - \frac{(1-\sigma)}{p+1}\right)\inf_{u\in N_\sigma}\|u\|_{H^1}^2,\end{aligned}$$

and by definition of D_σ we obtain $D_\sigma = \frac{p-1+2\sigma}{2(p+1)}\left(\frac{1-\sigma}{S_p^{p+1}}\right)^{2/(p-1)}$. □

We can also state the following properties of D_σ, which can be proved easily.

i) D_σ is strictly increasing on $\sigma \in \left(-\frac{p-1}{2}, 0\right) \cup (1, \infty)$ and strictly decreasing on $(0, 1)$.

ii) $\lim_{\sigma\to 1} D_\sigma = 0$, and $D_{\sigma_0} = 0$, where $\sigma_0 = -\frac{p-1}{2}$.

The following theorems show the invariance of I_σ under the flow of (2), (3) for $0 < E(0) < d$ and $E(0) = d$, respectively, and can be proved by contradiction as in [20].

Theorem 2.4. *Assume that $u_0 \in H^1(R^n)$, $u_1 \in L^2(R^n)$. Let $0 < E(0) < d$. Then the sign of I_σ is invariant under the flow of (2), (3) for $\sigma \in (\sigma_1, \sigma_2]$, where σ_1 and σ_2 are the corresponding minimal negative and minimal positive roots of equation $D_\sigma = E(0)$.*

Theorem 2.5. *Let all the assumptions of Theorem 2.4 hold and that $E(0) = d$. Then the sign of I_0 (recall that when $E(0) = d$, we have $\sigma_1 = \sigma_2 = 0$) is invariant with respect to (2), (3) for every $t \in [0, \infty)$.*

Now, we give a lemma for $\sigma > 1$, which states similar results to Lemmas 2.1, 2.2, and can be proved similarly.

Lemma 2.6. *Assume that $u \in H^1(R^n)$. Let $\sigma > 1$. If $I_\sigma(u) > 0$, then $\|u\|_{H^1} > s(\sigma)$. If $I_\sigma(u) = 0$, then $\|u\|_{H^1} \geq s(\sigma)$ or $\|u\|_{H^1} = 0$, where $s(\sigma) = \left(\frac{\sigma-1}{S_p^{p+1}}\right)^{1/(p-1)}$. Moreover, if $\|u\|_{H^1} < s(\sigma)$, then $I_\sigma(u) \leq 0$ and $I_\sigma(u) = 0$ if and only if $\|u\|_{H^1} = 0$.*

Theorem 2.7. *Assume that $u_0 \in H^1(R^n)$, $u_1 \in L^2(R^n)$. If $E(0) > 0$, then $I_\sigma(u(t)) \leq 0$ for every $t > 0$ and $\sigma \geq \sigma_m$, where σ_m is the maximal positive root of $D_\sigma = E(0)$.*

Proof. We give the proof of the theorem for $\sigma = \sigma_m$ and $\sigma > \sigma_m$ separately. First, we prove the theorem for $\sigma = \sigma_m$. By contradiction, assume that there exists some $t' > 0$ such that $I_{\sigma_m}(u(t')) > 0$. By Lemma 2.1, we have $\|u\|_{H^1} > 0$ and there exists a value $\sigma > \sigma_m$ such that $I_\sigma(u(t')) = 0$. Then, by (4), $D_{\sigma_m} = E(0) \geq J(u(t')) \geq \inf_{u\in N_\sigma} J(u) = D_\sigma$. By definition of D_σ, for $\sigma > \sigma_m > 1$ we have $D_\sigma > D_{\sigma_m}$. A contradiction occurs, which proves the theorem for $\sigma = \sigma_m$. For $\sigma \geq \sigma_m$, $I_{\sigma_m}(u(t)) \geq I_\sigma(u(t))$ implies that the theorem is true for every $\sigma \geq \sigma_m$. □

The following Corollary gives a more precise result for subcritical initial energy.

Corollary 2.8. *Suppose* $u_0 \in H^1(R^n)$, $u_1 \in L^2(R^n)$. *Let* $0 < E(0) < d$ *and* $I_0(u_0) > 0$. *Then,*

$$0 < I_0(u(t)) \le \sigma_m \|u\|_{H^1}^2 \tag{11}$$

for every $t > 0$.

Proof. We know that for $I_0(u(t)) > 0$, the solution $u(x,t)$ of problem (2), (3) is globally defined. Since $E(0) = D_{\sigma_m}$ for some $\sigma_m > 1$ then by Theorem 2.7 we have $I_{\sigma_m}(u(t)) \le 0$ for every $t \in [0,\infty)$. Thus we get the inequality (11) from below and from above. □

Remark 2.9. *We tried to characterize the behavior of solutions for* $E(0) > d$ *in terms of initial displacement. We constituted the new functional* $I_\sigma(u)$ *and proved the sign invariance of* $I_\sigma(u)$ *for* $0 < E(0) < d$ *and* $E(0) = d$. *But the case* $E(0) > d$ *is still an open question, because from Theorem 2.7, we concluded that in this case* $I_\sigma(u)$ *is always non-positive.*

3. Main Results

We will introduce our new functional which will be used for global existence of solutions with high energy initial data.

$$\widetilde{M}(v,\omega) = \left(\|\nabla v\|^2 + \|v\|^2\right) - \|v\|_{p+1}^{p+1} - (\omega,\omega) \tag{12}$$

for every $v \in H^1$ and $\omega \in L^2$. For simplicity we denote

$$M(u,t) = \widetilde{M}(u(.,t), u_t(.,t)).$$

The sign invariance of this new functional can be stated as follows.

Theorem 3.1. *Let* $u_0 \in H^1(R^n)$, $u_1 \in L^2(R^n)$ *and* $E(0) > 0$. *For* $\sigma > \sigma_m$, *assume that*

$$(u_1,u_0) + \frac{1}{2}\|u_0\|^2 + \frac{(p+1)\sigma}{p-1+(p+3)\sigma}E(0) \le 0. \tag{13}$$

If $M(u,0)$ *is positive, then* $M(u,t)$ *is positive for every* $t \in [0,\infty)$.

Proof. [Proof]We prove the theorem by contradiction. Let us define

$$\theta(t) = \|u\|^2 + \int_0^t \|u\|^2 d\tau.$$

Then

$$\theta'(t) = 2(u_t,u) + \|u\|^2,$$

$$\begin{aligned}\theta''(t) &= 2\|u_t\|^2 + 2(u_{tt},u) + 2(u_t,u)\\ &= 2\|u_t\|^2 + 2\left[\|u\|_{p+1}^{p+1} - \|\nabla u\|^2 - \|u\|^2 - (u_t,u)\right] + 2(u_t,u)\\ &= -2M(u,t).\end{aligned}$$

To get a contradiction, let us assume that there exists some $t' > 0$ such that $M(u,t') = 0$. Since $\theta''(t) < 0$, we conclude that $\theta'(t)$ is strictly decreasing on $[0,t')$. Moreover, (13) implies $\theta'(0) < 0$ and therefore $\theta'(t) < 0$ in

$[0,t']$, from which follows that $\theta(t)$ is strictly decreasing on $[0,t']$. By the energy identity and $M(u,t')=0$, we have

$$E(0) \geq \frac{1}{2}\|u_t(t')\|^2 + \frac{p-1}{2(p+1)}\left(\|\nabla u\|^2 + \|u\|^2\right) + \frac{1}{p+1}I(u(t'))$$
$$= \left(\frac{1}{2} + \frac{1}{p+1}\right)\|u_t(t')\|^2 + \frac{p-1}{2(p+1)}\left(\|\nabla u\|^2 + \|u\|^2\right). \tag{14}$$

Theorem 2.7 and $M(u,t')=0$ yield

$$\left(\|\nabla u\|^2 + \|u\|^2\right) \geq \sigma_m^{-1} I_0(u(t')) \geq \sigma^{-1}\|u_t(t')\|^2.$$

The use of this inequality in (14) gives

$$E(0) \geq \left(\frac{1}{2} + \frac{1}{p+1} + \frac{p-1}{2(p+1)\sigma}\right)\|u_t(t')\|^2$$
$$= \frac{(p+3)\sigma + p - 1}{2(p+1)\sigma}\left[\|(u_t(t') + u(t'))\|^2\right.$$
$$\left.-2(u_t(t'), u(t')) - \|u(t')\|^2\right].$$

From the monotonocity of $\theta(t)$ and $\theta'(t)$, we get

$$E(0) > \frac{(p+3)\sigma + p - 1}{(p+1)\sigma}\left[-(u_1, u_0) - \frac{1}{2}\|u_0\|^2\right]$$

which contradicts with (13). Thus the proof is completed. □

Theorem 3.2. *Let $1 < p < \infty$ for $n = 2$; $1 < p < \frac{n+2}{n-2}$ for $n \geq 3$ and $u_0 \in H^1(R^n)$, $u_1 \in L^2(R^n)$. Suppose that $E(0) > 0$, $M(u,0) > 0$ and (13) holds for some $\sigma > \sigma_m$. Then, the weak solution of problem (2),(3) is globally defined for every $t \in [0,\infty)$.*

Proof. [Proof]The proof of this theorem follows from adding some arguments to the local existence result of Proposition 1.1 of [25]. $M(u,0) > 0$ implies from the sign preserving property of $M(u,t)$ that $M(u,t) > 0$, thereby $I_0(u) > 0$ for every $t > 0$. From enegy identity, we have

$$E(0) \geq \frac{1}{2}\|u_t\|^2 + \frac{p-1}{2(p+1)}\left(\|\nabla u\|^2 + \|u\|^2\right) + \frac{1}{p+1}I(u)$$
$$\geq \frac{1}{2}\|u_t\|^2 + \frac{p-1}{2(p+1)}\left(\|\nabla u\|^2 + \|u\|^2\right).$$

Therefore $\|u\|_{H^1}$ and $\|u_t\|_{L^2}$ are bounded for every $t > 0$. The previously mentioned local existence theory completes the proof. □

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