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Global Malmquist Productivity Index Based on Preference Common-Weights

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Abstract.
Data Envelopment Analysis is a linear programming technique for assessing the efficiency and productivity of decision making units (DMUs). Over the last decade, DEA has gained considerable attention as a managerial tool for measuring performance. The flexibility in selecting the weights in standard DEA models deters the comparison among DMUs on a common base. Moreover, these weights are unsuitable to measure the preferences of a decision maker (DM). For dealing with these two difficulties simultaneously; we use preference common weights. This paper uses preference common weights for time-series evaluations to calculate the global Malmquist productivity index (MPI) so that the productivity of changes of all DMUs have a common basis for comparison, and DM's preference information is incorporated in calculating global MPI. The Malmquist Productivity Index (MPI) suggests a convenient way of measuring the productivity change of a given unit between two consequent time periods.

1. Introduction

Data envelopment analysis(DEA) introduced by Charnes et al. (1978) is a mathematical programming technique that calculates the relative efficiency of multiple decision making units(DMUs) based on observed inputs and outputs, which may be expressed indifferent types of metrics. The original DEA model assesses the relative efficiency of a DMU as the ratio of weighted outputs to weighted inputs, where the models elect weights for each DMU to present it in the most favorable light (see Amirteimoori et al.; 2005, Amirteimoori and Shafiei; 2006). By doing so it identifies its relative efficiency with respect to an "efficiency frontier" that is defined by all the assessed DMUs. However, in real-world applications, virtually unconstrained weights are usually unacceptable (Roll and Golany; 1993). Likewise, large differences in the weight values for different DMUs may be a concern. Restricted DEA approaches such as the concept of common weights were developed to allow some control over the weights in the model (Amirteimoori and Kordrostami; 2009). However, none of the common weight approaches are suitable to measure the preferences of a DM.

In addition to comparing the relative efficiency of a set of DMUs at a specific time, the conventional DEA can also be used to calculate the productivity growth of a DMU over the time.

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The computation of productivity growth by efficiency measures was introduced by Caves et al. (1982) and developed by Nishimizu and Page (1982) and by Fre et al. (1994c), in the context of parametric and non-parametric efficiency measurement, respectively. The Fre et al. (1994c) approach has become known as the measurement of productivity of changes through Malmquist indices. The Malmquist index has been defined by using distance functions. The distance functions allow us to describe a multi-input, multi-output production technology without the need to specify the producer behavior (such as cost minimization or profit maximization). The non-parametric Malmquist approach has been applied mostly in the analysis of productivity growth, technical progress, and efficiency change at the firm level, for example, hospitals (Fre et al.,1989), pharmacies (Fre et al.,1992; Suyanto and Salim, 2013), electricity (Hjalmarssonan and Veiderpass, 1992), natural gas (Price and Weyman-Jones, 1996), mobile operators (Usero and Asimakopoulos, 2013), services sector (Lee, 2013), agriculture (Taminia et al., 2012), Chinese state enterprises (Zheng et al., 2003), Energy (Moroney, 1990), OECD countries (Yoruk and Zaim, 2005), Public Enterprise (Zaim and Taskin, 1997) and airports (Abbot and Wu, 2002). Fre et al. (1994b) also used this method to compare the productivity growth in the industrialized countries.

The objective of this paper is to design a Malmquist productivity index, which is based on a preference common-weights DEA model, such as the productivity change of all DMUs over time can be compared on a common basis, and the DM's preference information can be incorporated.

The rest of this paper is organized as follows: The idea of the preference common-weights DEA model based on DM's preference information proposed by Jahanshahloo et al. (2011) is reviewed in Section 2. The idea of common-weights based on DM's preference is extended to calculate the MPI in Section 3, and the global MPI proposed by Pastor and Lovell (2005) in Section 4. In Section 5, an example is designed for illustration. The article is sum up with conclusions and some suggestions given in Section 6.

2. Preference Common Weights in DEA

Let each DMU_j in the set of n DMUs be characterized by its input/output data collected into the row vector (x_j, y_j), where we expect all entries are nonnegative and at least one input and one output are positive. Each unit has m inputs and s outputs. Suppose x_{ij} denotes the consumed amount of the ith input measure and y_{rj} denotes the amount produced of the rth output measure by the jth DMU. The production possibility set (PPS) of most widely used DEA model, CCR (the proposed model by Charnes, Cooper and Rhodes; 1978) with constant returns to scale characteristic, is defined as semi-positive vectors (x,y) as follows:

$$T_c = \{(x, y)|x \geq \sum_{j=1}^{n} \lambda_j x_j, y \geq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0, j = 1, 2, ..., n\}.$$

Here, $lambda_j$ is the jth unknown decision variable for connecting inputs and outputs in the convex combination.
A subset of T_c, consisting of the following set of input-output vectors:

$EF = \left\{\binom{x}{y} \in T_c \middle| \text{ there is no } \binom{\bar{x}}{\bar{y}} \in T_c \text{ such that } \binom{-\bar{x}}{\bar{y}} \geq \binom{-x}{y} \text{ and } \binom{\bar{x}}{\bar{y}} \neq \binom{x}{y}\right\}$ is referred to as "an efficiency frontier". Fre et al. (1994a) provide the following definition related to the return to scale (see Banker and Thrall; 1992) for an equivalent definition. The technology exhibits constant returns to scale (CRS) if $\alpha T = T, \alpha > 0$; it exhibits non-increasing returns to scale (NIRS) if $\beta T \subseteq T, 0 < \beta \leq 1$; it exhibits non-decreasing returns to scale (NDRS) if $\gamma T \subseteq T, \gamma \geq 1$ (or equivalently if $T \subseteq \delta T, 0 < \delta \leq 1$). On the basis of the above RTS definition, we say that a DMU exhibits decreasing returns to scale (DRS) if it exhibits NIRS but not CRS and increasing returns to scale (IRS) if it exhibits neither CRS nor DRS. Seiford and Zhu (1999) have proposed that the efficient frontier obtained by the CCR model exhibits CRS and the efficient frontier obtained by the BCC model exhibits variable returns to scale (VRS), i.e. IRS, CRS and DRS are all allowed in the BCC model.

Classical DEA models evaluate DMUs and specify reference points on the assumption that inputs have to be minimized, and outputs have to be maximized. However, based on this assumption, specification of reference points in DEA models could be expressed as an MOLP problem which their constraints are same as the constraints which define PPS of standard DEA models, but applied to minimization of input variables and maximization of output variables. Then, Jahanshahloo et al. (2011) proposed following MOLP model, which is intellectually consistent with the DEA philosophy, to obtain an efficient reference point for the virtual DMU (x, y):

$$\begin{aligned} & max \ \{-x_1, -x_2, ..., -x_m, y_1, y_2, ..., y_p\} \\ & s.t. \ \ (x_1, x_2, ..., x_m, y_1, y_2, ..., y_p) \in T_c \end{aligned} \tag{1}$$

When this model is solved by Zionts-Wallenius (1976) approach, a proper set of preference weights could be specified that reflect the relative degree of DM's underlying value structure about inputs and outputs. In other words, they produced a preference common weights and then efficiency score of DMU_j, $j = 1, 2, ..., n$, can be obtained by using these common weights as $E_j = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}$.
In Zionts-Wallenius (1976) a man-machine interactive mathematical programming method was presented for solving the multiple criteria problem involving a single decision maker. It was assumed that all decision-relevant criteria or objective functions were concave functions to be maximized, and that the constraint set was convex. The overall utility function was assumed to be unknown explicitly to the decision maker, but was assumed to be implicitly a linear function, and more generally a concave function of the objective functions. To solve a problem involving multiple objectives the decision maker was requested to provide answers to yes and no questions regarding certain trade offs that he likes or dislikes.

3. Preference Common Weights Malmquist Productivity Index (MPI)

The MPI is a bilateral index that can be used to compare the production technology of two economies or the productivity change of one economy over time. It was introduced by Caves et al. (1982) that named it after Malmquist (1953), who proposed to construct quantity indices as ratios of distance functions for use in consumption analysis. MPI has a number of desirable features. It does not require input prices or output prices in their construction, which makes it particularly useful in situations in which prices are distorted or non-existent. It does not require a behavioral assumption such as cost minimization or profit maximization, which makes it useful in situations in which producers' objectives differ, either are unknown or are unachieved. Besides, the MPI is easy to compute, as Färe et al. (1992) have demonstrated, and its various decompositions provide insight into the sources of productivity change. A functional representation of production technology is provided by the following input distance function:

$$D(x_o, y_o) = inf\{\theta | (\theta x_k, y_k) \in PPS\} \tag{2}$$

This function, gives us the efficiency score of the unit $o \in \{1, 2, ..., n\}$ relative to other units. Model (2) has not the measurability of efficiency changes in two different times. Färe et al. (1994b) showed that it is possible to decompose the Malmquist productivity index into two components: an index of the change in efficiency and an index of technological change. We need to solve four linear programming models to calculate Malmquist index.

$$\begin{aligned} & D_k^t(x_k^t, y_k^t) = Min \ \ \theta \\ s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{ik}^t, \quad i = 1, 2, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rk}^t, \quad r = 1, 2, ..., s, \\ & \lambda_j \ge 0 \quad j = 1, 2, ..., n. \end{aligned} \tag{3}$$

Which $D_k^t(x_k^t, y_k^t)$ is the efficiency score of DMU_k in the time of t and by putting $t+1$ instead of t in the model (3), the efficiency score of DMU_k in time $t+1$ show by $D_k^{t+1} = (x_k^{t+1}, y_k^{t+1})$ is attainable.
We can calculate the efficiency score of DMU_k in time $t+1$ based on the frontier in time t, $D_k^t = (x_k^{t+1}, y_k^{t+1})$, from the model (4):

$$\begin{aligned} D_k^t(x_k^{t+1}, y_k^{t+1}) &= Min\ \ \theta \\ s.t.\ \ \sum_{j=1}^{n} \lambda_j x_{ij}^t &\leq x_{ik}^{t+1}, i = 1,2,...,m \\ \sum_{j=1}^{n} \lambda_j y_{rj}^t &\geq y_{rk}^{t+1}, r = 1,2,...,s \\ \lambda_j &\geq 0\ \ j = 1,2,...,n \end{aligned} \tag{4}$$

Where (x_k^t, y_k^t) and (x_k^{t+1}, y_k^{t+1}) are inputs/outputs components of DMU_k in the time t and next time, $t+1$, respectively.
We can calculate the efficiency score of DMU_k in the time t based on the frontier in time $t+1$, $D_k^{t+1}(x_k^t, y_k^t)$, by replacement of t and $t+1$. Sueyoshi (1998) introduced growth index for two times, t and t+1 as the efficiency score of the consideration unit in the time t+1 based on production technology in time t. Malmquist productivity index (MPI) based on the idea of Cave et al. (1982) is equal to the growth index divided by the efficiency score of the current time. In general, we solve the following linear programming in which is dual of model (4) for computation the growth index of kth unit in the time $t+1$ based on production technology in time t:

$$\begin{aligned} D_k^t(x_k^{t+1}, y_k^{t+1}) &= Max\ \ \sum_{r=1}^{s} u_r y_{rk}^{t+1} \\ s.t.\ \ &\sum_{i=1}^{m} v_i x_{ik}^{t+1} = 1 \\ \sum_{r=1}^{s} u_r y_{rj}^t - \sum_{i=1}^{m} v_i x_{ij}^t &\leq 0\ \ \ j = 1,2,...,n \\ u_r \geq 0 &\qquad r = 1,2,...,s \\ v_i \geq 0 &\qquad i = 1,2,...,m \end{aligned} \tag{5}$$

Notice that growth index can be greater than one. Thus, MPI provided by Cave et al. (1982), which is displayed with the sign of MPI^{CCD} is equal to:

$$MPI^{CCD} = \frac{D_k^t(x_k^{t+1}, y_k^{t+1})}{D_k^t(x_k^t, y_k^t)}$$

Where $D_k^t(x_k^t, y_k^t)$ is the calculated efficiency score from model (3).
Since the weights used for calculating $D_k^t(x_k^{t+1}, y_k^{t+1})$ and $D_k^t(x_k^t, y_k^t)$ are different, the MPI, which is the ratio of $D_k^t(x_k^{t+1}, y_k^{t+1})$ to $D_k^t(x_k^t, y_k^t)$, is not calculated on the same basis. Now, we calculate MPI by the ratio of $D_k^{t+1}(x_k^{t+1}, y_k^{t+1})$ to $D_k^{t+1}(x_k^t, y_k^t)$ and we show it as $\widetilde{MPI}^{CCD}$. To sum up MPI^{CCD} calculation is based on production technology of the period t and $\widetilde{MPI}^{CCD}$ calculation is based on production technology during $t+1$. We see that both indexes ($\widetilde{MPI}^{CCD}$ and MPI^{CCD}) compare $(x_k^(t+1), y_k^(t+1)\,)$ to $(x_k^t, y_k^t\,)$, but they use different benchmarks. In order to provide the same basis for a consistent comparison and to incorporate preference information in DEA, the preference common-weights (PCW) introduced in section 2 can be applied. Since these preference common weights have this merit that DM preferences are considered

for obtaining common weights; we call this index as preference common-weights MPI. We use PCW for obtaining growth index and then MPI. To sum up, the procedure is to apply model (1) to find the PCW (u^*, v^*) primarily and then, the efficiency of DMU_k at the current time is calculated as:

$$E_k^{PCW} = \sum_{r=1}^{s} u_r^* y_{rk}^t / \sum_{i=1}^{m} v_i^* x_{ik}^t$$

And the growth index is calculated as:

$$\widetilde{E_k}^{PCW} = \sum_{r=1}^{s} u_r^* y_{rk}^{t+1} / \sum_{i=1}^{m} v_i^* x_{ik}^{t+1}$$

and finally, we get MPI based on preference common weights for DMU_k from the following ratio:

$$MPI^{PCW} = \frac{\widetilde{E_k}^{PCW}}{E_k^{PCW}}$$

4. Global Malmquist Productivity Index (global MPI)

Consider n DMUs (DMU_j, $j = 1, 2, ..., n$) and $t = 1, 2, ..., T$ time periods. Each DMU consume inputs$X \in R_+^m$ to produce outputs $Y \in R_+^s$. we define two PPS. A contemporaneous PPS is defined as $T_c^t = \{(x^t, y^t) | x^t can produce y^t\}$ for $t = 1, 2, ..., T$. A global PPS is defined as $T_c^G = conv\{T_c^1, ..., T_c^T\}$. The subscript c indicates that both PPSs satisfy constant return to scale. Conv is abbreviation of convex set produced by $T_c^1 \bigcup ... \bigcup T_c^T$. A contemporaneous Malmquist productivity index is defined on T_c^s for DMU_k as:

$$MPI^s = \frac{D_k^s(x_k^{t+1}, y_k^{t+1})}{D_k^s(x_k^t, y_k^t)} \quad s = t, t+1. \tag{6}$$

Since $MPI^t \neq MPI^{t+1}$ without restrictions on the two PPSs, in order to avoid choosing an arbitrary benchmark, Färe et al. (1989) specified the contemporaneous MPI as the geometric mean form as:

$$MPI^{FGNZ} = (MPI^t \times MPI^{t+1})^{\frac{1}{2}} = ((\frac{D_k^t(x_k^{t+1}, y_k^{t+1}}{D_k^t(x_k^t, y_k^t)}) \times (\frac{D_k^{t+1}(x_k^{t+1}, y_k^{t+1})}{D_k^{t+1}(x_k^t, y_k^t)}))^{\frac{1}{2}} \tag{7}$$

The MPI^{FGNZ} solves the disparity problem of using different base times in measuring productivity changes. However, it is not circular (Pastor and Lovell; 2005). Considering that, Pastor and Lovell (2005) proposed a global MPI which uses the data of all DMUs at all times to construct the frontier for calculating the MPI. A global Malmquist productivity index for DMU_k is defined on T_c^G as:

$$MPI^G = \frac{D_k^G(x_k^{t+1}, y_k^{t+1})}{D_k^G(x_k^t, y_k^t)} \tag{8}$$

Where the input distance functions $D_k^G(x, y) = min\{\theta | (\theta x_k, y_k) \in T_c^G\}$
To get the preference global Malmquist productivity index, at first, we calculate the preference common weight for all DMUs at all times. In other words, if we have n units and T times, we compute preference common weight for nT units. Suppose that we use the same weights such as v_i^* and u_r^* for calculating the efficiency of all units in all times that these are the preference common weights, we can obtain the global MPI based on PCW for DMU_k between period t and $t + 1$ as follows:

$$GMPI^{PCW} = \frac{\sum_{r=1}^{s} u_r^* y_{rk}^{t+1} / \sum_{i=1}^{m} v_i^* x_{ik}^{t+1}}{\sum_{r=1}^{s} u_r^* y_{rk}^t / \sum_{i=1}^{m} v_i^* x_{ik}^t} \tag{9}$$

5. An Illustrative Example

In the current paper, we employ the Malmquist productivity index for evaluating 25 bank branches for two period times by using the methodology developed above. This section describes the data and results. Table 1 shows inputs and outputs of these banks at time period t and Table 2 shows inputs and outputs of these banks at time period t+1. Each bank contains four inputs and four outputs.

Table 1: The data for 25 bank branches at time t

	DEA Inputs				DEA Outputs			
Bank NO.	I_1	I_2	I_3	I_4	O_1	O_2	O_3	O_4
1	9.38	34.33	3143	1	782449	250068	833004	101067.81
2	143.03	171.31	15747	31	652125	31068	338152	91873.74
3	1580.38	466.93	221300	51	4040385	701735	2424385	437267.97
4	735.84	513.55	252217	51	3409633	428514	2677324	1646423.34
5	53.44	340.92	71071	45	1224900	256802	1367283	379950.67
6	2535.54	254.98	43010	35	2156648	293247	2920857	1169735.81
7	1139.39	394.01	61841	45	3809482	374144	1665672	308032.42
8	4660.1	461.67	155558	51	3644455	1962969	4118313	834094.56
9	848.66	266.95	48677	48	799361	72616	574892	168866.35
10	0.99	141.63	18366	34	405501	12942	108368	35236.01
11	798.5	720.61	40157	141	2625840	106282	685899	221782.3
12	303.31	502.46	36920	98	1052064	35297	548749	149980.8
13	3.45	27.65	1146	6	43683	32492	5688	1415.17
14	51.1	182.14	15419	45	428114	16377	139570	41807.41
15	0.9	182.6	20131	48	339768	18213	164653	51807.24
16	839.56	666.05	83938	144	1874197	133063	1125711	475364.41
17	0.48	129.13	14954	33	259330	8657	162732	74421.66
18	0.11	44.09	11139	14	143857	4894	149931	55683.43
19	807.95	408.53	124600	89	1094242	85205	642039	246534.15
20	113.99	104.74	16822	28	259817	14174	201210	93034.64
21	29.42	147.54	18932	34	312741	20805	166018	57174.19
22	53.31	144.14	29997	30	282343	14441	264833	95249.99
23	71.81	409.25	27173	97	807147	25803	501600	166854.03
24	423.58	446.41	46793	80	935800	62753	608261	179386.4
25	4.74	127.53	14612	30	247707	6882	167085	59788

Table 2: The data for 25 bank branches at time t+1

	DEA Inputs				DEA Outputs			
Bank NO.	I_1	I_2	I_3	I_4	O_1	O_2	O_3	O_4
1	5482.32	33.17	3076	1	831750	268098	841725	177390.52
2	5451.58	169.88	17066	31	640258	33730	344091	97519.48
3	50633.58	464.89	217878	51	3936051	664068	2204346	482360.32
4	39453.29	513.04	252499	51	3394752	593271	3127005	1686407.04
5	10866.91	337.57	53427	45	1263973	323971	1412098	416052.9
6	29019.68	253.03	42722	35	2280451	409819	3130137	1267167.57
7	54914.42	395.71	61056	45	3911839	464597	1665863	318207.37
8	36562.67	463.52	162910	51	3531637	1881242	4202924	867748.67
9	8048.46	267.71	49346	48	805613	74521	574680	176427.9
10	2873.84	142.47	18292	34	399133	13851	109991	38641.25
11	33378.11	717.97	40561	141	2625625	142700	694756	236595.05
12	12108.04	502.29	37421	98	1052391	41562	552447	161991.54
13	267.45	26.96	1186	6	43451	51626	5689	1598.71
14	4115.57	181.66	15683	45	430320	23988	145108	43628.7
15	3489.83	183.46	19280	48	347016	17777	166013	54738.66
16	25014.61	668.66	84489	144	1896021	165423	1139098	499387.48
17	2106.56	129.02	14870	33	251780	11561	172155	78340.49
18	2116.09	44.33	10767	14	147085	4910	156281	58630.32
19	10539	409.08	127439	89	1098750	84289	649153	261217.3
20	1878.5	104.28	28090	28	258167	15995	198100	97676.95
21	2359.89	146.72	19663	34	315388	23664	172966	61480.53
22	3208.41	142.54	31008	30	283296	14941	266099	100696.8
23	5888.53	408.59	29186	97	805940	27801	511579	180677.36
24	8930.22	444.15	47296	80	942343	66846	629729	195533.16
25	1821.29	128.71	15341	30	245357	8563	175931	63273.62

Table 3: Three forms of MPI for the Bank case

	CCD base t			PCW base t		PCW global	
Bank NO.	*Efficiency*	*Growth I*	*MPI*	*Efficiency*	*Growth I*	*MPI*	$GMPI^{PCW}$
1	1	1.8070	1.8070	0.9225	0.9037	0.9796	1.3529
2	0.6343	0.1936	0.3052	0.7043	0.6570	0.9328	0.4536
3	0.3797	0.3715	0.9784	0.4235	0.3920	0.9256	0.3657
4	1	0.8193	0.8193	0.9123	0.8978	0.9841	1.0023
5	0.5212	0.3043	0.5838	0.5934	0.5560	0.9370	0.7894
6	1	1.1116	1.1116	0.8763	0.8421	0.9610	1.1234
7	0.4107	0.4337	1.0560	0.4511	0.4367	0.9681	0.9566
8	0.6017	0.6045	1.0047	1	0.9231	0.9231	1.2453
9	0.1835	0.1807	0.9847	0.4341	0.2560	0.5897	1.1314
10	1	0.1229	0.1229	0.9536	0.8954	0.9390	0.4536
11	0.2627	1.2292	4.6791	0.3017	0.2960	0.9811	2.2425
12	0.1263	0.4715	3.7332	0.2560	0.2432	0.9500	1.3564
13	1	1.6295	1.6295	0.9430	0.9065	0.9613	0.8743
14	0.1115	0.9330	8.3677	0.4510	0.4123	0.9142	1.9879
15	1	0.0927	0.0927	0.8950	0.8256	0.9225	0.7643
16	0.2290	0.4261	1.8607	0.3541	0.3245	0.9164	0.9789
17	1	1.0317	1.0317	0.9672	0.9089	0.9397	1.3245
18	0.4358	0.3923	0.9002	0.5225	0.5036	0.9633	1.0045
19	0.4703	0.2068	0.4397	0.4156	0.3980	0.9577	0.9870
20	0.7445	0.2196	0.2950	0.4970	0.4786	0.9630	0.6787
21	0.1581	0.5248	3.3194	0.2154	0.2011	0.9336	1.5643
22	0.2245	0.1675	0.7461	0.2378	0.2032	0.8544	0.9521
23	0.2115	0.5507	2.6038	0.5690	0.5221	0.9176	1.3784
24	0.1314	0.3974	3.0244	0.4233	0.4102	0.9691	1.2536
25	0.9484	0.3190	0.3364	0.9730	0.9341	0.9600	0.8721
Average	0.5514	0.5816	1.6736	0.6117	0.5731	0.8929	0.8440

The efficiencies of the twenty five branches, under constant returns-to- scale, were calculated; the results are presented in the second column of Table 3. Seven banks are efficient. The growth indexes of the all branches were calculated by applying Model (4); the results are presented in the third column of Table 3.

The ratio of the growth index to the efficiency is the MPI^{CCD} and is shown in the fourth column of Table 3. where thirteen have a value > 1 and twelve have a value < 1, with an average MPI of 1.6736. Therefore, based on the conventional MPI, we concluded that, in general, time $t + 1$ had resulted in an improvement in performance. As discussed in the preceding sections, the efficiency and growth index of a DMU may have been calculated by using different frontier facets. It is inappropriate to use these numbers for comparison. To make them comparable and incorporate DM's preference information, the idea of the preference common-weights MPI be used. By applying Model (1), a single-facet frontier using time t as the base time is constructed. (u^*, v^*) is gradient vector of this supporting hyperplan. The efficiencies and growth indexes of the twenty five branches are calculated as it was presented in the sixth and seventh columns of Table 3, respectively. The ratio, which is the MPI^{PCW}, is presented in the seventh column. None of the ratios are greater than one, a result different from that obtained by conventional MPI shown in the fourth column. This indicates that the conventional MPI^{CCD} may produce unreliable results. The results in the fourth and seventh columns of Table 3 are both measures of the MPI. They both use time t as the base time. The only difference is that the former applied the conventional piecewise frontier DEA while the latter used a single-facet frontier DEA to calculate the efficiency and growth index. Obviously, if the base time is changed during time $t + 1$, the results would be different. Following the discussion of the preceding sections, the single- facet global MPI was calculated for each branch. The corresponding $GMPIs^{PCW}$ are listed in the last column. The numbers in parentheses are their ranks. Regarding the performance improvement, the values in the last column of Table 3 present that thirteen branches, have an MPI of > 1 and twelve branches, < 1. The average MPI of the twenty five branches is 0.8440. Hence, in general, the performances of the twenty five branches have declined during time $t + 1$. The conventional $MPIs^{CCD}$ in column four of Table 3 present that DMU_1, DMU_6, DMU_7, DMU_8, DMU_{11}, DMU_{12}, DMU_{13}, DMU_{14}, DMU_{16}, DMU_{17}, DMU_{21}, DMU_{23} and DMU_{24} have improved and that other DMUs have declined after the reorganization. Seventeen branches have the same direction of productivity change measured by the two methods. The results are similar, although those from the preference common-weights global MPI are more reliable. This case shows that allowing the MPI of a DMU at different times to be calculated using different frontier facets may produce misleading results. Even if a common single-facet frontier is used, the results may not be reliable if the base time is not selected properly. The global MPI of Pastor and Lovell (2005) combined with the concept of a single-facet frontier is suitable for measuring productivity changes.

6. Conclusion

MPI is used for time series evaluation that compares a DMU with itself at different time points. For time series evaluation with DEA, due to differences of the frontier facet used for calculating the index, the results from all DMUs are not comparable. To provide a common basis for comparison, this paper has adopted a single-facet frontier and all DMUs refer to the same frontier in calculating the index. The single-facet frontier is obtained by introducing an MOLP model which its objective functions are input/output variables subject to the defining constraints of production possibility set (PPS) in standard DEA models. Then by using the so-called Zionts-Wallenius approach, we aid DM in searching for the most preferred solution. A special feature of using this method is generating objective's weights as the DM's underlying value structure about objectives. In other words, considering the structure of objective functions of the proposed MOLP model, these weights be considered as the preference weights produced based on the DM's underlying value structure about inputs/outputs. Using these input/output weights, we introduce a hyperplane which all DMUs can be evaluated in common base. An example has showed that using the conventional piecewise frontier to calculate the MPI produce misleading results. The preference common-weights DEA model, on the other hand, produced reliable results. Moreover, the efficiencies and MPIs of all DMUs are comparable, and can thus be used for ranking. The case of bank branches in time t and $t+1$ is consistent with this example. The preference common-weights MPI using time t as the base time produced a result which is opposite to that obtained from the conventional MPI^{CCD}. That is, the performance of the branches during time $t + 1$ actually became worse. Interestingly, the preference common-weights global MPI approach produced a result similar to that obtained by conventional MPI^{CCD}, in that most branches showed an improvement in performance. This similarity is a coincidence. In order to obtain persuasive and reliable results, the preference common-weights global MPI is recommended. The cross-sectional evaluation identifies DMUs with satisfactory performance in a group of DMUs and the time-series evaluation judges whether a DMU is performing better than before. When these two types of comparison are conducted together, DMUs with satisfactory performance relative to others that are actually declining overtime can be detected. A DMU must perform not only better than others at the same time, but also more efficient than itself at different time points.

Recently, the DEA approach has been extended to measure the efficiency of systems composed of two stages (Cook et al; 2009) and of dynamic systems where the same process appears repeatedly (Tone and Tsutsui; 2010) and for a semi-oriented radial measure (SORM) to deal with negative data (Emrouznejad et al. 2010). How to apply the concept of preference common-weights MPI to these systems is a topic for future research. Although the discussion in this paper was for cases of constant returns-to-scale, the idea is readily applicable to variable returns-to-scale. In that case, the various kinds of decomposition of MPI reported in the literature can be explored.

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