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The Multivariable H-Function and the General Class of Srivastava Polynomials Involving the Generalized Mellin-Barnes Contour Integrals

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Abstract. In the present paper, we study and develop the definite integrals of Gradshteyn -Ryzhik given by Qureshi et al. [3]. The results are in general character and besides of this have been put in a compact form avoiding the occurrence of infinite series and thus making them useful in applications. Several other new and known results can also be obtained from our main theorems.

1. Introduction and Preliminaries

The multivariable H-function is defined and studied by Srivastava & Panda [12, p. 271, Eqn. (4.1)] in term of a multiple Mellin-Bernes type contour integral as

$$\begin{aligned} &H[z_1,...,z_r] \\ &= H^{0,n:m_1,n_1;...;m_r,n_r}_{p,q:p_1,q_1;...;p_r,q_r}\left[\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{c} \left(a_j;\alpha'_j,...,\alpha^{(r)}_j\right)_{1,p} : \left(c'_j,\gamma'_j\right)_{1,p_1};...;\left(c^{(r)}_j,\gamma^{(r)}_j\right)_{1,p_r} \\ \left(b_j;\beta'_j,...,\beta^{(r)}_j\right)_{1,q} : \left(d'_j,\delta'_j\right)_{1,q_1};...;\left(d^{(r)}_j,\delta^{(r)}_j\right)_{1,q_r} \end{array}\right] \\ &= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(\xi_1,...,\xi_r)\left\{\prod_{i=1}^{r}\theta_i(\xi_i)\,z_i^{\xi_i}\right\}d\xi_1...d\xi_r, \end{aligned} \tag{1}$$

where $\omega = \sqrt{-1}$; and

$$\phi(\xi_1,...\xi_r) = \frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha^{(i)}_j\xi_i\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha^{(i)}_j\xi_i\right)\ \prod_{j=1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta^{(i)}_j\xi_i\right)}, \tag{2}$$

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$$\theta_i(\xi_i) = \frac{\prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i\right) \prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} \xi_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)} \xi_i\right) \prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i\right)} \quad (i = 1, ..., r); \tag{3}$$

and $\mathcal{L}_j = \mathcal{L}_{\omega\tau_j\infty}$ represents the contours which start at the point $\tau_j - \omega\infty$ and terminate at the points $\tau_j + \omega\infty$ with $\tau_j \in \mathbb{R} = (-\infty, \infty)$ $(j = 1, \ldots, r)$.
In case $r = 2$, (1) reduces to the H-function of two variables.

For a detailed definition and convergence conditions of the multivariable H-function, the reader is referred to the original paper by Srivastava and Panda [12], Srivastava et al. [11], Saxena et al. [7], Saigo and Saxena [5], and Saigo et al. [6].

From Srivastava and Panda [13, p. 131], we have

$$H[z_1, \ldots, z_\tau] = O\left(|z_1|^{e_1} \ldots |z_\tau|^{e_r}\right) \left(\max_{1\le j\le r} \left\| z_j \right\| \to 0\right), \tag{4}$$

where

$$e_i = \min_{1\le j\le m_i} \left[\frac{Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}}\right] \quad (i = 1, \ldots, r). \tag{5}$$

For $n = p = q = 0$ the multivariable H-function breaks up into product of 'r' H-functions and consequently there holds the following result:

$$H_{0,0:p_1,q_1;\ldots;p_r,q_r}^{0,0:m_1,n_1;\ldots;m_r,n_r}\left[\begin{array}{c} z_1 \\ \vdots \\ z_r \end{array}\middle| \begin{array}{c} \left(c'_j, \gamma'_j\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(d'_j, \delta'_j\right)_{1,q_1}; \ldots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{array}\right] = \prod_{i=1}^{r} H_{p_i,q_i}^{m_i,n_i}\left[z \middle| \begin{array}{c} \left(c_j^{(i)}, \gamma_j^{(i)}\right)_{1,p_i} \\ \left(d_j^{(i)}, \delta_j^{(i)}\right)_{1,q_i} \end{array}\right], \tag{6}$$

where $H_{p,q}^{m,n}(\cdot)$ is the familiar H-function.

A general class of multivariable polynomials of real or complex variables $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}(x)$ is defined and studied by Srivastava [10] in the following form (also see, Kumar and Daiya [1]):

$$S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}(x) = \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i}}{k_i!} A_{n_i k_i} x^{k_i}, \tag{7}$$

where $n_1, \ldots, n_r = 0, 1, 2, \ldots; m_1, \ldots, m_r$ are arbitrary positive integers, the coefficients $A_{n_i k_i}$ $(n_i k_i \ge 0)$ are arbitrary constants real or complex.

Results required in the sequel:

For $a > 0$; $b \ge 0$; $c + 4ab > 0$; $\Re(\rho) + 1/2 > 0$ the following formulas is defined by Qureshi et al. [3, p.77, Eqn. (3.1)-(3.3)].

$$\int_0^\infty \left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-\rho-1} dx = \frac{\sqrt{\pi}}{2a\,(4ab + c)^{\rho+\frac{1}{2}}} \frac{\Gamma\left(\rho + \frac{1}{2}\right)}{\Gamma(\rho + 1)}. \tag{8}$$

For $a \ge 0$; $b > 0$; $c + 4ab > 0$; $\Re(\rho) + 1/2 > 0$,

$$\int_0^\infty \frac{1}{x^2} \left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-\rho-1} dx = \frac{\sqrt{\pi}}{2b\,(4ab + c)^{\rho+\frac{1}{2}}} \frac{\Gamma\left(\rho + \frac{1}{2}\right)}{\Gamma(\rho + 1)}. \tag{9}$$

For $a > 0$; $b > 0$; $c + 4ab > 0$; $\Re(\rho) + 1/2 > 0$,

$$\int_0^\infty \left(a + \frac{b}{x^2}\right)\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-\rho-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{\rho+\frac{1}{2}}} \frac{\Gamma\left(\rho + \frac{1}{2}\right)}{\Gamma(\rho+1)}. \tag{10}$$

The following formulas [9, p. 75] will be required in our investigation.

$$(1-y)^{a+b-c} \, {}_2F_1(2a, 2b, 2c; y) = \sum_{r=0}^{\infty} a_r \, y^r, \tag{11}$$

and

$${}_2F_1\left(a, b, c + \frac{1}{2}; x\right) {}_2F_1\left(c - a, \ c - b, c + \frac{1}{2}; \ x\right) = \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r \, x^r. \tag{12}$$

2. Main Results

Theorem 2.1. *Let* $a > 0$, $b \geq 0$, $c + 4ab > 0$, $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0 \quad (i = 1, \ldots, r)$, $-\frac{1}{2} < (a - b - c) < \frac{1}{2}$; *and* $X = \left(ax + \frac{b}{x}\right)^2 + c$ *the following formula holds:*

$$\begin{aligned}
&\int_0^\infty X^{-\lambda-1} {}_2F_1\left(a, b, c + \frac{1}{2}; \ X\right) {}_2F_1\left(c - a, \ c - b, \ c + \frac{1}{2}; \ X\right) \\
&\times \ S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H\left[Z_1 X^{-\eta_1}, \ \ldots, Z_r X^{-\eta_r}\right] = \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}} \\
&\times \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k} \mu_i l_i}} A_{n_i l_i} (y_i)^{l_i} \\
&\times H_{p+1,q+1:p_1,q_1;\ldots;p_r,q_r}^{0,n+1:m_1,n_1;\ldots;m_r,n_r} \left[\begin{array}{c} \frac{Z_1}{(4ab+c)^{\eta_1}} \\ \vdots \\ \frac{Z_r}{(4ab+c)^{\eta_r}} \end{array} \middle| \begin{array}{l} \left(-1/2-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(a_j; \alpha'_j, \ldots, \alpha_j^{(r)}\right)_{1,p} : \left(c'_j, \gamma'_j\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(b_j; \beta'_j, \ldots, \beta_j^{(r)}\right)_{1,q} : \left(d'_j, \delta'_j\right)_{1,q_1}; \ldots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{array} \right],
\end{aligned} \tag{13}$$

where e_i *is defined in (5)*

Proof. By virtue of equation (1), (7), (8) and (12), we have the following:

$$\begin{aligned}
&\int_0^\infty X^{-\lambda-1} {}_2F_1\left(a, b, c + \frac{1}{2}; X\right) {}_2F_1\left(c - a, c - b, c + \frac{1}{2}; X\right) S_{n_1, \ldots, n_r}^{m_1, \ldots, m_r}\left[\prod_{i=1}^{k} y_{i X^{-\mu_i}}\right] \\
&\times H\left[Z_1 X^{-\eta_1}, \ \ldots, Z_r X^{-\eta_r}\right] dX \\
&= \int_0^\infty X^{-\lambda-1} \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r x^r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i l_i} (y_i)^{l_i} (X^{-\mu_i})^{l_i} \\
&\times \frac{1}{(2\pi\omega)^r} \int_{\mathcal{L}_1} \cdots \int_{\mathcal{L}_r} \psi(\xi_1, \ \ldots, \xi_r) \left\{\prod_{i=1}^{r} \phi_i(\xi_i) \left[Z_i \, X^{-\eta_i}\right]^{\xi_i}\right\} d\xi_1 \ldots d\xi_r \\
&= \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i \, l_i}}{l_i!} A_{n_i l_i} (y_i)^{l_i}
\end{aligned}$$

$$\times \frac{1}{(2\pi\omega)^r} \int_{\mathcal{L}_1} \cdots \int_{\mathcal{L}_r} \psi(\xi_1 \ldots \xi_r) \left\{ \prod_{i=1}^{r} \phi_i(\xi_i) [Z_i]^{\xi_i} \right\} d\xi_1 \ldots d\xi_r \int_0^\infty X^{-\lambda+r-p-q-1} dX,$$

where $p = \sum_{i=1}^{k} \mu_i l_i$ and $q = \sum_{i=1}^{k} \eta_i \xi_i$

$$= \sum_{r=0}^{\infty} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i l_i} (y_i)^{l_i} \frac{1}{(2\pi\omega)^r} \int_{\mathcal{L}_1} \cdots \int_{\mathcal{L}_r} \psi(\xi_1, \ldots, \xi_r)$$

$$\times \left\{ \prod_{i=1}^{r} \phi_i(\xi_i) [Z_i]^{\xi_i} \right\} d\xi_1 \ldots d\xi_r \; \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda-r+p+q+1/2}} \frac{\Gamma(\lambda-r+p+q+1/2)}{\Gamma(1+\lambda-r+p+q)}$$

$$= \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!}$$

$$\times \frac{1}{(4ab+c)^p} A_{n_i l_i} (y_i)^{l_i} \frac{1}{(2\pi\omega)^r} \int_{\mathcal{L}_1} \cdots \int_{\mathcal{L}_r} \psi(\xi_1, \ldots, \xi_r) \left\{ \prod_{i=1}^{r} \phi_i(\xi_i) [Z_i]^{\xi_i} \right\} d\xi_1 \ldots d\xi_r$$

$$\times \frac{1}{(4ab+c)^q} \frac{\Gamma(\lambda-r+p+q+1/2)}{\Gamma(1+\lambda-r+p+q)}$$

$$= \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k} \mu_i l_i}}$$

$$\times A_{n_i l_i} (y_i)^{l_i} \frac{1}{(2\pi\omega)^r} \int_{\mathcal{L}_1} \cdots \int_{\mathcal{L}_r} \psi(\xi_1, \ldots, \xi_r) \left\{ \prod_{i=1}^{r} \phi_i(\xi_i) \left[\frac{Z_i}{(4ab+c)^{\eta_i}} \right]^{\xi_i} \right\} d\xi_1 \ldots d\xi_r$$

$$\times \frac{\Gamma(\lambda-r+p+q+1/2)}{\Gamma(1+\lambda-r+p+q)}$$

$$= \frac{\sqrt{\pi}}{2a\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r$$

$$\times \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k} \mu_i l_i}} A_{n_i l_i} (y_i)^{l_i}$$

$$\times H^{0,n+1:m_1,n_1;\ldots;m_r,n_r}_{p+1,q+1:p_1,q_1;\ldots;p_r,q_r} \left[\begin{array}{c} \frac{Z_1}{(4ab+c)^{\eta_1}} \\ \vdots \\ \frac{Z_r}{(4ab+c)^{\eta_r}} \end{array} \left| \begin{array}{l} \left(-1/2-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(a_j; \alpha'_j, \ldots, \alpha_j^{(r)}\right)_{1,p} : \left(c'_j, \gamma'_j\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(b_j; \beta'_j, \ldots, \beta_j^{(r)}\right)_{1,q} : \left(d'_j, \delta'_j\right)_{1,q_1}; \ldots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{array} \right. \right].$$

□

This completes the proof of Theorem 2.1.
If we set $n = p = q = 0$ then by virtue of the identity (6), we obtain

Corollary 2.2. *If $a \geq 0$; $b > 0$; $c + 4ab > 0$ and $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0$ $(i = 1, \ldots, r)$ and $-\frac{1}{2} < (a - b - c) < \frac{1}{2}$ and $X = \left(ax + \frac{b}{x}\right)^2 + c$ then there holds the following result:*

$$\int_0^\infty X^{-\lambda-1} {}_2F_1\left(a, b, c + \frac{1}{2}; X\right) {}_2F_1\left(c - a, c - b, c + \frac{1}{2}; X\right) S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]$$
$$\times \prod_{i=1}^{r} H_{p_i,q_i}^{m_i,n_i}\left[Z_i X^{-\eta_i} \middle| \begin{matrix} \left(c_j^{(i)}, \gamma_j^{(i)}\right)_{1,p_i} \\ \left(d_j^{(i)}, \delta_j^{(i)}\right)_{1,q_i} \end{matrix}\right]$$
$$= \frac{\sqrt{\pi}}{2a\,(4ab + c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab + c)^{-r}} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r$$
$$\times \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab + c)^{\sum_{i=1}^{k} \mu_i l_i}} A_{n_i l_i} (y_i)^{l_i}$$
$$\times H_{1,1:p_1,q_1;\ldots;p_r,q_r}^{0,1:m_1,n_1;\ldots;m_r,n_r}\left[\frac{Z_i}{(4ab + c)^{\eta_i}} \middle| \begin{matrix} \left(-1/2-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right): \left(c'_j, \gamma'_j\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right): \left(d'_j, \delta'_j\right)_{1,q_1}; \ldots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right]. \tag{14}$$

where e_i is defined in (5).

Following a similar procedure the following Theorem 2.3 and 2.5 can be proved.

Theorem 2.3. *Let $a \geq 0$, $b > 0$, $c + 4ab > 0$, $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0$ $(i = 1, \ldots, r)$, $-\frac{1}{2} < (a - b - c) < \frac{1}{2}$; and $X = \left(ax + \frac{b}{x}\right)^2 + c$ the following formula holds:*

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(a, b, c + \frac{1}{2}; X\right) {}_2F_1\left(c - a,\ c - b,\ c + \frac{1}{2}; X\right)$$
$$\times S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H\left[Z_1 X^{-\eta_1}, \ldots, Z_r X^{-\eta_r}\right] = \frac{\sqrt{\pi}}{2b\,(4ab + c)^{\lambda+1/2}}$$
$$\times \sum_{r=0}^{\infty} \frac{1}{(4ab + c)^{-r}} \frac{(c)_r}{\left(c + \frac{1}{2}\right)_r} a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab + c)^{\sum_{i=1}^{k} \mu_i l_i}} A_{n_i l_i} (y_i)^{l_i}$$
$$\times H_{p+1,q+1:p_1,q_1;\ldots;p_r,q_r}^{0,n+1:m_1,n_1;\ldots;m_r,n_r}\left[\begin{matrix} \frac{Z_1}{(4ab+c)^{\eta_1}} \\ \vdots \\ \frac{Z_r}{(4ab+c)^{\eta_r}} \end{matrix} \middle| \begin{matrix} \left(-1/2-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(a_j; \alpha'_j, \ldots, \alpha_j^{(r)}\right)_{1,p}: \left(c'_j, \gamma'_j\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s} \mu_i l_i; \eta_1, \ldots, \eta_r\right), \left(b_j; \beta'_j, \ldots, \beta_j^{(r)}\right)_{1,q}: \left(d'_j, \delta'_j\right)_{1,q_1}; \ldots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right]. \tag{15}$$

If we set $n = p = q = 0$ then by virtue of the identity (6), then we obtain the following:

Corollary 2.4. *If $a \geq 0$; $b > 0$; $c + 4ab > 0$ and $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0$ $(i = 1, \ldots, r)$ and*

$-\frac{1}{2} < (a-b-c) < \frac{1}{2}$ *and* $X = \left(ax + \frac{b}{x}\right)^2 + c$ *then there holds the following result:*

$$\begin{aligned}
&\int_0^\infty \frac{1}{x^2}\, X^{-\lambda-1}\, {}_2F_1\left(a, b, c+\frac{1}{2};\, X\right) {}_2F_1\left(c-a, c-b, c+\frac{1}{2};\, X\right) S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] \\
&\times \prod_{i=1}^{r} H_{p_i,q_i}^{m_i,n_i}\left[Z_i X^{-\eta_i} \,\middle|\, \begin{matrix} \left(c_j^{(i)}, \gamma_j^{(i)}\right)_{1,p_i} \\ \left(d_j^{(i)}, \delta_j^{(i)}\right)_{1,q_i} \end{matrix}\right] \\
&= \frac{\sqrt{\pi}}{2b\,(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\, a_r \\
&\times \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k} \mu_i l_i}}\, A_{n_i l_i}\, (y_i)^{l_i} \\
&\times H_{1,1:p_1,q_1;\dots;p_r,q_r}^{0,1:m_1,n_1;\dots;m_r,n_r}\left[\frac{Z_i}{(4ab+c)^{\eta_i}} \,\middle|\, \begin{matrix} \left(-1/2-\lambda+r-\sum_{i=1}^{s}\mu_i l_i;\, \eta_1,\dots,\eta_r\right): \left(c'_j, \gamma'_j\right)_{1,p_1}; \dots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s}\mu_i l_i;\, \eta_1,\dots,\eta_r\right): \left(d'_j, \delta'_j\right)_{1,q_1}; \dots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right].
\end{aligned} \tag{16}$$

Theorem 2.5. *Let* $a > 0$, $b > 0$, $c + 4ab > 0$, $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0 \quad (i = 1, \dots, r)$, $-\frac{1}{2} < (a-b-c) < \frac{1}{2}$; *and* $X = \left(ax + \frac{b}{x}\right)^2 + c$ *the following formula holds:*

$$\begin{aligned}
&\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(a, b, c+\frac{1}{2};\, X\right) {}_2F_1\left(c-a,\, c-b,\, c+\frac{1}{2};\, X\right) \\
&\times S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H\left[Z_1 X^{-\eta_1}, \dots, Z_r X^{-\eta_r}\right] = \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \\
&\times \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r}\, a_r \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k} \mu_i l_i}}\, A_{n_i l_i}\, (y_i)^{l_i} \\
&\times H_{p+1,q+1:p_1,q_1;\dots;p_r,q_r}^{0,n+1:m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} \frac{Z_1}{(4ab+c)^{\eta_1}} \\ \vdots \\ \frac{Z_r}{(4ab+c)^{\eta_r}} \end{matrix} \,\middle|\, \begin{matrix} \left(-1/2-\lambda+r-\sum_{i=1}^{s}\mu_i l_i;\eta_1,\dots,\eta_r\right), \left(a_j;\alpha'_j,\dots,\alpha_j^{(r)}\right)_{1,p}: \left(c'_j, \gamma'_j\right)_{1,p_1}; \dots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s}\mu_i l_i;\eta_1,\dots,\eta_r\right), \left(b_j;\beta'_j,\dots,\beta_j^{(r)}\right)_{1,q}: \left(d'_j, \delta'_j\right)_{1,q_1}; \dots; \left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r} \end{matrix}\right],
\end{aligned} \tag{17}$$

where e_i *is defined in (5)*

If we set $n = p = q = 0$ then by virtue of the identity (6), we obtain

Corollary 2.6. *If* $a > 0$; $b > 0$; $c + 4ab > 0$ *and* $\mu_i > 0$, $\eta \geq 0$, $\Re(\lambda) + \frac{1}{2} > 0$, $\Re(\rho + \mu_i e_i) > 0$ $(i = 1, \dots, r)$ *and*

$-\frac{1}{2} < (a-b-c) < \frac{1}{2}$ *and* $X = \left(ax + \frac{b}{x}\right)^2 + c$ *then there holds*

$$\int_0^\infty \left(a + \frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(a, b, c + \frac{1}{2}; X\right) {}_2F_1\left(c-a, c-b, c+\frac{1}{2}; X\right)$$
$$\times S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] \prod_{i=1}^{r} H_{p_i,q_i}^{m_i,n_i}\left[Z_i X^{-\eta_i} \left| \begin{matrix} \left(c_j^{(i)}, \gamma_j^{(i)}\right)_{1,p_i} \\ \left(d_j^{(i)}, \delta_j^{(i)}\right)_{1,q_i} \end{matrix} \right.\right]$$
$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \frac{1}{(4ab+c)^{-r}} \frac{(c)_r}{\left(c+\frac{1}{2}\right)_r} a_r$$
$$\times \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} \frac{1}{(4ab+c)^{\sum_{i=1}^{k}\mu_i l_i}} A_{n_i l_i} (y_i)^{l_i}$$
$$\times H_{1,1:p_1,q_1;\dots;p_r,q_r}^{0,1:m_1,n_1;\dots;m_r,n_r}\left[\frac{Z_i}{(4ab+c)^{\eta_i}} \left| \begin{matrix} \left(-1/2-\lambda+r-\sum_{i=1}^{s}\mu_i l_i; \eta_1,\dots,\eta_r\right):\left(c'_j,\gamma'_j\right)_{1,p_1};\dots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r} \\ \left(-\lambda+r-\sum_{i=1}^{s}\mu_i l_i; \eta_1,\dots,\eta_r\right):\left(d'_j,\delta'_j\right)_{1,q_1};\dots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r} \end{matrix} \right.\right], \tag{18}$$

where e_i *is defined in (5).*

Remark 2.7. *If we further take* $r = 1$ *in Corollaries 2.2, 2.4 and 2.6, then we can easily obtain the results in term of single H-function.*

3. Conclusion

In the present paper we investigate the generalized fractional calculus involving definite integrals of Gradshteyn-Ryzhik of the Multivariable H-function. We can also obtain the number of special functions as the special cases of our main results, which are related with Multivariable H-function.

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