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Fixed Point Results for Modified Various Contractions in Fuzzy Metric Spaces via α-Admissible

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Abstract. In this paper, we prove the existence of a fixed point for some new classes of α-admissible contraction mappings in fuzzy metric spaces. Our results generalize and extend some well-known results on the topic in the literature. Moreover, we present some examples to illustrate the usability of the obtained results.

1. Introduction

The concept of a fuzzy metric space was introduced by Kramosil and Michalek [1]. Afterwards, George and Veeramani [2] modified the concept of fuzzy metric space due to [1]. Later on, Gregori and Sapene[3] introduced fuzzy contraction mappings and proved a fixed point theorem in fuzzy metric space in the sense of George and Veeramani. In particular, Mihet [4] enlarged the class of fuzzy contractive mappings of Gregori and Sapene[3] in a complete non-Archimedean fuzzy metric space. Over the years, it has been generalized in different directions by several mathematicians (see [5–14] and the references therein).

On the other hand, Samet *et al.*[15] first introduced the concept of α-admissible mapping for single valued mapping and Asl *et al.* [16] extended the concept of admissible for single valued mappings to multi-valued mappings. Later on, Salimi *et al.*[17], established fixed point theorems for α-admissible contractions mapping with respect to η on metric space. Very recently Hussain *et al.* [18] generalized the notions of α-admissible mapping with respect to η for single-valued and set valued contraction mappings

In this paper, we modified the concept of α^*-η^*-admissible mapping for β and ψ contractions mappings type in fuzzy metric space. Moreover, some examples are given to illustrate the usability of obtained results.

2. Preliminaries

Firstly, we recall the basic definitions and properties about fuzzy metrics.

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Definition 2.1 ([20]). A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions :

(T1) $*$ is associative and commutative,
(T2) $*$ is continuous,
(T3) $a * 1 = a$ for all $a \in [0,1]$,
(T4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Remark 2.2. A t-norm $*$ is called positive, if $a * b > 0$ for all $a, b \in (0,1)$.

Examples of continuous t-norms are Lukasievicz t-norm, that is, $a *_L b = \max\{a + b - 1, 0\}$, product t-norm, that is, $a *_P b = ab$ and minimum t-norm, that is, $a *_M b = \min\{a, b\}$.

The concept of fuzzy metric space is defined by George and Veeramani [2] as follows.

Definition 2.3 ([2]). Let X be an arbitrary nonempty set, $*$ is a continuous t-norm, and M is a fuzzy set on $X \times X \times (0,\infty)$.The 3-tuple $(X, M, *)$ is called a fuzzy metric space if satisfying the following conditions, for each $x, y, z \in X$ and $t, s > 0$,

(M1) $M(x, y, t) > 0$,
(M2) $M(x, y, t) = 1$ if and only if $x = y$,
(M3) $M(x, y, t) = M(y, x, t)$,
(M4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(M5) $M(x, y, \cdot) : (0,\infty) \to [0,1]$ is continuous.

Remark 2.4. It is worth pointing out that $0 < M(x, y, t) < 1$ (for all $t > 0$) provided $x \neq y$, (see [21]).

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with a center $x \in X$ and a radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}.$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is a topology on X, called the topology induced by the fuzzy metric M. This topology is metrizable (see in[22]).

Example 2.5 ([2]). Let (X, d) be a metric space. Define $a * b = ab$ (or $a * b = \min\{a, b\}$) for all $a, b \in [0,1]$, and define $M : X \times X \times (0,\infty) \to [0,1]$ as

$$M(x, y, t) = \frac{t}{t + d(x, y)},$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space. We call this fuzzy metric induced by the metric d the standard fuzzy metric.

Now we give some examples of fuzzy metric space due to Gregori et al. [7].

Example 2.6 ([7]). Let X be a nonempty set, $f : X \to \mathbb{R}^+$ be a one-one function and $g : [0,\infty) \to \mathbb{R}^+$ be an increasing continuous function. For fixed $\alpha, \beta > 0$, define $M : X \times X \times (0,\infty) \to [0,1]$ as

$$M(x, y, t) = \left(\frac{(\min\{f(x), f(y)\})^\alpha + g(t)}{(\max\{f(x), f(y)\})^\alpha + g(t)} \right)^\beta,$$

for all $x, y \in X$ and $t > 0$. Then, $(X, M, *)$ is a fuzzy metric space on X where $*$ is the product t-norm.

Example 2.7 ([7]). Let (X, d) be a metric space and $g : \mathbb{R}^+ \to [0,\infty)$ be an increasing continuous function. Define $M : X \times X \times (0,\infty) \to [0,1]$ as

$$M(x, y, t) = e^{\left(-\frac{d(x,y)}{g(t)}\right)},$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space on X where $*$ is the product t-norm.

Example 2.8 ([7]). Let (X, d) be a bounded metric space with $d(x, y) < k$ (for all $x, y \in X$, where k is fixed constant in $(0, \infty)$) and $g : \mathbb{R}^+ \to (k, \infty)$ be an increasing continuous function. Define a function $M : X \times X \times (0, \infty) \to [0, 1]$ as

$$M(x, y, t) = 1 - \frac{d(x, y)}{g(t)},$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space on X wherein $*$ is a Lukasievicz t-norm.

Definition 2.9 ([2]). Let $(X, M, *)$ be a fuzzy metric space.

(1) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.
(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if, for each $0 < \epsilon < 1$ and $t > 0$, there exits $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$.
(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.
(4) A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

Lemma 2.10 ([6]). *Let $(X, M, *)$ be a fuzzy metric space. For all $x, y \in X$, $M(x, y, \cdot)$ is non-decreasing function.*

Definition 2.11. Let $(X, M, *)$ be a fuzzy metric space. Then the mapping M is said to be continuous on $X \times X \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t)$$

when $\{(x_n, y_n, t_n)\}$ is a sequence in $X^2 \times (0, \infty)$ which converges to a point $(x, y, t) \in X^2 \times (0, \infty)$, i.e.,

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \quad \text{and} \quad \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

Lemma 2.12 ([23]). *If $(X, M, *)$ be a fuzzy metric space, then M is a continuous function on $X \times X \times (0, \infty)$.*

On the other hand, the concept of α-admissible mapping introduced by Samet et al. [15] as follows.

Definition 2.13 ([15]). Let X be a nonempty set, $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$. We say that T is α-admissible mapping if for all $x, y \in X$, we have

$$\alpha(x, y) \geq 1 \Longrightarrow \alpha(Tx, Ty) \geq 1.$$

Next, Samet et al. [15] modified the concept of α-admissible mapping as follows.

Definition 2.14 ([17]). Let X be a nonempty set, $T : X \to X$ and $\alpha, \eta : X \times X \to [0, \infty)$. We say that T is α-admissible mapping with respect to η if for all $x, y \in X$, we have

$$\alpha(x, y) \geq \eta(x, y) \Longrightarrow \alpha(Tx, Ty) \geq \eta(Tx, Ty).$$

Note that, if we take $\eta(x, y) = 1$, then this definition reduces to Definition 2.13. Also, if we take $\alpha(x, y) = 1$, then we say that T is an η-subadmissible.

Definition 2.15. Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ and let $\alpha^* : X \times X \times (0, \infty) \to [0, \infty)$ be function. We say that T is an α^*-admissible mapping if, for all $x, y \in X$ and $t > 0$,

$$\alpha^*(x, y, t) \geq 1 \Longrightarrow \alpha^*(Tx, Ty, t) \geq 1.$$

Definition 2.16. Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ and let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ be two functions. We say that T is an α^*-η^*-admissible mapping if, for all $x, y \in X$ and $t > 0$,

$$\alpha^*(x, y, t) \geq \eta^*(x, y, t) \Longrightarrow \alpha^*(Tx, Ty, t) \geq \eta^*(Tx, Ty, t);$$

Note that, If $\eta^*(x, y, t) = 1$ then this definition reduces to Definition 2.15. Also, if we take $\alpha^*(x, y, t) = 1$, then we say that T is an η^*-subadmissible.

3. Modified α^*-η^*-β-Contractions in Fuzzy Metric Spaces

First, we introduce the following notion:

Definition 3.1. Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ and let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ be two functions. We say that T is a modified α^*-η^*-β-contractive mapping if there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$, for all $x, y \in X$ and $t > 0$,

$$\begin{aligned}&\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geq \eta^*(x, Tx, t)\eta^*(x, Tx, t)\\ &\implies \quad M(Tx, Ty, t) \geq \beta\Big(M(x, y, t)\Big)N(x, y, t),\end{aligned} \tag{3.1}$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\}.$$

Now, we are ready to state and prove our first main theorem.

Theorem 3.2. *Let $(X, M, *)$ be a complete fuzzy metric space. The mapping $T : X \to X$ is a modified α^*-η^*-β-contractive mapping. Suppose that the following assertions hold:*

(a) T is α^*-η^*-admissible mapping;
(b) there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geq \eta^*(x_0, Tx_0, t)$ for all $t > 0$;
(c) for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq \eta^*(x_n, x_{n+1}, t)$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geq \eta^*(x, Tx, t)$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

Proof. Let $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geq \eta^*(x_0, Tx_0, t)$ for all $t > 0$. Define a sequence a sequence $\{x_n\}$ in X such that $x_n = T^n x_0 = Tx_{n-1}$, for all $n \in \mathbb{N}$. If $x_n = x_{n+1}$, then $x_n = Tx_n$ and so x_n is a fixed point of T and we are finished. Assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Since T is α^*-η^*-admissible mapping and $\alpha^*(x_0, Tx_0, t) \geq \eta^*(x_0, Tx_0, t)$ we deduce that

$$\alpha^*(x_1, x_2, t) = \alpha^*(Tx_0, Tx_1, t) \geq \eta^*(Tx_0, Tx_1, t) = \eta^*(x_1, x_2, t).$$

So, we have

$$\alpha^*(x_0, Tx_0, t)\alpha^*(x_1, Tx_1, t) \geq \eta^*(x_0, Tx_0, t)\eta^*(x_1, Tx_1, t).$$

By continuing this process, we have $\alpha^*(x_n, Tx_n, t) \geq \eta^*(x_n, Tx_n, t)$. So, we get

$$\alpha^*(x_{n-1}, Tx_{n-1}, t)\alpha^*(x_n, Tx_n, t) \geq \eta^*(x_{n-1}, Tx_{n-1}, t)\eta^*(x_n, Tx_n, t)$$

for all $n \in \mathbb{N}$ and $t > 0$. Now, from (3.1), we obtain that

$$\begin{aligned} M(x_n, x_{n+1}, t) &= M(Tx_{n-1}, Tx_n, t) \\ &\geq \beta\Big(M(x_{n-1}, x_n, t)\Big)N(x_{n-1}, x_n, t) \end{aligned}$$

where

$$\begin{aligned} N(x_{n-1}, x_n, t) &= \min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, Tx_{n-1}, t), M(x_n, Tx_n, t)\}\Big\} \\ &= \min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\Big\} \end{aligned}$$

for all $n \in \mathbb{N}$ and $t > 0$. If $M(x_{n-1}, x_n, t) \leq M(x_n, x_{n+1}, t)$ for some $n \in \mathbb{N}$, then

$$\min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\Big\} = M(x_{n-1}, x_n, t).$$

Also, if $M(x_n, x_{n+1}, t) < M(x_{n-1}, x_n, t)$ for some $n \in \mathbb{N}$, then

$$\min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\Big\} = M(x_{n-1}, x_n, t).$$

That is, for all $n \in \mathbb{N}$ and $t > 0$, we have

$$\min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\Big\} = M(x_{n-1}, x_n, t).$$

Hence,

$$\begin{aligned} M(x_n, x_{n+1}, t) &\geq \beta\Big(M(x_{n-1}, x_n, t)\Big)M(x_{n-1}, x_n, t) \\ &\geq M(x_{n-1}, x_n, t) \end{aligned} \tag{3.2}$$

for all $n \in \mathbb{N}$ and $t > 0$. It follows that the sequence $\{M(x_n, x_{n+1}, t)\}$ is an increasing sequence in $(0, 1]$. Thus, there exists $l \in (0, 1]$ such that $\lim_{n\to\infty} M(x_n, x_{n+1}, t) = l$ for all $t > 0$. We shall prove that $l = 1$ for all $t > 0$. From (3.2), we have

$$\frac{M(x_n, x_{n+1}, t)}{M(x_{n-1}, x_n, t)} \geq \beta\Big(M(x_{n-1}, x_n, t)\Big) \geq 1,$$

which implies that $\lim_{n\to\infty} \beta\Big(M(x_{n-1}, x_n, t)\Big) = 1$. Regarding the property of the function β, which implies that $l = 1$ and we conclude that

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1. \tag{3.3}$$

Next, we prove that $\{x_n\}$ is a Cauchy sequence. Suppose, to the contrary, that $\{x_n\}$ is not a Cauchy sequence. Then there exist $\varepsilon \in (0, 1)$ and $t_0 > 0$ such that, for all $k \geq 1$, there are $m(k), n(k) \in \mathbb{N}$ with $m(k) > n(k) \geq k$ and

$$M(x_{n(k)}, x_{m(k)}, t_0) \leq 1 - \varepsilon.$$

Assume that $m(k)$ is the least integer exceeding $n(k)$ satisfying the above inequality, that is, equivalently,

$$M(x_{n(k)}, x_{m(k)-1}, t_0) > 1 - \varepsilon.$$

By the (M4), we derive that

$$\begin{aligned} 1 - \varepsilon &\geq M(x_{n(k)}, x_{m(k)}, t_0) \\ &\geq M(x_{n(k)}, x_{m(k)-1}, t_0) * M(x_{m(k)-1}, x_{m(k)}, t_0) \\ &> (1 - \varepsilon) * M(x_{m(k)-1}, x_{m(k)}, t_0) \end{aligned}$$

for all $k \in \mathbb{N}$. Taking the limit as $k \to \infty$ in the above inequality and using (3.3), we get

$$\lim_{k\to+\infty} M(x_{n(k)}, x_{m(k)}, t_0) = 1 - \varepsilon. \tag{3.4}$$

Again, by M(4), we find that

$$\begin{aligned} M(x_{n(k+1)}, x_{m(k+1)}, t_0) &\geq M(x_{n(k+1)}, x_{n(k)}, t_0) * M(x_{n(k)}, x_{m(k)}, t_0) \\ &\quad * M(x_{m(k)}, x_{m(k+1)}, t_0) \end{aligned}$$

and

$$\begin{aligned} M(x_{n(k)}, x_{m(k)}, t_0) &\geq M(x_{n(k)}, x_{n(k+1)}, t_0) * M(x_{n(k+1)}, x_{m(k+1)}, t_0) \\ &\quad * M(x_{m(k+1)}, x_{m(k)}, t_0). \end{aligned}$$

Taking the limit as $k \to \infty$ in the above inequality, together with (3.3) and (3.4), we deduce that

$$\lim_{k\to+\infty} M(x_{n(k+1)}, x_{m(k+1)}, t_0) = 1 - \varepsilon. \tag{3.5}$$

Since,

$$\alpha(x_{n(k)}, Tx_{n(k)}, t)\alpha(x_{m(k)}, Tx_{m(k)}, t) \leq \eta(x_{n(k)}, Tx_{n(k)}, t)\eta(x_{m(k)}, Tx_{m(k)}, t).$$

Then from (3.1), (3.3), (3.4) and (3.5), we have

$$M(x_{n(k+1)}, x_{m(k+1)}, t_0) \geq \beta\Big(M(x_{n(k)}, x_{m(k)}, t_0)\Big)N(x_{n(k)}, x_{m(k)}, t_0),$$

where

$$\begin{aligned} & N(x_{n(k)}, x_{m(k)}, t_0) \\ &= \min\Big\{M(x_{n(k)}, x_{m(k)}, t_0), \max\{M(x_{n(k)}, Tx_{n(k)}, t_0), M(x_{m(k)}, Tx_{m(k)}, t_0)\}\Big\}. \end{aligned}$$

Hence,

$$\frac{M(x_{n(k+1)}, x_{m(k+1)}, t_0)}{N(x_{n(k)}, x_{m(k)}, t_0)} \geq \beta(M(x_{n(k)}, x_{m(k)}, t_0)) \geq 1.$$

Letting $k \to \infty$ in the above inequality, we get

$$\lim_{k\to+\infty} \beta(M(x_{n(k)}, x_{m(k)}, t_0)) = 1.$$

That is,

$$1 - \varepsilon = \lim_{k\to+\infty} M(x_{n(k)}, x_{m(k)}, t_0) = 1.$$

So, $\varepsilon = 0$, which is contradiction. Thus, $\{x_n\}$ is a Cauchy sequence and $(X, M, *)$ complete then there exists $x^\star \in X$ such that $x_n \to x^\star$ as $n \to \infty$, that means $M(x_n, x^\star, t) = 1$ as $n \to \infty$, for each $t > 0$. By condition (c), we have $\alpha(x^*, Tx^\star, t) \geq \eta(x^*, Tx^\star, t)$. So, we get

$$\alpha(x_n, Tx_n, t)\alpha(x^\star, Tx^\star, t) \geq \eta(x_n, Tx_n, t)\eta(x^\star, Tx^\star, t)$$

for all $n \in \mathbb{N} \cup \{0\}$ and $t > 0$. By (3.1), we have

$$M(Tx^\star, Tx_n, t) = M(Tx^\star, x_{n+1}, t) \geq \beta(M(x^\star, x_n, t))N(x^\star, x_n, t)$$

where

$$\begin{aligned} & N(x^\star, x_n, t) \\ &= \min\Big\{M(x^\star, x_n, t), \max\{M(x^\star, Tx^\star, t), M(x_n, Tx_n, t)\}\Big\} \\ &= \min\Big\{M(x^\star, x_n, t), \max\{M(x^\star, Tx^\star, t), M(x_n, x_{n+1}, t)\}\Big\}. \end{aligned}$$

Hence,

$$\begin{aligned} M(Tx^\star, x^\star, t) &\geq M(Tx^\star, x_{n+1}, t) * M(x_{n+1}, x_n, t) * M(x_n, x^\star, t) \\ &\geq \beta(M(x^\star, x_n, t))N(x^\star, x_n, t) * M(x_{n+1}, x_n, t) * M(x_n, x^\star, t) \end{aligned}$$

Letting $n \to \infty$ in the above inequality, we get $M(Tx^\star, x^\star, t) = 1$, that is, $Tx^\star = x^\star$. This completes the proof.

By taking $\eta^*(x, y, t) = 1$ in Theorem 3.2, we have the following result.

Corollary 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be α^*-admissible mapping. Assume that there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$, and*

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geq 1 \Longrightarrow M(Tx, Ty, t) \geq \beta\Big(M(x, y, t)\Big)N(x, y, t),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\alpha^*(x_0, x_1, t) \geq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x_n, x, t) \geq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

Corollary 3.4. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ and let $\alpha^* : X \times X \times (0, \infty) \to [0, \infty)$ be α^*-admissible mapping. Assume that there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$ and*

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t)M(Tx, Ty, t) \geq \beta\Big(M(x, y, t)\Big)N(x, y, t),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\alpha^*(x_0, x_1, t) \geq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x_n, x, t) \geq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

By taking $\alpha^*(x, y, t) = 1$ in Theorem 3.2, we have the following result.

Corollary 3.5. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be η^*-subadmissible mapping. Assume that there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$ and*

$$\eta^*(x, Tx, t)\eta^*(x, Tx, t) \leq 1 \Longrightarrow M(Tx, Ty, t) \geq \beta\Big(M(x, y, t)\Big)N(x, y, t),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\eta^*(x_0, x_1, t) \leq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \subset X$ such that $\eta^*(x_n, x_{n+1}, t) \leq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\eta^*(x_n, x, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

Corollary 3.6. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be η^*-subadmissible mapping. Assume that there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$ and*

$$M(Tx, Ty, t) \geq \eta^*(x, Tx, t)\eta^*(x, Tx, t)\beta\Big(M(x, y, t)\Big)N(x, y, t),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\eta^*(x_0, x_1, t) \leq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \subset X$ such that $\eta^*(x_n, x_{n+1}, t) \leq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\eta^*(x_n, x, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

If we take $\alpha^*(x, y, t) = 1$ in Corollary 3.4 and $\eta^*(x, y, t) = 1$ in Corollary 3.6, we have the following result of Geragty type contraction in fuzzy metric space.

Corollary 3.7. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$. Assume that there exists a function $\beta : [0, 1] \to [1, \infty)$ such that, for any sequence $\{s_n\} \subset [0, 1]$ of positive reals, $\beta(s_n) \to 1$ implies $s_n \to 1$ and*

$$M(Tx, Ty, t) \geq \beta\Big(M(x, y, t)\Big)N(x, y, t),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Then T has a fixed point.

Now, we give an example to support Theorem 3.2.

Example 3.8. Let $X = [0, \infty)$ and define $d(x, y) = |x - y|$. Denote $a * b = ab$ for any $a, b \in [0, 1]$ and

$$M(x, y, t) = \Big(\frac{t}{t + 1}\Big)^{d(x,y)}$$

for $x, y \in X$ and $t > 0$. Then it easy to see that $(X, M, *)$ is complete fuzzy metric space [2]. Define the mapping $T : X \to X$ by

$$Tx = \begin{cases} \frac{2x}{3} & , x \in [0, 1], \\ \sinh x & , x \in (1, \infty). \end{cases}$$

Define $\alpha, \eta : X \times X \times (0, \infty) \to [0, \infty)$ by

$$\alpha^*(x, y, t) = \begin{cases} 2 & , 0 \leq x, y \leq 1, \\ 0 & , otherwise. \end{cases}$$

Also, define

$$\eta^*(x, y, t) = \begin{cases} 1 & , 0 \leq x, y \leq 1, \\ -2 & , otherwise. \end{cases}$$

We show that T is an α^*-η^*-admissible mapping. Let $x, y \in X$ with

$$\alpha^*(x, y, t) \geq \eta^*(x, y, t),$$

then $x, y \in [0, 1]$. On the other hand, for all $x, y \in [0, 1]$, we have $Tx \leq 1$. It follows that

$$\alpha^*(Tx, Ty, t) \geq \eta^*(Tx, Ty, t).$$

Hence, T is an α^*-η^*-admissible mapping. Also, $\alpha^*(0, T0, t) \geq \eta^*(0, T0, t)$. Let $\{x_n\}$ is a sequence in X such that $\alpha^*(x_n, x_{n+1}, t) \geq \eta^*(x_n, x_{n+1}, t)$ for all $n \in \mathbb{N} \cup \{0\}$ and $x_n \to x$ as $n \to \infty$, then $\{x_n\} \subset [0, 1]$, and hence $x \in [0, 1]$. This implies that $\alpha^*(x_n, x, t) \geq \eta^*(x_n, x, t)$ for all $n \in \mathbb{N}$ and $t > 0$. Now, let $\alpha^*(x, Tx, t) \geq \eta^*(x, Tx, t)$ and $\alpha^*(y, Ty, t) \geq \eta^*(y, Ty, t)$. Also, we get

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geq \eta^*(x, Tx, t)\eta^*(y, Ty, t).$$

Then $x, y \in [0,1]$. This implies that

$$\begin{aligned} M(Tx,Ty,t) &= \Big(\frac{t}{t+1}\Big)^{d(Tx,Ty)} \\ &= \Big(\frac{t}{t+1}\Big)^{|Tx-Ty|} \\ &= \Big(\frac{t}{t+1}\Big)^{|\frac{2x}{3}-\frac{2y}{3}|} \\ &\geq \beta\Big(\Big(\frac{t}{t+1}\Big)^{\frac{1}{3}|x-y|}\Big)\Big(\frac{t}{t+1}\Big)^{\frac{1}{3}|x-y|} \\ &\geq \beta\Big(\Big(\frac{t}{t+1}\Big)^{|x-y|}\Big)\Big(\frac{t}{t+1}\Big)^{|x-y|} \\ &= \beta\Big(\Big(\frac{t}{t+1}\Big)^{d(x,y)}\Big)\Big(\frac{t}{t+1}\Big)^{d(x,y)} \\ &= \beta\Big(M(x,y,t)\Big)M(x,y,t). \end{aligned}$$

Therefore, all of conditions in Theorem 3.2 are satisfied with $\beta : [0,1] \to [1,\infty)$. We can see that Corollary 3.3 and Corollary 3.4 can be applicable to this example. In this example, we have $0 \in X$ is a fixed point to T. This completes the proof.

Next, we show that contractive condition in Corollary 3.7 cannot be applied to this example. Indeed, for $x = 0$, $y = 3$, $\beta(s) = 1$ and $t > 0$, we obtain

$$M(T0, T3, t) = (\frac{t}{t+1})^{\sinh 3} < (\frac{t}{t+1})^{3} = M(0,3,t),$$

Therefore, Corollary 3.7 cannot be applied to this case.

4. Modified α^*-η^*-ψ-Contractions in Fuzzy Metric Spaces

Let Ψ be the class of all mappings $\psi : [0,1] \to [0,1]$ such that ψ is continuous, nondecreasing and $\psi(s) > s$ for all $s \in [0,1]$.

Definition 4.1. Let $(X, M, *)$ be a fuzzy metric space and $\psi \in \Psi$. A mapping $T : X \to X$ and let $\alpha^*, \eta^* : X \times X \times (0,\infty) \to [0,\infty)$ be two functions. We say that T is a modified α^*-η^*-ψ-contractive mapping if the following implication takes place:

$$\begin{aligned} &\alpha^*(x,Tx,t)\alpha^*(y,Ty,t) \geq \eta^*(x,Tx,t)\eta^*(x,Tx,t) \\ &\quad \Longrightarrow \quad M(Tx,Ty,t) \geq \psi\Big(N(x,y,t))\Big), \end{aligned} \tag{4.1}$$

where

$$N(x,y,t) = \min\Big\{M(x,y,t), \max\{M(x,Tx,t), M(y,Ty,t)\}\Big\}.$$

Now, we prove some fixed point theorem for modified α^*-η^*-ψ-contraction mapping in fuzzy metric spaces.

Theorem 4.2. *Let $(X, M, *)$ be a complete fuzzy metric space. The mapping $T : X \to X$ is a modified α^*-η^*-ψ-contractive mapping. Suppose that the following assertions hold:*

(a) T is α^*-η^*-admissible mapping;

(b) there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geq \eta^*(x_0, Tx_0, t)$ for all $t > 0$;

(c) for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq \eta^*(x_n, x_{n+1}, t)$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geq \eta^*(x, Tx, t)$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

Proof. Following the same lines in the proof of Theorem 3.2, we have

$$\alpha^*(x_n, Tx_n, t) \geq \eta^*(x_n, Tx_n, t).$$

So, we get

$$\alpha^*(x_{n-1}, Tx_{n-1}, t)\alpha^*(x_n, Tx_n, t) \geq \eta^*(x_{n-1}, Tx_{n-1}, t)\eta^*(x_n, Tx_n, t)$$

for all $n \in \mathbb{N}$ and $t > 0$. It follows from 4.2, we have

$$\begin{aligned} M(x_n, x_{n+1}, t) &= M(Tx_{n-1}, Tx_n, t) \\ &\geq \psi\Big(N(x_{n-1}, x_n, t))\Big) \end{aligned}$$

where

$$\begin{aligned} N(x_{n-1}, x_n, t) &= \min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, Tx_{n-1}, t), M(x_n, Tx_n, t)\}\Big\} \\ &= \min\Big\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\Big\} \\ &= M(x_{n-1}, x_n, t) \end{aligned}$$

for all $n \in \mathbb{N}$ and $t > 0$. Hence,

$$M(x_n, x_{n+1}, t) \geq \psi\Big(M(x_{n-1}, x_n, t)\Big) > M(x_{n-1}, x_n, t)$$

for all $n \in \mathbb{N}$ and $t > 0$. Hence, $\{M(x_n, x_{n+1}, t)\}$ is an increasing sequence in $(0, 1]$. Thus, there exists $l \in (0, 1]$ such that $\lim_{n\to\infty} M(x_n, x_{n+1}, t) = l$ for all $t > 0$. Now, we prove that $l = 1$ for all $t > 0$. From (4.2), we have

$$M(x_n, x_{n+1}, t) \geq \psi\Big(M(x_{n-1}, x_n, t)\Big).$$

Since, ψ is continuous, $l \geq \psi(l)$. This implies that $l = 1$ and therefore

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = 1$$

for all $n \in \mathbb{N}$ and $t > 0$.

Next, we prove that $\{x_n\}$ is a Cauchy sequence. Since,

$$\alpha(x_{n(k)}, Tx_{n(k)}, t)\alpha(x_{m(k)}, Tx_{m(k)}, t) \leq \eta(x_{n(k)}, Tx_{n(k)}, t)\eta(x_{m(k)}, Tx_{m(k)}, t).$$

By (4.2), (3.3), (3.4) and (3.5), we have

$$M(x_{n(k+1)}, x_{m(k+1)}, t_0) \geq \psi\Big(N(x_{n(k)}, x_{m(k)}, t_0)\Big) \geq 1,$$

where

$$\begin{aligned} &N(x_{n(k)}, x_{m(k)}, t_0) \\ &\quad= \min\Big\{M(x_{n(k)}, x_{m(k)}, t_0), \max\{M(x_{n(k)}, Tx_{n(k)}, t_0), M(x_{m(k)}, Tx_{m(k)}, t_0)\}\Big\}. \end{aligned}$$

Letting $k \to +\infty$ in the above inequality, we get

$$1 - \varepsilon \geq \psi(1 - \varepsilon) > 1 - \varepsilon$$

which is contradiction. Thus, $\{x_n\}$ is a Cauchy sequence and $(X, M, *)$ complete then there exists $x^\star \in X$ such that $x_n \to x^\star$ as $n \to \infty$, that means $M(x_n, x^\star, t) = 1$ as $n \to \infty$, for each $t > 0$. By condition (c), we have $\alpha(x^*, Tx^*, t) \geq \eta(x^*, Tx^*, t)$. So, we get

$$\alpha(x_n, Tx_n, t)\alpha(x^\star, Tx^\star, t) \geq \eta(x_n, Tx_n, t)\eta(x^\star, Tx^\star, t)$$

for all $n \in \mathbb{N} \cup \{0\}$ and $t > 0$. By (3.1), we have

$$M(Tx^\star, Tx_n, t) = M(Tx^\star, x_{n+1}, t) \geq \psi(N(x^\star, x_n, t))$$

where

$$\begin{aligned} & N(x^\star, x_n, t) \\ = & \min\Big\{M(x^\star, x_n, t), \max\{M(x^\star, Tx^\star, t), M(x_n, Tx_n, t)\}\Big\} \\ = & \min\Big\{M(x^\star, x_n, t), \max\{M(x^\star, Tx^\star, t), M(x_n, x_{n+1}, t)\}\Big\}. \end{aligned}$$

Hence,

$$\begin{aligned} M(Tx^\star, x^\star, t) & \geq M(Tx^\star, x_{n+1}, t) * M(x_{n+1}, x_n, t) * M(x_n, x^\star, t) \\ & \geq \psi(N(x^\star, x_n, t)) * M(x_{n+1}, x_n, t) * M(x_n, x^\star, t) \end{aligned}$$

Letting $n \to \infty$ in the above inequality, we get $M(Tx^\star, x^\star, t) = 1$, that is, $Tx^\star = x^\star$. This completes the proof.

By taking $\eta^*(x, y, t) = 1$ in Theorem 4.2, we have the following result.

Corollary 4.3. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be α^*-admissible mapping and*

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geq 1 \Longrightarrow M(Tx, Ty, t) \geq \psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\}.$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\alpha^*(x_0, x_1, t) \geq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \in X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x_n, x, t) \geq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

Corollary 4.4. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be α^*-admissible mapping and*

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t)M(Tx, Ty, t) \geq \psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\}.$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\alpha^*(x_0, x_1, t) \geq 1$ for all $t > 0$;

(b) for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geq 1$, for all $n \in \mathbb{N}$, $t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x_n, x, t) \geq 1$ for all $n \in \mathbb{N}$ and $t > 0$.

Then T has a fixed point.

By taking $\alpha^*(x, y, t) = 1$ in Theorem 4.2, we have the following result.

Corollary 4.5. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be η^*-subadmissible mapping and*

$$\eta^*(x, Tx, t)\eta^*(x, Tx, t) \leq 1 \Longrightarrow M(Tx, Ty, t) \geq \psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold:

(a) there exists $x_0 \in X$ such that $\eta^*(x_0, x_1, t) \leq 1$ for all $t > 0$;

(b) *for any sequence* $\{x_n\} \subset X$ *such that* $\eta^*(x_n, x_{n+1}, t) \leq 1$, *for all* $n \in \mathbb{N}$, $t > 0$ *and* $x_n \to x$ *as* $n \to \infty$, *then* $\eta^*(x_n, x, t) \leq 1$ *for all* $n \in \mathbb{N}$ *and* $t > 0$.

Then T has a fixed point.

Corollary 4.6. *Let* $(X, M, *)$ *be a complete fuzzy metric space. A mapping* $T : X \to X$ *be* η^**-subadmissible mapping and*

$$M(Tx, Ty, t) \geq \eta^*(x, Tx, t)\eta^*(x, Tx, t)\psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ *and* $t > 0$. *Suppose that the following assertions hold:*

(a) *there exists* $x_0 \in X$ *such that* $\eta^*(x_0, x_1, t) \leq 1$ *for all* $t > 0$;

(b) *for any sequence* $\{x_n\} \subset X$ *such that* $\eta^*(x_n, x_{n+1}, t) \leq 1$, *for all* $n \in \mathbb{N}$, $t > 0$ *and* $x_n \to x$ *as* $n \to \infty$, *then* $\eta^*(x_n, x, t) \leq 1$ *for all* $n \in \mathbb{N}$ *and* $t > 0$.

Then T has a fixed point.

If we take $\alpha^*(x, y, t) = 1$ in Corollary 4.4 and $\eta^*(x, y, t) = 1$ in Corollary 4.6, we have the following result of Mihet [4] type contraction in fuzzy metric space.

Corollary 4.7. *Let* $(X, M, *)$ *be a complete fuzzy metric space. A mapping* $T : X \to X$. *and*

$$M(Tx, Ty, t) \geq \psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\Big\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\Big\},$$

for all $x, y \in X$ *and* $t > 0$. *Then T has a fixed point.*

Now, we give an example to support Theorem 4.2.

Example 4.8. Let $X = [0, \infty)$ and define $a * b = ab$ for any $a, b \in [0, 1]$. The fuzzy metric

$$M(x, y, t) = e^{(-\frac{|x-y|}{t})}$$

for $x, y \in X$ and $t > 0$. Define the mapping $T : X \to X$ by

$$Tx = \begin{cases} \frac{x^2}{4} & , x \in [0, 1], \\ 2x^2 + 1 & , x \in (1, \infty). \end{cases}$$

Let $\psi(s) = \sqrt{s}$ for $s \in [0, 1]$, then $\psi \in \Psi$. Define $\alpha, \eta : X \times X \times (0, \infty) \to [0, \infty)$ by $\alpha \equiv 1$,

$$\eta^*(x, y, t) = \begin{cases} \frac{1}{3} & , x, y \in [0, 1], \\ 1 & , x, y \in (1, \infty). \end{cases}$$

We show that T is an η^*-subadmissible mapping. Let $x, y \in X$ with

$$\eta^*(x, y, t) \leq 1,$$

then $x, y \in [0, 1]$. On the other hand, for all $x, y \in [0, 1]$, we have $Tx \geq 1$. It follows that

$$\eta^*(Tx, Ty, t) \leq 1.$$

Hence, T is an η^*-subadmissible mapping. Also, $\eta^*(1, T1, t) \leq 1$. Let $\{x_n\}$ is a sequence in X such that $\alpha^*(x_n, x_{n+1}, t) \geq \eta^*(x_n, x_{n+1}, t)$ for all $n \in \mathbb{N} \cup \{0\}$ and $x_n \to x$ as $n \to \infty$, then $\{x_n\} \subset [0, 1]$, which implies that

$x \geq 1$. This implies that $\eta^*(x_n, x, t) \leq 1$ for all $n \in \mathbb{N}$ and $t > 0$. Now, let $\eta^*(x, Tx, t) \leq 1$ and $\eta^*(y, Ty, t) \leq 1$. Also, we get

$$\eta^*(x, Tx, t)\eta^*(y, Ty, t) \leq 1.$$

Then $x, y \in [0, 1]$. This implies that

$$M(Tx, Ty, t) = e^{(-\frac{|Tx-Ty|}{t})} = e^{(-\frac{|x^2-y^2|}{4t})} \geq e^{(-\frac{|x-y|}{2t})} = \psi(M(x, y, t)).$$

Therefore, all of conditions in Corollary 4.5 and Corollary 4.6 can be applicable to this example. In this example, we have $0 \in X$ is a fixed point to T. This completes the proof.

Next, we show that contractive condition in Corollary 3.7 cannot be applied to this example. Indeed, for $x = 0$, $y = 2$, $\psi(s) > s$ and $t = 1$, we obtain

$$M(T0, T6, t) = e^{(-\frac{|0-9|}{t})} = e^{(-\frac{9}{t})} < e^{(-\frac{1}{t})} = \psi(M(0, 2, t)).$$

Therefore, Corollary 3.7 cannot be applied to this case.

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