Filomat 30:8 (2016), 2147–2153
DOI 10.2298/FIL1608147C

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

# Existence of Nonoscillatory Solutions of Higher Order Neutral Differential Equations

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**Abstract.** This article is concerned with nonoscillatory solutions of higher order nonlinear neutral differential equations with deviating and distributed deviating arguments. By using Knaster-Tarski fixed point theorem, new sufficient conditions are established. Illustrative example is given to show applicability of results.

## 1. Introduction

In this paper, we give some new sufficient conditions for the existence of nonoscillatory solutions of the following higher-order nonlinear neutral differential equations

$$\left[r(t)\left[[x(t)-p_1(t)x(t-\tau)]^{(n-1)}\right]^{\gamma}\right]' + (-1)^n Q_1(t)G(x(t-\sigma)) = 0, \tag{1}$$

$$\left[r(t)\left[[x(t)-p_1(t)x(t-\tau)]^{(n-1)}\right]^{\gamma}\right]' + (-1)^n \int_c^d Q_2(t,\xi)G(x(t-\xi))d\xi = 0 \tag{2}$$

and

$$\left[r(t)\left[\left[x(t)-\int_a^b p_2(t,\xi)x(t-\xi)d\xi\right]^{(n-1)}\right]^{\gamma}\right]' + (-1)^n \int_c^d Q_2(t,\xi)G(x(t-\xi))d\xi = 0, \tag{3}$$

where $n \geqslant 2$ is a positive integer, $\gamma$ is a ratio of odd positive integers, $\tau > 0$, $\sigma \geqslant 0$, $d > c \geqslant 0$, $b > a \geqslant 0$, $r \in C([t_0,\infty),(0,\infty))$, $p_1 \in C([t_0,\infty),[0,\infty))$, $p_2 \in C([t_0,\infty)\times[a,b],[0,\infty))$, $Q_1 \in C([t_0,\infty),[0,\infty))$, $Q_2 \in C([t_0,\infty)\times[c,d],[0,\infty))$, $G \in C(\mathbb{R},\mathbb{R})$, $xG(x) > 0$ for $x \neq 0$.

During the last two decades, a good deal of work has been done on the existence of nonoscillatory solutions of first, second and higher order neutral differential equations. In [6, 8, 16, 21] and [15] the authors have studied existence of nonoscillatory solutions of first order neutral differential equations and system of first order neutral differential equations, respectively. In [9, 13, 19, 20] the authors have considered existence of nonoscillatory solutions of second order neutral differential equations. Finally, in [4, 5, 10, 17, 18, 22] and [7] the authors have studied existence of nonoscillatory solutions of higher order neutral differential

2010 *Mathematics Subject Classification*. Primary 34K40; Secondary 34K11
*Keywords*. Neutral equations, Fixed point, Nonoscillatory solution.
Received: 20 May 2014; Accepted: 24 September 2014
Communicated by Jelena Manojlović
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equations and system of higher order neutral differential equations, respectively. For related books, we refer the reader to [1–3, 11, 12, 14]. Our motivation for present article came from the recent work of Candan [6], the author studied existence of nonoscillatory solutions of first order neutral differential equations. In this article we extend Candan's [6] results to higher order neutral differential equations. Also since $\gamma$ makes the left part of the equations (1)-(3) nonlinear, our results include and extend some well known results in the literature.

Let $m_1 = \max\{\tau, \sigma\}$. By a solution of (1) we understand a function $x \in C([t_1 - m_1, \infty), \mathbb{R})$, for some $t_1 \geqslant t_0$, such that $x(t) - p_1(t)x(t-\tau)$ is $n-1$ times continuously differentiable and $r(t)\left[[x(t) - p_1(t)x(t-\tau)]^{(n-1)}\right]^{\gamma}$ is continuously differentiable on $[t_1, \infty)$ and (1) is satisfied for $t \geqslant t_1$.

Similarly, let $m_2 = \max\{\tau, d\}$. By a solution of (2) we mean a function $x \in C([t_1 - m_2, \infty), \mathbb{R})$, for some $t_1 \geqslant t_0$, such that $x(t) - p_1(t)x(t-\tau)$ is $n-1$ times continuously differentiable and $r(t)\left[[x(t) - p_1(t)x(t-\tau)]^{(n-1)}\right]^{\gamma}$ is continuously differentiable on $[t_1, \infty)$ and (2) is satisfied for $t \geqslant t_1$.

Finally, let $m_3 = \max\{b, d\}$. By a solution of (3) we understand a function $x \in C([t_1 - m_3, \infty), \mathbb{R})$, for some $t_1 \geqslant t_0$, such that $x(t) - \int_a^b p_2(t,\xi)x(t-\xi)d\xi$ is $n-1$ times continuously differentiable and $r(t)\left[[x(t) - \int_a^b p_2(t,\xi)x(t-\xi)d\xi]^{(n-1)}\right]^{\gamma}$ is continuously differentiable on $[t_1, \infty)$ and (3) is satisfied for $t \geqslant t_1$.

As is customary, a solution of (1)-(3) is said to be oscillatory if it has arbitrarily large zeros. Otherwise, it is called nonoscillatory.

The following fixed point theorem will be used in proofs.

**Theorem 1.1.** *(Knaster-Tarski Fixed Point Theorem [1]). Let X be partially ordered Banach space with ordering $\leqslant$. Let M be subset of X with the following properties: The infimum of M belongs to M and every nonempty subset of M has a supremum which belongs to M. Let $T : M \to M$ be an increasing mapping, i.e., $x \leqslant y$ implies $Tx \leqslant Ty$. Then T has a fixed point in M.*

## 2. Main Results

**Theorem 2.1.** *Suppose that G is nondecreasing, $0 \leqslant p_1(t) \leqslant p < 1$ and*

$$\int_{t_0}^{\infty} s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^{s} Q_1(u)du\right]^{\frac{1}{\gamma}} ds < \infty. \tag{4}$$

*Then (1) has a bounded nonoscillatory solution.*

*Proof.* From condition (4) there exists $t_1 > t_0$ with

$$t_1 \geqslant t_0 + \max\{\tau, \sigma\} \tag{5}$$

sufficiently large such that

$$\frac{1}{(n-2)!}\int_{t}^{\infty} s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^{s} Q_1(u)du\right]^{\frac{1}{\gamma}} ds \leqslant \frac{(1-p)M_2 - \alpha}{[G(M_2)]^{\frac{1}{\gamma}}}, \quad t \geqslant t_1, \tag{6}$$

where $\alpha$ and $M_2$ are positive constants such that

$$0 < M_1 \leqslant \alpha < (1-p)M_2.$$

Let $\Phi$ be the partially ordered Banach space of all bounded, continuous and real-valued functions $x$ on $[t_0, \infty)$ with the sup norm and usual pointwise ordering $\leqslant$: for given $x_1, x_2 \in \Phi$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$\Omega = \{x \in \Phi : M_1 \leqslant x(t) \leqslant M_2, \quad t \geqslant t_0\}.$$

If $\tilde{x}_1(t) = M_1$, $t \geqslant t_0$, then $\tilde{x}_1 \in \Omega$ and $\tilde{x}_1 = \inf \Omega$. In addition, if $\emptyset \subset \Omega^* \subset \Omega$, then

$$\Omega^* = \{x \in \Phi : \lambda \leqslant x(t) \leqslant \mu, \quad M_1 \leqslant \lambda, \quad \mu \leqslant M_2, \quad t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : M_1 \leqslant \mu \leqslant M_2, \quad t \geqslant t_0\}$. Then $\tilde{x}_2 \in \Omega$ and $\tilde{x}_2 = \sup \Omega^*$. For $x \in \Omega$, we define

$$(Tx)(t) = \begin{cases} \alpha + p_1(t)x(t-\tau) + \frac{1}{(n-2)!}\int_t^\infty (s-t)^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s Q_1(u)G(x(u-\sigma))du\right]^{\frac{1}{\gamma}} ds, & t \geqslant t_1, \\ (Tx)(t_1), & t_0 \leqslant t \leqslant t_1. \end{cases}$$

Thus $Tx$ is a real-valued continuous function on $[t_0, \infty)$ for every $x \in \Omega$. For $t \geqslant t_1$ and $x \in \Omega$, by making use of (6), we obtain

$$\begin{aligned} (Tx)(t) &\leqslant \alpha + pM_2 + \frac{[G(M_2)]^{\frac{1}{\gamma}}}{(n-2)!}\int_t^\infty (s-t)^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s Q_1(u)du\right]^{\frac{1}{\gamma}} ds \\ &\leqslant \alpha + pM_2 + \frac{[G(M_2)]^{\frac{1}{\gamma}}}{(n-2)!}\int_t^\infty s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s Q_1(u)du\right]^{\frac{1}{\gamma}} ds \\ &\leqslant \alpha + pM_2 + [G(M_2)]^{\frac{1}{\gamma}}\left[\frac{(1-p)M_2 - \alpha}{[G(M_2)]^{\frac{1}{\gamma}}}\right] \\ &= M_2 \end{aligned}$$

and

$$(Tx)(t) \geqslant \alpha \geqslant M_1.$$

Hence, $Tx \in \Omega$ for every $x \in \Omega$. Since $G$ is nondecreasing, $T$ is an increasing mapping. That is, for any $x_1, x_2 \in \Omega$ with $x_1 \leqslant x_2$ implies $Tx_1 \leqslant Tx_2$. Hence, the mapping $T$ satisfies the assumptions of Knaster-Tarski's fixed point theorem and therefore there exists a positive $x \in \Omega$ such that $Tx = x$. This completes the proof. □

**Theorem 2.2.** *Suppose that $G$ is nonincreasing, $1 < p \leqslant p_1(t) \leqslant p_0 < \infty$ and (4) holds; then (1) has a bounded nonoscillatory solution.*

*Proof.* From condition (4) there exists $t_1 > t_0$ with

$$t_1 + \tau \geqslant t_0 + \sigma \tag{7}$$

sufficiently large such that

$$\frac{1}{(n-2)!}\int_t^\infty s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s Q_1(u)du\right]^{\frac{1}{\gamma}} ds \leqslant \frac{\alpha - (p_0 - 1)M_3}{[G(M_3)]^{\frac{1}{\gamma}}}, \quad t \geqslant t_1, \tag{8}$$

where $\alpha$ and $M_3$ are positive constants such that

$$(p_0 - 1)M_3 < \alpha \leqslant (p-1)M_4.$$

Let $\Phi$ be the partially ordered Banach space of all bounded, continuous and real-valued functions $x$ on $[t_0, \infty)$ with the sup norm and usual pointwise ordering $\leqslant$: for given $x_1, x_2 \in \Phi$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$\Omega = \{x \in \Phi : M_3 \leqslant x(t) \leqslant M_4, \quad t \geqslant t_0\}.$$

If $\tilde{x}_1(t) = M_3$, $t \geqslant t_0$, then $\tilde{x}_1 \in \Omega$ and $\tilde{x}_1 = \inf \Omega$. In addition, if $\emptyset \subset \Omega^* \subset \Omega$, then

$$\Omega^* = \{x \in \Phi : \lambda \leqslant x(t) \leqslant \mu, \quad M_3 \leqslant \lambda, \quad \mu \leqslant M_4, \quad t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : M_3 \leqslant \mu \leqslant M_4, \quad t \geqslant t_0\}$. Then $\tilde{x}_2 \in \Omega$ and $\tilde{x}_2 = \sup \Omega^*$. For $x \in \Omega$, we define

$$(Tx)(t) = \begin{cases} \frac{1}{p_1(t+\tau)}\Big\{\alpha + x(t+\tau) \\ \quad -\frac{1}{(n-2)!}\int_{t+\tau}^\infty (s-t-\tau)^{n-2}\left[\frac{1}{r(s)}\int_{t_1+\tau}^s Q_1(u)G(x(u-\sigma))du\right]^{\frac{1}{\gamma}} ds\Big\}, & t \geqslant t_1, \\ (Tx)(t_1), & t_0 \leqslant t \leqslant t_1. \end{cases}$$

Thus $Tx$ is a real-valued continuous function on $[t_0, \infty)$ for every $x \in \Omega$. For $t \geqslant t_1$ and $x \in \Omega$, using (8), we obtain

$$\begin{aligned}(Tx)(t) &\leqslant \frac{1}{p}\Big[M_4 + \alpha\Big] \leqslant \frac{1}{p}\Big[M_4 + (p-1)M_4\Big]\\ &= M_4\end{aligned}$$

and

$$\begin{aligned}(Tx)(t) &\geqslant \frac{1}{p_1(t+\tau)}\Big\{\alpha + M_3 - \frac{[G(M_3)]^{\frac{1}{\gamma}}}{(n-2)!}\int_{t+\tau}^{\infty}(s-t-\tau)^{n-2}\left[\frac{1}{r(s)}\int_{t_1+\tau}^{s} Q_1(u)du\right]^{\frac{1}{\gamma}} ds\Big\}\\ &\geqslant \frac{1}{p_1(t+\tau)}\Big\{\alpha + M_3 - \frac{[G(M_3)]^{\frac{1}{\gamma}}}{(n-2)!}\int_{t}^{\infty} s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^{s} Q_1(u)du\right]^{\frac{1}{\gamma}} ds\Big\}\\ &\geqslant \frac{1}{p_1(t+\tau)}\Big\{\alpha + M_3 - [G(M_3)]^{\frac{1}{\gamma}}\left[\frac{\alpha-(p_0-1)M_3}{[G(M_3)]^{\frac{1}{\gamma}}}\right]\Big\}\\ &\geqslant \frac{1}{p_0}[p_0 M_3]\\ &= M_3.\end{aligned}$$

Hence, $Tx \in \Omega$ for every $x \in \Omega$. Since $G$ is nonincreasing, $T$ is an increasing mapping. That is, for any $x_1, x_2 \in \Omega$ with $x_1 \leqslant x_2$ implies $Tx_1 \leqslant Tx_2$. Hence, the mapping $T$ satisfies the assumptions of Knaster-Tarski's fixed point theorem and therefore there exists a positive $x \in \Omega$ such that $Tx = x$. This completes the proof. □

**Theorem 2.3.** *Suppose that $G$ is nondecreasing, $0 \leqslant p_1(t) \leqslant p < 1$ and*

$$\int_{t_0}^{\infty} s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^{s}\int_{c}^{d} Q_2(u,\xi)d\xi du\right]^{\frac{1}{\gamma}} ds < \infty. \tag{9}$$

*Then (2) has a bounded nonoscillatory solution.*

*Proof.* From condition (9) there exists $t_1 > t_0$ with

$$t_1 \geqslant t_0 + \max\{\tau, d\}$$

sufficiently large such that

$$\frac{1}{(n-2)!}\int_{t}^{\infty} s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^{s}\int_{c}^{d} Q_2(u,\xi)d\xi du\right]^{\frac{1}{\gamma}} ds \leqslant \frac{(1-p)M_6 - \alpha}{[G(M_6)]^{\frac{1}{\gamma}}}, \quad t \geqslant t_1,$$

where $\alpha$ and $M_6$ are positive constants such that

$$0 < M_5 \leqslant \alpha < (1-p)M_6.$$

Let $\Phi$ be the partially ordered Banach space of all bounded, continuous and real-valued functions $x$ on $[t_0, \infty)$ with the sup norm and usual pointwise ordering $\leqslant$: for given $x_1, x_2 \in \Phi$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$\Omega = \{x \in \Phi : M_5 \leqslant x(t) \leqslant M_6, \quad t \geqslant t_0\}.$$

If $\tilde{x}_1(t) = M_5$, $t \geqslant t_0$, then $\tilde{x}_1 \in \Omega$ and $\tilde{x}_1 = \inf \Omega$. In addition, if $\emptyset \subset \Omega^* \subset \Omega$, then

$$\Omega^* = \{x \in \Phi : \lambda \leqslant x(t) \leqslant \mu, \quad M_5 \leqslant \lambda, \quad \mu \leqslant M_6, \quad t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : M_5 \leqslant \mu \leqslant M_6, \quad t \geqslant t_0\}$. Then $\tilde{x}_2 \in \Omega$ and $\tilde{x}_2 = \sup \Omega^*$. For $x \in \Omega$, we define

$$(Tx)(t) = \begin{cases} \alpha + p_1(t)x(t-\tau) + \frac{1}{(n-2)!}\int_t^\infty (s-t)^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s \int_c^d Q_2(u,\xi)G(x(u-\xi))d\xi du\right]^{\frac{1}{\gamma}} ds, & t \geqslant t_1, \\ (Tx)(t_1), & t_0 \leqslant t \leqslant t_1. \end{cases}$$

Hence $Tx$ is a real-valued continuous function on $[t_0, \infty)$ for every $x \in \Omega$. Since the rest of the proof is similar to that of Theorem 2.1, it is omitted and the proof is complete. □

**Theorem 2.4.** *Suppose that $G$ is nonincreasing, $1 < p \leqslant p_1(t) \leqslant p_0 < \infty$ and (9) holds; then (2) has a bounded nonoscillatory solution.*

*Proof.* From condition (9) there exists $t_1 > t_0$ with

$$t_1 + \tau \geqslant t_0 + d$$

sufficiently large such that

$$\frac{1}{(n-2)!}\int_t^\infty s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s \int_c^d Q_2(u,\xi)d\xi du\right]^{\frac{1}{\gamma}} ds \leqslant \frac{\alpha-(p_0-1)M_7}{[G(M_7)]^{\frac{1}{\gamma}}}, \quad t \geqslant t_1,$$

where $\alpha$ and $M_7$ are positive constants such that

$$(p_0-1)M_7 < \alpha \leqslant (p-1)M_8.$$

Let $\Phi$ be the partially ordered Banach space of all bounded, continuous and real-valued functions $x$ on $[t_0, \infty)$ with the sup norm and usual pointwise ordering $\leqslant$: for given $x_1, x_2 \in \Phi$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$\Omega = \{x \in \Phi : M_7 \leqslant x(t) \leqslant M_8, \quad t \geqslant t_0\}.$$

If $\tilde{x}_1(t) = M_7$, $t \geqslant t_0$, then $\tilde{x}_1 \in \Omega$ and $\tilde{x}_1 = \inf \Omega$. In addition, if $\emptyset \subset \Omega^* \subset \Omega$, then

$$\Omega^* = \{x \in \Phi : \lambda \leqslant x(t) \leqslant \mu, \quad M_7 \leqslant \lambda, \quad \mu \leqslant M_8, \quad t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : M_7 \leqslant \mu \leqslant M_8, \quad t \geqslant t_0\}$. Then $\tilde{x}_2 \in \Omega$ and $\tilde{x}_2 = \sup \Omega^*$. For $x \in \Omega$, we define

$$(Tx)(t) = \begin{cases} \frac{1}{p_1(t+\tau)}\Big\{\alpha + x(t+\tau) & \\ -\frac{1}{(n-2)!}\int_{t+\tau}^\infty (s-t-\tau)^{n-2}\left[\frac{1}{r(s)}\int_{t_1+\tau}^s \int_c^d Q_2(u,\xi)G(x(u-\xi))d\xi du\right]^{\frac{1}{\gamma}} ds\Big\}, & t \geqslant t_1, \\ (Tx)(t_1), & t_0 \leqslant t \leqslant t_1. \end{cases}$$

Clearly $Tx$ is a real-valued continuous function on $[t_0, \infty)$ for every $x \in \Omega$. Since the rest of the proof is similar to that of Theorem 2.2, it is omitted and the theorem is proved. □

**Theorem 2.5.** *Suppose that $G$ is nondecreasing, $0 \leqslant \int_a^b p_2(t,\xi)d\xi \leqslant p < 1$ and (9) holds; then (3) has a bounded nonoscillatory solution.*

*Proof.* From condition (9) there exists $t_1 > t_0$ with

$$t_1 \geqslant t_0 + \max\{b, d\}$$

sufficiently large such that

$$\frac{1}{(n-2)!}\int_t^\infty s^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s \int_c^d Q_2(u,\xi)d\xi du\right]^{\frac{1}{\gamma}} ds \leqslant \frac{(1-p)M_{10}-\alpha}{[G(M_{10})]^{\frac{1}{\gamma}}}, \quad t \geqslant t_1,$$

where $\alpha$ and $M_{10}$ are positive constants such that

$$0 < M_9 \leqslant \alpha < (1-p)M_{10}.$$

Let $\Phi$ be the partially ordered Banach space of all bounded, continuous and real-valued functions $x$ on $[t_0,\infty)$ with the sup norm and usual pointwise ordering $\leqslant$: for given $x_1, x_2 \in \Phi$, $x_1 \leqslant x_2$ means that $x_1(t) \leqslant x_2(t)$ for $t \geqslant t_0$. Set

$$\Omega = \{x \in \Phi : M_9 \leqslant x(t) \leqslant M_{10}, \quad t \geqslant t_0\}.$$

If $\tilde{x}_1(t) = M_9$, $t \geqslant t_0$, then $\tilde{x}_1 \in \Omega$ and $\tilde{x}_1 = \inf \Omega$. In addition, if $\emptyset \subset \Omega^* \subset \Omega$, then

$$\Omega^* = \{x \in \Phi : \lambda \leqslant x(t) \leqslant \mu, \quad M_9 \leqslant \lambda, \quad \mu \leqslant M_{10}, \quad t \geqslant t_0\}.$$

Let $\tilde{x}_2(t) = \mu_0 = \sup\{\mu : M_9 \leqslant \mu \leqslant M_{10}, \quad t \geqslant t_0\}$. Then $\tilde{x}_2 \in \Omega$ and $\tilde{x}_2 = \sup \Omega^*$. For $x \in \Omega$, we define

$$(Tx)(t) = \begin{cases} \alpha + \int_a^b p_2(t,\xi)x(t-\xi)d\xi \\ +\frac{1}{(n-2)!}\int_t^\infty (s-t)^{n-2}\left[\frac{1}{r(s)}\int_{t_1}^s \int_c^d Q_2(u,\xi)G(x(u-\xi))d\xi du\right]^{\frac{1}{\gamma}} ds, & t \geqslant t_1, \\ (Tx)(t_1), & t_0 \leqslant t \leqslant t_1. \end{cases}$$

Obviously $Tx$ is a real-valued continuous function on $[t_0,\infty)$ for every $x \in \Omega$. The rest of the proof is similar to the proof of Theorem 2.1, therefore it is omitted. Hence the theorem is proved. □

**Example 2.6.** *Consider the equation*

$$\left[e^t\left[\left[x(t) - \frac{5}{e}x(t-1)\right]^{(2)}\right]^3\right]' - \frac{128e^{-t}}{e^2}x(t-2) = 0, \tag{10}$$

*and note that $n = 3$, $\gamma = 3$, $r(t) = e^t$, $p_1(t) = \frac{5}{e}$, $Q_1(t) = \frac{128e^{-t}}{e^2}$ and $G(x) = x$. A straightforward verification yields that the conditions of Theorem 2.2 are satisfied. We note that $x(t) = e^{-t}$ is a nonoscillatory solution of (10).*

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