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Some Properties of Certain Family of Multiplier Transforms

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Abstract. The main purpose of this paper is to derive some inequality properties, convolution properties, subordination and superordination properties, and sandwich-type results of a certain family of multiplier transforms.

1. Introduction and Preliminaries

Let Σ_p denote the class of functions of the form

$$f(z) = z^{-p} + \sum_{k=0}^{\infty} a_k z^{k+1-p} \quad (p \in \mathbb{N} := \{1, 2, 3, \ldots\}), \tag{1.1}$$

which are *analytic* in the *punctured* open unit disk

$$\mathbb{U}^* := \{z : \ z \in \mathbb{C} \ \text{ and } \ 0 < |z| < 1\} =: \mathbb{U}\backslash\{0\}.$$

Let $\mathcal{H}(\mathbb{U})$ be the linear space of all analytic functions in $\mathbb{U}$. For a positive integer number n and $a \in \mathbb{C}$, we let

$$\mathcal{H}[a, n] := \{f \in \mathcal{H}(\mathbb{U}) : \ f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots\}.$$

Let $f, \ g \in \Sigma_p$, where f is given by (1.1) and g is defined by

$$g(z) = z^{-p} + \sum_{k=0}^{\infty} b_k z^{k+1-p}.$$

Then the Hadamard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) := z^{-p} + \sum_{k=0}^{\infty} a_k b_k z^{k+1-p} =: (g * f)(z).$$

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Let $\mathbb{P}(\beta)$ denote the class of functions of the form

$$p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k,$$

which are analytic and convex in $\mathbb{U}$ and satisfy the condition

$$\Re(p(z)) > \beta \quad (z \in \mathbb{U};\ 0 \leqq \beta < 1).$$

For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$, and write

$$f(z) \prec g(z) \quad (z \in \mathbb{U}),$$

if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0 \ \text{ and } \ |\omega(z)| < 1 \quad (z \in \mathbb{U})$$

such that

$$f(z) = g\big(\omega(z)\big) \quad (z \in \mathbb{U}).$$

Indeed, it is known that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Longrightarrow f(0) = g(0) \ \text{ and } \ f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Longleftrightarrow f(0) = g(0) \ \text{ and } \ f(\mathbb{U}) \subset g(\mathbb{U}).$$

In a recent paper, El-Ashwah [9] defined the multiplier transform $\mathcal{D}^{n,l}_{\lambda,p}$ of functions $f \in \Sigma_p$ by

$$\mathcal{D}^{n,l}_{\lambda,p} f(z) := z^{-p} + \sum_{k=0}^{\infty} \left(\frac{\lambda + l(k+1)}{\lambda}\right)^n a_k z^{k+1-p} \quad (z \in \mathbb{U}^*;\ \lambda > 0;\ l \geqq 0;\ n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\};\ p \in \mathbb{N}). \tag{1.2}$$

It should be remarked that the operators $\mathcal{D}^{n,1}_{\lambda,1}$ and $\mathcal{D}^{n,1}_{1,1}$ are the multiplier transforms introduced and investigated, respectively, by Sarangi and Uralegaddi [19], and Uralegaddi and Somanatha [27, 28]. Analogous to $\mathcal{D}^{n,l}_{\lambda,p}$, we here define a new multiplier transform $\mathcal{I}^{n,l}_{\lambda,p,\mu}$.

By setting

$$f^{n,l}_{\lambda,p}(z) := z^{-p} + \sum_{k=0}^{\infty} \left(\frac{\lambda + l(k+1)}{\lambda}\right)^n z^{k+1-p} \quad (z \in \mathbb{U}^*;\ n \geqq 0;\ l \geqq 0;\ \lambda > 0;\ p \in \mathbb{N}), \tag{1.3}$$

we define a new function $f^{n,l}_{\lambda,p,\mu}(z)$ in terms of the Hadamard product (or convolution):

$$f^{n,l}_{\lambda,p}(z) * f^{n,l}_{\lambda,p,\mu}(z) = \frac{1}{z^p(1-z)^\mu} \quad (z \in \mathbb{U}^*;\ \lambda > 0;\ \mu > 0;\ n \geqq 0;\ l \geqq 0;\ p \in \mathbb{N}). \tag{1.4}$$

Then, analogous to $\mathcal{D}^{n,l}_{\lambda,p}$, we have

$$\mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) := f^{n,l}_{\lambda,p,\mu}(z) * f(z) \quad \left(z \in \mathbb{U}^*;\ f \in \Sigma_p\right), \tag{1.5}$$

where (and throughout this paper unless otherwise mentioned) the parameters $n,\ l,\ p,\ \lambda$ and μ are constrained as follows:

$$n \geqq 0;\ \ l \geqq 0;\ \ p \in \mathbb{N};\ \ \lambda > 0 \ \text{ and } \ \mu > 0.$$

We can easily find from (1.3), (1.4) and (1.5) that

$$\mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) = z^{-p} + \sum_{k=0}^{\infty} \frac{(\mu)_{k+1}}{(k+1)!} \left(\frac{\lambda}{\lambda + l(k+1)} \right)^{n} a_k z^{k+1-p} \quad (z \in \mathbb{U}^*), \tag{1.6}$$

where $(\mu)_k$ is the Pochhammer symbol defined by

$$(\mu)_k := \begin{cases} 1 & (k = 0), \\ \mu(\mu+1)\cdots(\mu+k-1) & (k \in \mathbb{N}). \end{cases}$$

We observe that the operator $\mathcal{I}^{n,1}_{\lambda,1,\mu}$ $(n \in \mathbb{N}_0)$ was introduced by Cho *et al.* [6] (see also [17]). Moreover, the operator $\mathcal{I}^{0,1}_{\lambda,1,\mu}$ was investigated recently by Yuan *et al.* [32].

It is readily verified from (1.6) that

$$z\left(\mathcal{I}^{n,l}_{\lambda,p,\mu} f\right)'(z) = \mu \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) - (\mu + p)\mathcal{I}^{n,l}_{\lambda,p,\mu} f(z), \tag{1.7}$$

and

$$lz\left(\mathcal{I}^{n+1,l}_{\lambda,p,\mu} f\right)'(z) = \lambda \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) - (\lambda + p\,l)\mathcal{I}^{n+1,l}_{\lambda,p,\mu} f(z). \tag{1.8}$$

For some recent investigations of meromorphic functions and integral operators, see (for example) the works of [1–4, 7, 8, 10, 12, 13, 16, 20, 21, 23, 24, 26, 29–32] and the references cited therein.

In order to derive our main results, we need the following definition and lemmas.

Definition 1.1. (See [15]) Denote by Q the set of all functions f that are analytic and injective on $\overline{\mathbb{U}} - E(f)$, where

$$E(f) = \left\{ \varepsilon \in \partial\mathbb{U} : \lim_{z \to \varepsilon} f(z) = \infty \right\},$$

and such that $f'(\varepsilon) \neq 0$ for $\varepsilon \in \partial\mathbb{U} - E(f)$.

Lemma 1.2. (See [11]) *Let the function* Ω *be analytic and convex (univalent) in* $\mathbb{U}$ *with* $\Omega(0) = 1$. *Suppose also that the function* Θ *given by*

$$\Theta(z) = 1 + c_1 z + c_2 z^2 + \cdots$$

is analytic in $\mathbb{U}$. *If*

$$\Theta(z) + \frac{z\Theta'(z)}{c} \prec \Omega(z) \quad (c \neq 0,\ \Re(c) \geqq 0;\ z \in \mathbb{U}), \tag{1.9}$$

then

$$\Theta(z) \prec \chi(z) = cz^{-c} \int_0^z t^{c-1}\Omega(t)dt \prec \Omega(z) \quad (z \in \mathbb{U}),$$

and χ *is the best dominant of* (1.9).

Lemma 1.3. (See [25]) *Let*

$$\psi_j(z) \in \mathbb{P}(\gamma_j) \quad (0 \leqq \gamma_j < 1;\ j = 1, 2).$$

Then

$$(\psi_1 * \psi_2)(z) \in \mathbb{P}(\gamma_3) \quad (\gamma_3 = 1 - 2(1 - \gamma_1)(1 - \gamma_2)).$$

The result is the best possible.

Lemma 1.4. (See [22]) *Let*

$$\mathfrak{p}(z) = 1 + \mathfrak{p}_1 z + \mathfrak{p}_2 z^2 + \cdots \in \mathbb{P}(\beta) \quad (0 \leqq \beta < 1).$$

Then

$$\Re(\mathfrak{p}(z)) > 2\beta - 1 + \frac{2(1-\beta)}{1+|z|}.$$

Lemma 1.5. (See [18]) *The function*

$$(1-z)^{\nu} \equiv e^{\nu \log(1-z)} \quad (\nu \neq 0)$$

is univalent in $\mathbb{U}$ *if and only if* ν *is either in the closed disk* $|\nu - 1| \leqq 1$ *or in the closed disk* $|\nu + 1| \leqq 1$.

Lemma 1.6. (See [14]) *Let* q *be univalent in* $\mathbb{U}$, *and let* θ *and* ϕ *be analytic in the domain* $\mathbb{D}$ *containing* $q(\mathbb{U})$ *with* $\phi(\omega) \neq 0$ *when* $\omega \in q(\mathbb{U})$. *Setting*

$$Q(z) = zq'(z)\phi(q(z)) \quad \text{and} \quad h(z) = \theta(q(z)) + Q(z).$$

Suppose also that

1. Q *is starlike univalent in* $\mathbb{U}$;
2. $\Re\left(\frac{zh'(z)}{Q(z)}\right) = \Re\left(\frac{\theta'(q(z))}{\phi(q(z))} + \frac{zQ'(z)}{Q(z)}\right) > 0 \quad (z \in \mathbb{U}).$

If p *is analytic in* $\mathbb{U}$ *with* $p(0) = q(0)$, $p(\mathbb{U}) \subseteq \mathbb{D}$, *and*

$$\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z)),$$

then $p \prec q$, *and* q *is the best dominant.*

Lemma 1.7. (See [5]) *Let* q *be convex univalent in* $\mathbb{U}$, *and let* ϑ *and* φ *be analytic in the domain* $\mathbb{D}$ *containing* $q(\mathbb{U})$. *Suppose that*

1. $\Re\left(\frac{\vartheta'(q(z))}{\varphi(q(z))}\right) > 0$ *for* $z \in \mathbb{U}$;
2. $zq'(z)\varphi(q(z))$ *is starlike univalent in* $\mathbb{U}$.

If $p \in \mathcal{H}[q(0), 1] \cap Q$ *with* $p(\mathbb{U}) \subseteq \mathbb{D}$, *and* $\vartheta(p(z)) + zp'(z)\varphi(p(z))$ *is univalent in* $\mathbb{U}$, *and*

$$\vartheta(q(z)) + zq'(z)\varphi(q(z)) \prec \vartheta(p(z)) + zp'(z)\varphi(p(z)),$$

then $q \prec p$, *and* q *is the best subordinant.*

In the present paper, we aim at proving some inequality properties, convolution properties, subordination and superordination properties, and sandwich-type results of the multiplier transform $\mathcal{I}^{n,l}_{\lambda,p,\mu}$.

2. Main Results

We begin by stating our first inequality property given by Theorem 2.1 below.

Theorem 2.1. *Let $\delta < 1$ and $-1 \leqq B < A \leqq 1$. If $f \in \Sigma_p$ satisfies the condition*

$$z^p \left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \delta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right] \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.1}$$

then

$$\Re\left(\left(z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right)^{\frac{1}{m}}\right) > \left(\frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{\frac{1}{m}} \quad (m \geqq 1). \tag{2.2}$$

The result is sharp.

Proof. Suppose that

$$p(z) := z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) \quad (z \in \mathbb{U};\ f \in \Sigma_p). \tag{2.3}$$

Then p is analytic in $\mathbb{U}$ with $p(0) = 1$. Combining (1.7) and (2.3), we find that

$$z^p \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) = p(z) + \frac{zp'(z)}{\mu}. \tag{2.4}$$

From (2.1), (2.3) and (2.4), we get

$$p(z) + \frac{1-\delta}{\mu} zp'(z) \prec \frac{1+Az}{1+Bz}. \tag{2.5}$$

By Lemma 1.2, we obtain

$$p(z) \prec \frac{\mu}{1-\delta} z^{-\frac{\mu}{1-\delta}} \int_0^z t^{\frac{\mu}{1-\delta}-1}\left(\frac{1+At}{1+Bt}\right)dt, \tag{2.6}$$

or equivalently,

$$z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) = \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(\frac{1+Au\omega(z)}{1+Bu\omega(z)}\right)du, \tag{2.7}$$

where ω is analytic in $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ $(z \in \mathbb{U})$.

Since $\delta < 1$ and $-1 \leqq B < A \leqq 1$, we deduce from (2.7) that

$$\Re\left(z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right) > \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(\frac{1-Au}{1-Bu}\right)du. \tag{2.8}$$

By noting that

$$\Re\left(\varrho^{\frac{1}{m}}\right) \geqq \left(\Re(\varrho)\right)^{\frac{1}{m}} \quad (\varrho \in \mathbb{C},\ \Re(\varrho) \geqq 0;\ m \geqq 1), \tag{2.9}$$

the assertion (2.2) of Theorem 2.1 follows immediately from (2.8) and (2.9).

To show the sharpness of (2.2), we consider the function $f \in \Sigma_p$ defined by

$$z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) = \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(\frac{1+Auz}{1+Buz}\right)du. \tag{2.10}$$

For the function f defined by (2.10), we easily find that

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1}f(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)\right]=\frac{1+Az}{1+Bz}\quad(z\in\mathbb{U}),$$

and

$$z^p\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)\to\frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(\frac{1-Au}{1-Bu}\right)du\quad(z\to-1).$$

This evidently completes the proof of Theorem 2.1. □

We remark that all the corollaries of this paper are trivial consequences and direct applications of the main results, so the details of proof of these corollaries are omitted.

With the aid of (1.8), by similarly applying the method of proof of Theorem 2.1, we get the following result.

Corollary 2.2. *Let* $\delta<1$ *and* $-1\leqq B<A\leqq 1$. *If* $f\in\Sigma_p$ *satisfies the condition*

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)+\delta\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z)\right]\prec\frac{1+Az}{1+Bz}\quad(z\in\mathbb{U}),$$

then

$$\Re\left(\left(z^p\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z)\right)^{\frac{1}{m}}\right)>\left(\frac{\lambda}{l(1-\delta)}\int_0^1 u^{\frac{\lambda}{l(1-\delta)}-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{\frac{1}{m}}\quad(m\geqq 1).$$

The result is sharp.

For the function $f\in\Sigma_p$ given by (1.1), we here recall the integral operator

$$J_\upsilon:\ \Sigma_p\longrightarrow\Sigma_p\,,$$

defined by

$$J_\upsilon f(z):=\frac{\upsilon-p}{z^\upsilon}\int_0^z t^{\upsilon-1}f(t)dt\quad(\upsilon>p).\tag{2.11}$$

Theorem 2.3. *Let* $\delta<1$, $\upsilon>p$ *and* $-1\leqq B<A\leqq 1$. *Suppose also that* J_υ *is given by* (2.11). *If* $f\in\Sigma_p$ *satisfies the condition*

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}J_\upsilon f(z)\right]\prec\frac{1+Az}{1+Bz}\quad(z\in\mathbb{U}),\tag{2.12}$$

then

$$\Re\left(\left(z^p\mathcal{I}^{n,l}_{\lambda,p,\mu}J_\upsilon f(z)\right)^{\frac{1}{m}}\right)>\left(\frac{\upsilon-p}{1-\delta}\int_0^1 u^{\frac{\upsilon-p}{1-\delta}-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{\frac{1}{m}}\quad(m\geqq 1).\tag{2.13}$$

The result is sharp.

Proof. We easily find from (2.11) that

$$(\upsilon-p)\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)=\upsilon\mathcal{I}^{n,l}_{\lambda,p,\mu}J_\upsilon f(z)+z\left(\mathcal{I}^{n,l}_{\lambda,p,\mu}J_\upsilon f\right)'(z).\tag{2.14}$$

Suppose that

$$\mathfrak{q}(z):=z^p\mathcal{I}^{n,l}_{\lambda,p,\mu}J_\upsilon f(z)\quad(z\in\mathbb{U};\ f\in\Sigma_p).\tag{2.15}$$

It follows from (2.12), (2.14) and (2.15) that

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}J_v f(z)\right]=\mathfrak{q}(z)+\frac{1-\delta}{v-p}z\mathfrak{q}'(z)\prec\frac{1+Az}{1+Bz}.$$

The remainder of the proof of Theorem 2.3 is similar to that of Theorem 2.1, we therefore choose to omit the analogous details involved. □

Theorem 2.4. *Let* $\delta<1$ *and* $-1\leqq B_j<A_j\leqq 1$ $(j=1,2)$. *If* $F\in\Sigma_p$ *is defined by*

$$\mathcal{I}^{n,l}_{\lambda,p,\mu}F(z)=\mathcal{I}^{n,l}_{\lambda,p,\mu}f_1(z)*\mathcal{I}^{n,l}_{\lambda,p,\mu}f_2(z), \tag{2.16}$$

and each of the functions $f_j\in\Sigma_p$ $(j=1,2)$ *satisfies the condition*

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1}f_j(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}f_j(z)\right]\prec\frac{1+A_jz}{1+B_jz}\quad(z\in\mathbb{U}), \tag{2.17}$$

then

$$\Re\left(z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1}F(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}F(z)\right]\right)>1-\frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1-\frac{\mu}{1-\delta}\int_0^1\frac{u^{\frac{\mu}{1-\delta}-1}}{1+u}du\right). \tag{2.18}$$

The result is sharp when $B_1=B_2=-1$.

Proof. Suppose that $f_j\in\Sigma_p$ $(j=1,2)$ satisfy the conditions (2.17). By setting

$$\psi_j(z):=z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1}f_j(z)+\delta\mathcal{I}^{n,l}_{\lambda,p,\mu}f_j(z)\right]\quad(z\in\mathbb{U};\ j=1,2), \tag{2.19}$$

it follows from (2.17) and (2.19) that

$$\psi_j\in\mathbb{P}\left(\gamma_j\right)\quad\left(\gamma_j=\frac{1-A_j}{1-B_j};\ j=1,2\right).$$

Combining (1.3) and (2.19), we get

$$\mathcal{I}^{n,l}_{\lambda,p,\mu}f_j(z)=\frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}\psi_j(t)dt\quad(j=1,2). \tag{2.20}$$

For the function $f\in\Sigma_p$ given by (2.16), we find from (2.20) that

$$\begin{aligned}\mathcal{I}^{n,l}_{\lambda,p,\mu}F(z)&=\mathcal{I}^{n,l}_{\lambda,p,\mu}f_1(z)*\mathcal{I}^{n,l}_{\lambda,p,\mu}f_2(z)\\&=\left(\frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}\psi_1(t)dt\right)*\left(\frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}\psi_2(t)dt\right)\\&=\frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}\psi(t)dt,\end{aligned} \tag{2.21}$$

where

$$\psi(z)=\frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}(\psi_1*\psi_2)(t)dt. \tag{2.22}$$

By noting that $\psi_1\in\mathbb{P}(\gamma_1)$ and $\psi_2\in\mathbb{P}(\gamma_2)$, it follows from Lemma 1.3 that

$$(\psi_1*\psi_2)(z)\in\mathbb{P}(\gamma_3)\quad(\gamma_3=1-2(1-\gamma_1)(1-\gamma_2)).$$

By Lemma 1.4, we know that

$$\Re((\psi_1 * \psi_2)(z)) > 2\gamma_3 - 1 + \frac{2(1-\gamma_3)}{1+|z|}. \tag{2.23}$$

In view of (2.17), (2.22) and (2.23), we deduce that

$$\begin{aligned}
&\Re\left(z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1}F(z) + \delta\mathcal{I}^{n,l}_{\lambda,p,\mu}F(z)\right]\right)\\
&\quad = \Re(\psi(z)) = \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\Re((\psi_1 * \psi_2)(uz))du\\
&\quad \geqq \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left(2\gamma_3 - 1 + \frac{2(1-\gamma_3)}{1+u|z|}\right)du\\
&\quad = 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1 - \frac{\mu}{1-\delta}\int_0^1 \frac{u^{\frac{\mu}{1-\delta}-1}}{1+u}du\right).
\end{aligned}$$

When $B_1 = B_2 = -1$, we consider the functions $f_j \in \Sigma_p$ $(j = 1,2)$ which are satisfied the conditions (2.17) and given by

$$\mathcal{I}^{n,l}_{\lambda,p,\mu}f_j(z) = \frac{\mu}{1-\delta}z^{-\frac{\mu}{1-\delta}}\int_0^z t^{\frac{\mu}{1-\delta}-1}\left(\frac{1+A_jt}{1-t}\right)dt \quad (j=1,2). \tag{2.24}$$

It follows from (2.19), (2.20), (2.22) and (2.24) that

$$\psi(z) = \frac{\mu}{1-\delta}\int_0^1 u^{\frac{\mu}{1-\delta}-1}\left[1-(1+A_1)(1+A_2) + \frac{(1+A_1)(1+A_2)}{1-uz}\right]du. \tag{2.25}$$

Thus, we have

$$\psi(z) \to 1-(1+A_1)(1+A_2)\left(1 - \frac{\mu}{1-\delta}\int_0^1 \frac{u^{\frac{\mu}{1-\delta}-1}}{1+u}du\right) \quad (z \to -1).$$

The proof of Theorem 2.4 is evidently completed. □

By virtue of (1.8), by applying the similar method of the proof of Theorem 2.4, we obtain the following result.

Corollary 2.5. *Let $\delta < 1$ and $-1 \leqq B_j < A_j \leqq 1$ $(j = 1,2)$. If $F \in \Sigma_p$ is defined by* (2.16) *and each of the functions $f_j \in \Sigma_p$ $(j = 1,2)$ satisfies the condition*

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}f_j(z) + \mu\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f_j(z)\right] \prec \frac{1+A_jz}{1+B_jz} \quad (z \in \mathbb{U}),$$

then

$$\Re\left(z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}F(z) + \delta\mathcal{I}^{n+1,l}_{\lambda,p,\mu}F(z)\right]\right) > 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1 - \frac{\lambda}{l(1-\delta)}\int_0^1 \frac{u^{\frac{\lambda}{l(1-\delta)}-1}}{1+u}du\right).$$

The result is sharp when $B_1 = B_2 = -1$.

Theorem 2.6. *Let $0 \leqq \sigma < 1$. Suppose that $\gamma \in \mathbb{C}$ with $\gamma \neq 0$ and satisfy either $\left|2\mu\gamma(1-\sigma)+1\right| \leqq 1$ or $\left|2\mu\gamma(1-\sigma)-1\right| \leqq 1$. If f satisfies the condition*

$$\Re\left(\frac{\mathcal{I}^{n,l}_{\lambda,p,\mu+1}f(z)}{\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z)}\right) > \sigma, \tag{2.26}$$

then

$$\left(z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right)^\gamma \prec \frac{1}{(1-z)^{2\mu\gamma(1-\sigma)}} = q(z),$$

and q is the best dominant.

Proof. Suppose that

$$\mathbb{P}(z) = \left(z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right)^\gamma \quad (z \in \mathbb{U}). \tag{2.27}$$

Combining (1.7), (2.26) and (2.27), we have

$$1 + \frac{z\mathbb{P}'(z)}{\mu\gamma\mathbb{P}(z)} \prec \frac{1+(1-2\sigma)z}{1-z} \quad (z \in \mathbb{U}). \tag{2.28}$$

If we take

$$q(z) = \frac{1}{(1-z)^{2\mu\gamma(1-\sigma)}}, \quad \theta(\varpi) = 1 \text{ and } \phi(\varpi) = \frac{1}{\mu\gamma\varpi},$$

then q is univalent by the condition of the theorem and Lemma 1.5. Further, it is easy to show that q, $\theta(\omega)$ and $\phi(\omega)$ satisfy the conditions of Lemma 1.6. Since

$$Q(z) = zq'(z)\phi(q(z)) = \frac{2(1-\sigma)z}{1-z}$$

is univalent starlike in $\mathbb{U}$ and

$$h(z) = \theta(q(z)) + Q(z) = \frac{1+(1-2\sigma)z}{1-z}$$

satisfies the conditions of Lemma 1.6. Thus, the assertion of Theorem 2.6 follows immediately from (2.28). The proof is evidently completed. □

By similarly applying the method of proof of Theorem 2.6, we easily get the following result.

Corollary 2.7. *Let $0 \leqq \sigma < 1$. Suppose that $\gamma \in \mathbb{C}$ with $\gamma \neq 0$ and satisfy either $\left|2\gamma\lambda(1-\sigma)+l\right| \leqq l$ or $\left|2\gamma\lambda(1-\sigma)-l\right| \leqq l$. If f satisfies the condition*

$$\Re\left(\frac{\mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)}{\mathcal{I}^{n+1,l}_{\lambda,p,\mu} f(z)}\right) > \sigma,$$

then

$$\left(z^p \mathcal{I}^{n+1,l}_{\lambda,p,\mu} f(z)\right)^\gamma \prec \frac{1}{(1-z)^{[2\gamma\lambda(1-\sigma)]/l}} = \widetilde{q}(z),$$

and $\widetilde{q}$ is the best dominant.

Theorem 2.8. *Let q be convex univalent in $\mathbb{U}$ and $q(0) = 1$ with*

$$\Re\left(1 + \frac{zq''(z)}{q'(z)}\right) > \max\left\{0, -\Re\left(\frac{\mu}{1-\delta}\right)\right\}. \tag{2.29}$$

If $f \in \Sigma_p$ satisfies the subordination

$$z^p \left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \delta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right] \prec q(z) + \frac{1-\delta}{\mu} z q'(z), \tag{2.30}$$

then

$$z^p \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z) \prec q(z), \tag{2.31}$$

and q is the best dominant.

Proof. Suppose that the function p is given by (2.3). Then, we find from (2.3) and (2.30) that

$$p(z) + \frac{1-\delta}{\mu} z p'(z) \prec q(z) + \frac{1-\delta}{\mu} z q'(z).$$

By Lemma 1.7, we readily get the assertion (2.31) of Theorem 2.8. □

Theorem 2.9. *Let q be univalent in $\mathbb{U}$. Suppose also that q satisfies the condition*

$$\Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) > 0. \tag{2.32}$$

Let

$$\rho(z) = 1 + \gamma\kappa \left(p + \frac{\xi z \left(\mathcal{I}^{n,l}_{\lambda,p,\mu+1} f\right)'(z) + \eta z \left(\mathcal{I}^{n,l}_{\lambda,p,\mu} f\right)'(z)}{\xi \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \eta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)}\right) \quad (\gamma \neq 0;\ \kappa \neq 0;\ \xi + \eta \neq 0). \tag{2.33}$$

If

$$\rho(z) \prec 1 + \gamma \frac{zq'(z)}{q(z)},$$

then

$$\left(\frac{z^p \left(\xi \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \eta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right)}{\xi + \eta}\right)^{\kappa} \prec q(z), \tag{2.34}$$

and q is the best dominant.

Proof. We consider the function $\mathfrak{h}$ defined by

$$\mathfrak{h}(z) := \left(\frac{z^p \left(\xi \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \eta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)\right)}{\xi + \eta}\right)^{\kappa} \quad (\kappa \neq 0;\ \xi + \eta \neq 0). \tag{2.35}$$

Differentiating both sides of (2.35) logarithmically, we get

$$\frac{z\mathfrak{h}'(z)}{\mathfrak{h}(z)} = \kappa \left(p + \frac{\xi z \left(\mathcal{I}^{n,l}_{\lambda,p,\mu+1} f\right)'(z) + \eta z \left(\mathcal{I}^{n,l}_{\lambda,p,\mu} f\right)'(z)}{\xi \mathcal{I}^{n,l}_{\lambda,p,\mu+1} f(z) + \eta \mathcal{I}^{n,l}_{\lambda,p,\mu} f(z)}\right).$$

By setting $\theta(\varpi) = p$ and $\phi(\varpi) = \frac{\gamma}{\varpi}$ $(\gamma \neq 0)$, we observe that $\theta(\varpi)$ is analytic in $\mathbb{C}$ and that $\phi(\varpi) \neq 0$ is analytic in $\mathbb{C} \setminus \{0\}$.

Furthermore, we assume that

$$Q(z) := zq'(z)\phi(q(z)) = \gamma \frac{zq'(z)}{q(z)},$$

and

$$h(z) := \theta(q(z)) + Q(z) = p + \gamma \frac{zq'(z)}{q(z)}.$$

From (2.32), we see that $Q(z)$ is starlike univalent in $\mathbb{U}$, and

$$\Re\left(\frac{zh'(z)}{Q(z)}\right) = \Re\left(1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right) > 0.$$

Thus, an application of Lemma 1.6 to (2.33) yields the desired result. □

By means of (1.8), and using the similar methods of the proof of Theorem 2.8 and Theorem 2.9, respectively, we get the following results.

Corollary 2.10. *Let q be convex univalent in $\mathbb{U}$ and $q(0) = 1$ with*

$$\Re\left(1 + \frac{zq''(z)}{q'(z)}\right) > \max\left\{0, \ -\Re\left(\frac{\lambda}{l(1-\delta)}\right)\right\}. \tag{2.36}$$

If $f \in \Sigma_p$ satisfies the subordination

$$z^p\left[(1-\delta)\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z) + \delta\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z)\right] \prec q(z) + \frac{l(1-\delta)}{\lambda}zq'(z),$$

then

$$z^p\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z) \prec q(z),$$

and q is the best dominant.

Corollary 2.11. *Let q be univalent in $\mathbb{U}$. Suppose also that q satisfies the condition* (2.32). *Let*

$$\chi(z) = 1 + \gamma\kappa\left(p + \frac{\xi z\left(\mathcal{I}^{n,l}_{\lambda,p,\mu}f\right)'(z) + \eta z\left(\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f\right)'(z)}{\xi\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z) + \eta\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z)}\right) \quad (\gamma \neq 0;\ \kappa \neq 0;\ \xi + \eta \neq 0). \tag{2.37}$$

If

$$\chi(z) \prec 1 + \gamma\frac{zq'(z)}{q(z)},$$

then

$$\left(\frac{z^p\left(\xi\mathcal{I}^{n,l}_{\lambda,p,\mu}f(z) + \eta\mathcal{I}^{n+1,l}_{\lambda,p,\mu}f(z)\right)}{\xi + \eta}\right)^{\kappa} \prec q(z),$$

and q is the best dominant.

In what follows, we prove some superordination results involving the multiplier transform $\mathcal{I}^{n,l}_{\lambda,p,\mu}$.

Theorem 2.12. *Let q be convex univalent in $\mathbb{U}$ and $\Re(\delta) < 1$. Also let*

$$z^p \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) \in \mathcal{H}[q(0),1] \cap Q$$

and

$$z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu+1}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z)\right]$$

is univalent in $\mathbb{U}$. If

$$q(z) + \frac{1-\delta}{\mu} z q'(z) \prec z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu+1}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z)\right], \tag{2.38}$$

then

$$q(z) \prec z^p \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z), \tag{2.39}$$

and q is the best subordinant.

Proof. Let $f \in \Sigma_p$ and suppose that p is defined by (2.3). We easily find that

$$q(z) + \frac{1-\delta}{\mu} z q'(z) \prec p(z) + \frac{1-\delta}{\mu} z p'(z). \tag{2.40}$$

Thus, by means of (2.40) and Lemma 1.7, we readily get the assertion (2.39) of Theorem 2.12. □

In view of (1.8) and Lemma 1.7, and by similarly applying the method of proof of Theorem 2.12, we can get the following result.

Corollary 2.13. *Let q be convex univalent in $\mathbb{U}$ and $\Re(\delta) < 1$. Also let*

$$z^p \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z) \in \mathcal{H}[q(0),1] \cap Q$$

and

$$z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right]$$

is univalent in $\mathbb{U}$. If

$$q(z) + \frac{l(1-\delta)}{\lambda} z q'(z) \prec z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right],$$

then

$$q(z) \prec z^p \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z),$$

and q is the best subordinant.

In view of Lemma 1.7 and (1.8), and by similarly applying the method of proof of Theorem 2.12, we get the following results.

Corollary 2.14. *Let q be convex univalent in $\mathbb{U}$. Also let*

$$\left(\frac{z^p\left(\xi \mathcal{I}_{\lambda,p,\mu+1}^{n,l} f(z) + \eta \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z)\right)}{\xi+\eta}\right)^{\kappa} \in \mathcal{H}[q(0),1] \cap Q \quad (\kappa \neq 0;\ \xi+\eta \neq 0)$$

and ρ be defined by (2.33) *is univalent in* $\mathbb{U}$. *If*

$$1+\gamma\frac{zq'(z)}{q(z)} \prec \rho(z) \quad (\gamma \neq 0),$$

then

$$q(z) \prec \left(\frac{z^p\left(\xi\mathcal{I}_{\lambda,p,\mu+1}^{n,l}f(z)+\eta\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z)\right)}{\xi+\eta}\right)^{\kappa},$$

and q is the best subordinant.

Corollary 2.15. *Let q be convex univalent in* $\mathbb{U}$. *Also let*

$$\left(\frac{z^p\left(\xi\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z)+\eta\mathcal{I}_{\lambda,p,\mu}^{n+1,l}f(z)\right)}{\xi+\eta}\right)^{\kappa} \in \mathcal{H}[q(0),1]\cap Q \quad (\kappa \neq 0;\ \xi+\eta \neq 0)$$

and χ be defined by (2.37) *is univalent in* $\mathbb{U}$. *If*

$$1+\gamma\frac{zq'(z)}{q(z)} \prec \chi(z) \quad (\gamma \neq 0),$$

then

$$q(z) \prec \left(\frac{z^p\left(\xi\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z)+\eta\mathcal{I}_{\lambda,p,\mu}^{n+1,l}f(z)\right)}{\xi+\eta}\right)^{\kappa},$$

and q is the best subordinant.

Finally, combining the above mentioned subordination and superordination results, we easily get the following sandwich-type results.

Corollary 2.16. *Let q_1, q_2 be convex univalent in* $\mathbb{U}$ *and* $\Re(\delta) < 1$. *Suppose that q_2 satisfies* (2.29) *and*

$$z^p\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z) \in \mathcal{H}[q(0),1]\cap Q.$$

Let

$$z^p\left[(1-\delta)\mathcal{I}_{\lambda,p,\mu+1}^{n,l}f(z)+\delta\mathcal{I}_{\lambda,p,\mu}^{n,l}\right]$$

is univalent in $\mathbb{U}$. *If*

$$q_1(z)+\frac{1-\delta}{\mu}zq_1'(z) \prec z^p\left[(1-\delta)\mathcal{I}_{\lambda,p,\mu+1}^{n,l}f(z)+\delta\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z)\right] \prec q_2(z)+\frac{1-\delta}{\mu}zq_2'(z),$$

then

$$q_1(z) \prec z^p\mathcal{I}_{\lambda,p,\mu}^{n,l}f(z) \prec q_2(z),$$

where q_1 and q_2 are, respectively, the best subordinant and the best dominant.

Corollary 2.17. *Let q_3 and q_4 be convex univalent in $\mathbb{U}$, and $\Re(\delta) < 1$. Suppose that q_4 satisfies* (2.36) *and*

$$z^p \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z) \in \mathcal{H}[q(0), 1] \cap Q.$$

Let

$$z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right]$$

is univalent in $\mathbb{U}$. If

$$q_3(z) + \frac{l(1-\delta)}{\lambda} z q_3'(z) \prec z^p \left[(1-\delta)\mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \delta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right] \prec q_4(z) + \frac{l(1-\delta)}{\lambda} z q_4'(z),$$

then

$$q_3(z) \prec z^p \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z) \prec q_4(z),$$

where q_3 and q_4 are, respectively, the best subordinant and the best dominant.

Corollary 2.18. *Let q_5 be convex univalent and q_6 be univalent in $\mathbb{U}$. Suppose that q_6 satisfies* (2.32), *and*

$$\left(\frac{z^p \left(\xi \mathcal{I}_{\lambda,p,\mu+1}^{n,l} f(z) + \eta \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z)\right)}{\xi + \eta}\right)^{\kappa} \in \mathcal{H}[q(0), 1] \cap Q \quad (\kappa \neq 0;\ \xi + \eta \neq 0).$$

Let ρ be defined by (2.33) *is univalent in $\mathbb{U}$. If*

$$1 + \gamma \frac{z q_5'(z)}{q_5(z)} \prec \rho(z) \prec 1 + \gamma \frac{z q_6'(z)}{q_6(z)} \quad (\gamma \neq 0),$$

then

$$q_5(z) \prec \left(\frac{z^p \left(\xi \mathcal{I}_{\lambda,p,\mu+1}^{n,l} f(z) + \eta \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z)\right)}{\xi + \eta}\right)^{\kappa} \prec q_6(z),$$

where q_5 and q_6 are, respectively, the best subordinant and the best dominant.

Corollary 2.19. *Let q_7 be convex univalent and q_8 be univalent in $\mathbb{U}$. Suppose that q_8 satisfies* (2.32), *and*

$$\left(\frac{z^p \left(\xi \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \eta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right)}{\xi + \eta}\right)^{\kappa} \in \mathcal{H}[q(0), 1] \cap Q \quad (\kappa \neq 0;\ \xi + \eta \neq 0).$$

Let χ be defined by (2.37) *is univalent in $\mathbb{U}$. If*

$$1 + \gamma \frac{z q_7'(z)}{q_7(z)} \prec \chi(z) \prec 1 + \gamma \frac{z q_8'(z)}{q_8(z)} \quad (\gamma \neq 0),$$

then

$$q_7(z) \prec \left(\frac{z^p \left(\xi \mathcal{I}_{\lambda,p,\mu}^{n,l} f(z) + \eta \mathcal{I}_{\lambda,p,\mu}^{n+1,l} f(z)\right)}{\xi + \eta}\right)^{\kappa} \prec q_8(z),$$

where q_7 and q_8 are, respectively, the best subordinant and the best dominant.

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