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New Weighted Sum Model

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Abstract. In this paper, a new weighting sum method for multi-criteria decision making is presented. The main advantage of this method is that it is easier for understanding and it can effectively be handled by a decision maker, so the obtained solution best suits his goal and his understanding of the problem.

1. Introduction

All criteria in a multi-criteria decision making problem can be classified into two categories. Criteria that are to be maximized (highest value is best value) and criteria that are to be minimized (lowest value is best value). An ideal solution to a multi-criteria decision making problem would maximize all criteria of the first type and minimize all criteria of the second type, but usually this solution is not possible to obtain. So, the question is what would be the best satisfying solution? This question may not always have a conclusive or unique answer. Whether a solution is satisfying depends on the level of the decision makers expectation. The goal of every analyst (the person using the method) is for his suggestion to be accepted as a valid solution by the decision maker. In order to reach that goal (and for the analyst to justify his work and achieve success), it is required to include the decision maker as much as possible, directly or indirectly, in the calculation process for solving the problem. Sometimes, this is not possible because of the limited time or the structure and complexity of the method from mathematical point of view. Therefore, it is necessary to find a solution reaching method where the decision maker can give his subjective judgments and to be included successfully and with ease.

There are many multi-criteria decision making methods available in the literature. Some of the most commonly used approaches are the Weighted sum model [7], the Weighted product model [2, 13], the analytic hierarchy process [17–20], the ELECTRE method [3, 16], the TOPSIS method [9], the PROMETHEE method [1], the VIKOR method [14], etc. There are also a number of papers, which are devoted to comparison of their characteristics and performances. Multi-criteria decision making methods differ in normalization processes performed to convert all criteria into a same unit [4, 11, 12], weighting techniques used for determination of the criteria importance [10, 22], method of aggregation of value functions assigned to each criterion [5], etc. This affects the core complexity of a method.

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In this article, we will present a new multi-criteria decision method where it is relatively easy to include the decision maker, his personal preference and his view on the problem. The method is consisted of two main parts, normalization and weighing processes. Finally, to illustrate the feasibility of our approach, we will apply our proposed method on a real application problem.

For an overview of the available methods for solving multi-criteria decision problems we refer to Figueira et al. [6], Hwang and Yoon [9], Radojičić and Žižović [15], Triantaphyllou [21] and Zeleny [23], and for an insight into practical applications we refer to [8].

2. Normalization of the Multi-Criteria Model

We will observe a multi-criteria model for ranking m alternatives $(A_1, \ldots, A_m)$ by n criteria $(C_1, \ldots, C_n)$ presented in Table 1. In this model, the degree in which alternative A_i $(i = 1, \ldots, m)$ satisfies criterion C_j, $(j = 1, \ldots, n)$ is denoted by a_{ij}. Without lost of generality, we can assume that the criteria are ordered based on importance, from the most important criterion C_1 to the least important criterion C_n.

Table 1: Decision matrix

	C_1	C_2	$\cdots$	C_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}		a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	a_{m1}	a_{m2}	$\cdots$	a_{mn}

For different criteria, the performance values of alternatives can be measured by different units. In order to have a valid comparisons, all elements of decision matrix need to be transferred into a same unit (the interval [0,1] is usually used as the basic unit interval). A lot of normalization methods have been developed. Some of most popular are vector normalization method, linear max-min normalization, linear sum based normalization, linear max normalization, Gaussian normalization, etc. The review of the literature on normalization methods can be found in [11].

Applying different normalization methods on a decision-making matrix can lead to different numerical results and finally affect alternatives order of preference. Therefore, a normalization method affects the quality of decision-making (see for example [12]). In this section, we will present one new normalization method and we will point out some of its advantages over most commonly known normalization methods.

Values of alternatives with respect to every criterion C_j, $(j = 1, \ldots, n)$ are given in the jth column of Table 1. Clearly, these values can have different importance for the decision maker. Some values are extremely important for the decision maker, some are acceptable, some are barely acceptable, while some values are totally unacceptable. It is therefore, logical to define levels of acceptability for possible alternative values (this can be done even before observing the model itself in Table 1).

For each criterion C_j, $(j = 1, \ldots, n)$ of maximization type, the decision maker defines r, $(r \in \mathbb{N})$ values $Q_{j1} > Q_{j2} > \cdots > Q_{jr}$, such that Q_{j1} is assumed to be the decision makers ideal alternative value, Q_{j2} represents the lower limit of ideal values for decision maker, $Q_{j,r-1}$ represents the upper limit for barely acceptable values for decision maker, and Q_{jr} is a lowest acceptable value for the decision maker.

Analogously, for each criterion of minimization type, the decision maker defines r, $(r \in \mathbb{N})$ values $Q_{j1} > Q_{j2} > \cdots > Q_{jr}$ whose meanings are in reverse order with respect to previous list, i.e. Q_{j1} is assumed to be the decision maker nadir alternative value, while Q_{jr} is the ideal one.

For a criterion C_j, $(j = 1, \ldots, n)$ of maximization type, these values Q_{jk}, $(1 \le k \le r)$ specify $r - 1$ intervals

$$I_{j1} = [Q_{j2}, Q_{j1}],\ I_{j2} = (Q_{j3}, Q_{j2}],\ \cdots, I_{j,r-1} = (Q_{jr}, Q_{j,r-1}],$$

and for a criterion of minimization type the corresponding intervals are

$$I_{j1} = [Q_{j2}, Q_{j1}),\ I_{j2} = [Q_{j3}, Q_{j2}),\ \cdots, I_{j,r-1} = [Q_{jr}, Q_{j,r-1}].$$

In general, the number of these intervals can be different for different criteria and it depends on the decision maker preference on the criterion.

In the case that C_j is a criterion of maximization type, let $p_{jk} : I_{jk} \to (0,1]$, $(j = 1,\ldots,n, k = 1,2,\ldots,r-1)$ be a non-decreasing functions given by the decision maker and the analysts such that they satisfy the condition $p_{jk}(x) > p_{j,k+1}(y)$, for every $x \in I_{jk}$ and $y \in I_{j,k+1}$, $(k = 1,2,\ldots,r-2)$. Further, let the functions $f_j : \mathbb{R}_{\geq 0} \to [0,1]$ be such that

$$f_j(a) = \begin{cases} 1, & a \geq Q_{j1}; \\ p_{jk}(a), & a \in I_{jk}, \quad k = 1,2,\ldots,r-1; \\ 0, & a \leq Q_{jr}. \end{cases} \tag{1}$$

In the case that C_j is a criterion of minimization type, then functions $p^*_{jk} : I_{jk} \to (0,1]$, $(j = 1,\ldots,n, k = 1,2,\ldots,r-1)$ are required to be non-increasing and to satisfy $p_{jk}(x) < p_{j,k+1}(y)$, for every $x \in I_{jk}$ and $y \in I_{j,k+1}$, $(k = 1,2,\ldots,r-2)$. Further, let the functions $f^*_j : \mathbb{R}_{\geq 0} \to [0,1]$ be defined as

$$f^*_j(a) = \begin{cases} 0, & a \geq Q_{j1}; \\ p^*_{jk}(a), & a \in I_{jk}, \quad k = 1,2,\ldots,r-1; \\ 1, & a \leq Q_{jr}. \end{cases} \tag{2}$$

Now, we normalize the decision matrix given in Table 1, by

$q_{ij} = f_j(a_{ij})$, if C_j is a criterion of maximization type;

$q_{ij} = f^*_j(a_{ij})$, if C_j is a criterion of minimization type.

By this procedure, we obtain normalized decision matrix given by Table 2. All values in this matrix are elements of the real unit interval [0,1]. Moreover, all the criteria of minimization type are converted into the criteria of maximization type.

Table 2: Normalized decision matrix

	C_1	C_2	$\cdots$	C_n
A_1	q_{11}	q_{12}	$\cdots$	q_{1n}
A_2	q_{21}	q_{22}	$\cdots$	q_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	q_{m1}	q_{m2}	$\cdots$	q_{mn}

3. Weighted Coefficients

In this section, we will present a new procedure for calculation of weighting coefficients in a multi-criteria decision making model. This procedure is based on the pairwise comparisons between the most important criterion C_1 and the remaining $n-1$ criteria $C_2,\ldots,C_n$.

Let $p_{1k} \in (0,100]$, $(k = 2,\cdots,n)$ be the value of importance of the criterion C_1 with respect to the criterion C_k. Then $p_{k1} = 100 - p_{1k}$ represent the value of importance of criterion C_k with respect to criterion C_1. These values are given in Table 3.

Table 3: The most important criterion w.r.t. other criteria

p_{12}	p_{13}	p_{14}	$\cdots$	p_{1n}
p_{21}	p_{31}	p_{41}	$\cdots$	p_{n1}

Clearly, the following holds

$$50 \leq p_{12} \leq p_{13} \leq \cdots \leq p_{1n} < 100 \qquad \text{and} \qquad 50 \geq p_{21} \geq p_{31} \geq \cdots \geq p_{n1} > 0.$$

As we assumed above, criteria $C_1, \ldots, C_n$ are listed in order of importance, and thus C_i is of greater or equal importance as C_{i+1}, for each $i = 1, \ldots, n-1$. Now, using Table 3, it is possible to calculate the value of importance of the criterion C_2 with respect to criteria $C_3, \ldots, C_n$, further, the value of importance of criterion C_3 with respect to criteria $C_4, \ldots, C_n$, etc., and finally the value of importance of criterion C_{n-1} with respect to criterion C_n. This procedure is given by following recursive formula:

$$p_{j+1,k} = \frac{100(p_{jk} : p_{kj}) : (p_{j,j+1} : p_{j+1,j})}{1 + (p_{jk} : p_{kj}) : (p_{j,j+1} : p_{j+1,j})}, \quad p_{k,j+1} = 100 - p_{j+1,k}, \quad \text{for } j = 2, \ldots, n-1,\ k = j+1, \ldots, n. \tag{3}$$

In this way we can form a triangular Table 4 (which also includes Table 3).

Table 4: Pairwise comparison of criteria

p_{12}	p_{13}	p_{14}	$\cdots$	$p_{1,n-1}$	p_{1n}
p_{21}	p_{31}	p_{41}	$\cdots$	$p_{n-1,1}$	p_{n1}
	p_{23}	p_{24}	$\cdots$	$p_{2,n-1}$	p_{2n}
	p_{32}	p_{42}	$\cdots$	$p_{n-1,2}$	p_{n2}
		p_{34}	$\cdots$	$p_{3,n-1}$	p_{3n}
		p_{43}	$\cdots$	$p_{n-1,3}$	p_{n3}
				$\cdots$	
				$p_{n-2,n-1}$	$p_{n-2,n}$
				$p_{n-1,n-2}$	$p_{n,n-2}$
					$p_{n-1,n}$
					$p_{n,n-1}$

Theorem 3.1. *The following holds for every* $j = 1, 2, \ldots, n-1$*:*

$$50 \leq p_{j,j+1} \leq p_{j,j+2} \leq \cdots \leq p_{j,n} < 100 \quad \textit{and} \quad 50 \geq p_{j+1,j} \geq p_{j+2,j} \geq \cdots \geq p_{n,j} > 0. \tag{4}$$

Proof. Clearly, the assertion holds for $j = 1$ and suppose that the assertion holds for some $j-1 \in \{1, 2, \ldots n-1\}$. Then by (4) we obtain

$$p_{j,j+1} = \frac{100\delta}{1+\delta}, \qquad \text{for } \delta = \frac{p_{j-1,j+1}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+1,j-1}}.$$

Since $p_{j-1,j} \leq p_{j-1,j+1}$ and $p_{j,j-1} \geq p_{j+1,j}$, holds $\delta \geq 1$, and therefore $p_{j,j+1} = 100 - \frac{100}{1+\delta} \geq 50$. Further, from $p_{j-1,j+k} \leq p_{j-1,j+k+1}$ and $p_{j+k,j-1} \geq p_{j+k+1,j}$, for every $k = 1, 2, \ldots, n-j-1$ it follows that

$$\frac{p_{j-1,j+k+1}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+k+1,j-1}} \geq \frac{p_{j-1,j+k}}{p_{j-1,j}} \cdot \frac{p_{j,j-1}}{p_{j+k,j-1}},$$

which implies $p_{j,j+k+1} \geq p_{j,j+k}$. The inequality $p_{j,n} < 100$ is satisfied according to (3). Thus, $50 \leq p_{j,j+1} \leq p_{j,j+2} \leq \cdots \leq p_{j,n} < 100$ holds.

Further, from $p_{kj} = 100 - p_{jk}$, for $k = j+1, \ldots, n-j = 1$, we have $50 \geq p_{j+1,j} \geq p_{j+2,j} \geq \cdots \geq p_{n,j} > 0$. □

The weighted coefficient W_j is given by

$$W_j = \frac{\sum_{k=1,k\neq j}^{n} p_{jk}}{50n(n-1)}, \tag{5}$$

for each criterion C_j ($j = 1, 2, \ldots, n$) Also, the following statement is true for weighted coefficients.

Theorem 3.2. *The following properties of the weighted coefficients* W_j $(j = 1, 2, \dots, n)$ *are satisfied:*

(i) $\sum_{j=1}^{n} W_j = 1,$

(ii) $W_1 \geq W_2 \geq \cdots \geq W_n.$

Proof. (i) Using (5) we have

$$\sum_{j=1}^{n} W_j = \sum_{j=1}^{n} \frac{\sum_{k=1,k\neq j}^{n} p_{jk}}{50n(n-1)} = \frac{1}{50n(n-1)} \cdot \{(p_{12} + p_{13} + \cdots p_{1n}) + (p_{21} + p_{23} + \cdots p_{2n}) +$$

$$+ \cdots + (p_{n1} + p_{n2} + \cdots p_{n,n-1})\} = \frac{1}{50n(n-1)} \cdot \{(p_{12} + p_{21}) + (p_{13} + p_{31}) + \cdots + (p_{n-1,n} + p_{n,n-1})\} =$$

$$= \frac{1}{50n(n-1)} \cdot \frac{100n(n-1)}{2} = 1.$$

(ii) This assertion holds by definition of weighted coefficients (5) and Theorem 3.1. □

Corollary 3.3. $W_1 = \cdots = W_n = \frac{1}{n}$ *if and only if* $p_{ij} = 50$*, for all* $i, j = 1, 2 \dots n$*,* $i \neq j$*.*

Proof. Follows immediately by (5). □

4. Ranking of Alternatives

For $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, let q_{ij} be the normalized performance value of the alternative A_i by the criterion C_j and let W_j be the weighted coefficient associated to the criterion C_j according to formula (5). By multiplying q_{ij} with weight W_j, we obtain preference value e_{ij} associated to criterion C_j, i.e.

$$e_{ij} = W_j * q_{ij}, \qquad \text{for all } i = 1, \dots, m, \ j = 1, \dots, n. \tag{6}$$

In this way, we form Table 5.

Table 5: Preference values associated to criteria

	C_1	C_2	$\cdots$	C_n
A_1	e_{11}	e_{12}	$\cdots$	e_{1n}
A_2	e_{21}	e_{22}	$\cdots$	e_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	e_{m1}	e_{m2}	$\cdots$	e_{mn}

Further, we sum up the values e_{ij} $(j = 1, 2, \dots, n)$ to obtain the overall value of the alternative A_i $(i = 1, 2, \dots, m)$, i.e.

$$V(A_i) = \sum_{j=1}^{n} e_{ij}. \tag{7}$$

The ranking of alternatives A_i $(i = 1, 2, \dots, m)$ is based on the aggregation value function (7) and fulfilment of criteria in order of their importance. In other words, for two alternatives A_i and A_j $(i, j = 1, 2, \dots m)$ we say that A_j is preferred over A_i, in notation $A_j \to A_i$, if:

$V(A_i) < V(A_j)$ or

$V(A_i) = V(A_j)$, $e_{i1} < e_{j1}$ or

$$V(A_i) = V(A_j),\ e_{i1} = e_{j1},\ e_{i2} < e_{j2} \text{ or}$$

$$\ldots$$

$$V(A_i) = V(A_j),\ e_{i1} = e_{j1},\ \ldots,\ e_{i,n-1} = e_{j,n-1},\ e_{in} < e_{jn}.$$

Two alternatives A_i i A_j $(i, j = 1, 2, \ldots m)$ are equivalents if all their values are equal, i.e. $e_{ik} = e_{jk}$, for all $k = 1, 2, \ldots, n$.

It is well known fact that many multi-criteria decision making methods suffer from phenomena called rank reversal. Namely, the order of alternatives can be changed when an alternative is added to the model. The main reason for rank reversal is the use of an inappropriate normalization method. Actually, all most commonly used methods such as vector normalization method, linear max-min normalization, linear sum based normalization, linear max normalization, Gaussian normalization are based on vector normalization, or choice of ideal or nadir solutions created from alternatives, which basically depend on all alternatives included into the model. Normalization process presented in this paper depends only on treated alternative and hypothetical ideal (nadir) solution given by the decision maker. This proves the following.

Theorem 4.1. *The rank of alternatives from the set* $\{A_1, A_2, \ldots, A_n\}$ *remains the same in the case that the starting set of alternatives is expanded by a new alternative* A.

Proof. Let $\mathcal{A} = \{A_1, A_2, \ldots, A_n\}$ and $\mathcal{B} = \{A_1, A_2, \ldots, A_n, A\}$, and let $V_{\mathcal{A}}$ and $V_{\mathcal{B}}$ denote the value functions on $\mathcal{A}$ and $\mathcal{B}$ (respectively) calculated by (7), and $\to_{\mathcal{A}}$ and $\to_{\mathcal{B}}$ denote the preference order relations on $\mathcal{A}$ and $\mathcal{B}$ (respectively). Then $V_{\mathcal{A}}(A_i) = V_{\mathcal{B}}(A_i)$, for all $i = 1, 2, \cdots, n$. Therefore, for every $i, j = 1, 2, \cdots, n$ holds $A_i \to_{\mathcal{A}} A_j$ if and only if $A_i \to_{\mathcal{B}} A_j$. □

Corollary 4.2. *Adding a new alternative into the multi-criteria model can not lead to a rank reversal.*

Example 4.3. In this example, we will rank five alternatives A_1, A_2, A_3, A_4 and A_5 by four criteria C_1, C_2, C_3 and C_4. This multi-criteria decision making model is given by Table 6. The criterion C_1 is dominant. The criteria C_1, C_2 and C_3 are to be maximized and criterion C_4 is to be minimized.

Table 6: Decision matrix of Example 4.3

	C_1	C_2	C_3	C_4
A_1	10	7	6	9
A_2	8	8	7	8
A_3	6	6	10	8
A_4	7	6	9	7
A_5	4	10	3	5

We will assume that the following values are given by decision maker

$$Q_{j1} = 10, \quad Q_{j2} = 8.5, \quad Q_{j3} = 6.5, \quad Q_{j4} = 4.5, \quad Q_{j5} = 2, \quad \text{for } j = 1, 2, 3,$$

$$Q_{41} = 10, \quad Q_{42} = 8.5, \quad Q_{43} = 6.5, \quad Q_{44} = 3.5, \quad Q_{45} = 1.$$

Thus we have four intervals of importance for all criteria. Also, it is assumed that the decision maker has provided the values of importance in relation to to the first criterion (Table 7).

Table 7: The most important criterion w.r.t. other criteria in Example 4.3

$p_{12} = 50$	$p_{13} = 60$	$p_{14} = 70$
$p_{21} = 50$	$p_{31} = 40$	$p_{41} = 30$

In this example, we will use linear functions for normalization process. In relation to a maximizing criterion C_j ($j = 1, 2, 3$), the functions f_j are defined in the following way:

$$f_j(a_{ij}) = \begin{cases} 1, & a_{ij} \geq Q_{j1}; \\ \dfrac{a_{ij} - Q_{j5}}{Q_{j1} - Q_{j5}}, & a_{ij} \in I_{jk}, \quad k = 1, 2, 3, 4; \\ 0, & a_{ij} \leq Q_{i5}. \end{cases} \tag{8}$$

and in relation to the minimizing C_4 criterion, the function f_4^* can be defined by:

$$f_j^*(a_{ij}) = \begin{cases} 0, & a_{ij} \geq Q_{j1}; \\ \dfrac{Q_{j1} - a_{ij}}{Q_{j1} - Q_{j5}}, & a_{ij} \in I_{jk}, \quad k = 1, 2, 3, 4; \\ 1, & a_{ij} \leq Q_{j5}. \end{cases} \tag{9}$$

In this way, we obtain normalized decision matrix given by Table 8.

Table 8: Normalized decision matrix of Example 4.3

	C_1	C_2	C_3	C_4
A_1	1	0.625	0.5	0.111
A_2	0.75	0.75	0.625	0.222
A_3	0.5	0.5	1	0.222
A_4	0.625	0.5	0.875	0.333
A_5	0.25	1	0.125	0.556

Further, starting with Table 7 and using formula (3), we obtain Table 9. Now, by (5) we have

Table 9: Pairwise comparison of criteria in Example 4.3

$p_{12} = 50$	$p_{13} = 60$	$p_{14} = 70$
$p_{21} = 50$	$p_{31} = 40$	$p_{41} = 30$
	$p_{23} = 60$	$p_{24} = 70$
	$p_{32} = 40$	$p_{42} = 30$
		$p_{34} = 61$
		$p_{43} = 39$

$W_1 = 0.300, \quad W_2 = 0.300, \quad W_3 = 0.235 \quad \text{and} \quad W_4 = 0.165.$

Finally, by (6) we obtain Table 10 with overall values of the alternatives.

Table 10: Preference values associated to criteria and overall scores of alternatives in Example 4.3

	C_1	C_2	C_3	C_4	Σ
A_1	0.300	0.188	0.118	0.018	$V(A_1) = 0.624$
A_2	0.225	0.225	0.147	0.037	$V(A_2) = 0.634$
A_3	0.150	0.150	0.235	0.037	$V(A_3) = 0.572$
A_4	0.188	0.150	0.206	0.055	$V(A_4) = 0.599$
A_5	0.075	0.300	0.029	0.092	$V(A_5) = 0.496$

Since all values $V(A_i)$ ($i = 1, 2, \ldots, 5$) are different we have the total order of alternatives

$$A_2 \quad \to \quad A_1 \quad \to \quad A_4 \quad \to \quad A_3 \quad \to \quad A_5,$$

and therefore, A_2 is the best alternative solution.

5. Conclusion

The suggested method allows a high level of influence of personal preferences of the decision maker and helps him to find a solution that best suits his goal and his understanding of the problem. This results in higher quality of decisions reached. Also, the advantage of this method is that it is easier to understand and it can effectively handle both qualitative and quantitative data. It can be expected that this method will be applicable in many areas (science, technology, business decision making, military doctrine, etc.) because introducing new alternatives does not require additional calculations and comparisons to previously introduced alternatives and does not change the established order. It can be noticed that complexity of methods such as AHP, PPOMETHEE, and others, depends on number of mutually comparisons of alternatives, while this is not a case with the complexity of the proposed new method, since it linearly depends on the number of alternatives included into the model.

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