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C-Class Functions and Pair $(\mathcal{F}, h)$ upper Class on Common Best Proximity Points Results for New Proximal C-Contraction mappings

A.H.Ansari[a], Geno Kadwin Jacob[b], D. Chellapillai[b]

[a]Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran.
[b]Department of Mathematics, Bharathidasan University,Trichy- 620 024, Tamil Nadu, India.

Abstract. In this paper, using the concept of $C-$class and Upper class functions we prove the existence of unique common best proximity point. Our main result generalizes results of Kumam et al. [[17]] and Parvaneh et al. [[21]].

To the memory of Professor Lj. Ćirić (1935–2016)

1. Introduction and Preliminaries

Consider a pair (A, B) of nonempty subsets of a metric space (X, d). Assume that f is a function from A into B. An element $w \in A$ is said to be a best proximity point whenever $d(w, fw) = d(A, B)$,where

$$d(A, B) = \inf\{d(s, t) : s \in A, t \in B\}.$$

Best proximity point theory of non-self functions was initiated by Fan [1] and Kirk et al. [[16]]; see also [[19][15][11][13] [4][8][9][24][25][20][18]].

Definition 1.1. *Consider non-self functions $f_1, f_2, \dots, f_n : A \to B$. We say the a point $s \in A$ is a common best proximity point of $f_1, f_2, \dots, f_n$ if*

$$d(s, f_1(s)) = d(s, f_2(s)) = \cdots = d(s, f_n(s)) = d(A, B).$$

Definition 1.2. *([17])Let (X, d) be a metric space and $\emptyset \neq A, B \subset X$. We say the pair (A, B) has the V-property if for every sequence $\{t_n\}$ of B satisfying $d(s, t_n) \to d(s, B)$ for some $s \in A$, there exists a $t \in B$ such that $d(s, t) = d(s, B)$.*

Definition 1.3. *([5]) A continuous function $F : [0, \infty)^2 \to \mathbb{R}$ is called C-class function if for any $s, t \in [0, \infty)$, the following conditions hold:*
(1) $F(s, t) \leq s$;
(2) $F(s, t) = s$ implies that either $s = 0$ or $t = 0$.

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Email addresses: analsisamirmath2@gmail.com (A.H.Ansari), genomaths@gmail.com (Geno Kadwin Jacob), chellapillai2@gmail.com (D. Chellapillai)

An extra condition on F that $F(0,0) = 0$ could be imposed in some cases if required. The letter C will denote the class of all C- functions.

Example 1.4. *([5]) Following examples show that the class* $\mathbf{C}$ *is nonempty:*

1. $F(s,t) = s - t$.
2. $F(s,t) = ms$,for some $m \in (0,1)$.
3. $F(s,t) = \frac{s}{(1+t)^r}$ for some $r \in (0,\infty)$.
4. $F(s,t) = \log(t + a^s)/(1+t)$, for some $a > 1$.
5. $F(s,t) = \frac{s}{\Gamma(1/2)} \int_0^\infty \frac{e^{-x}}{\sqrt{x}+t}\, dx$, where Γ is the Euler Gamma function.

Definition 1.5. *[6, 7]We say that the function* $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ *is a function of subclass of type I, if* $x \geq 1 \Longrightarrow h(1,y) \leq h(x,y)$ *for all* $y \in \mathbb{R}^+$.

Example 1.6. *[6, 7]Define* $h\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ *by:*

(a) $h(x,y) = (y+l)^x, l > 1;$
(b) $h(x,y) = (x+l)^y, l > 1;$
(c) $h(x,y) = x^n y, n \in \mathbb{N};$
(d) $h(x,y) = y;$
(e) $h(x,y) = \frac{1}{n+1}\left(\sum_{i=0}^n x^i\right) y, n \in \mathbb{N};$
(f) $h(x,y) = \left[\frac{1}{n+1}\left(\sum_{i=0}^n x^i\right) + l\right]^y, l > 1, n \in \mathbb{N}$

for all $x, y \in \mathbb{R}^+$*. Then h is a function of subclass of type I.*

Definition 1.7. *[6, 7]Let* $h, \mathcal{F}\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$*, then we say that the pair* $(\mathcal{F}, h)$ *is an upper class, if h is a function of subclass of type I and: (i)* $0 \leq s \leq 1 \Longrightarrow \mathcal{F}[s,t] \leq \mathcal{F}[1,t]$*, (ii)* $h(1,y) \leq \mathcal{F}[1,t] \Longrightarrow y \leq t$ *for all* $t, y \in \mathbb{R}^+$.

Example 1.8. *[6, 7]Define* $h, \mathcal{F}\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ *by:*

(a) $h(x,y) = (y+l)^x, l > 1$ *and* $\mathcal{F}[s,t] = st + l;$
(b) $h(x,y) = (x+l)^y, l > 1$ *and* $\mathcal{F}[s,t] = (1+l)^{st};$
(c) $h(x,y) = x^m y, m \in \mathbb{N}$ *and* $\mathcal{F}[s,t] = st;$
(d) $h(x,y) = y$ *and* $\mathcal{F}[s,t] = t;$
(d) $h(x,y) = \frac{1}{n+1}\left(\sum_{i=0}^n x^i\right) y, n \in \mathbb{N}$ *and* $\mathcal{F}[s,t] = st;$
(e) $h(x,y) = \left[\frac{1}{n+1}\left(\sum_{i=0}^n x^i\right) + l\right]^y, l > 1, n \in \mathbb{N}$ *and* $\mathcal{F}[s,t] = (1+l)^{st}$

for all $x, y, s, t \in \mathbb{R}^+$*. Then the pair* $(\mathcal{F}, h)$ *is an upper class of type I.*

Let Φ_u denote the class of the functions $\varphi : [0,+\infty) \times [0,+\infty) \to [0,+\infty)$ which satisfy the following conditions:

(a) φ continuous ;
(b) $\varphi(u,v) > 0, (u,v) \neq (0,0)$ and $\varphi(0,0) \geq 0$.

Let Ψ_a be a set of all continuous functions $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying the following conditions:

(ψ_1) ψ is continuous and strictly increasing.

(ψ_2) $\psi(t) = 0$ if and only of $t = 0$.

Also we denote by Ψ the family of all continuous functions from $[0,+\infty) \times [0,+\infty)$ to $[0,+\infty)$ such that $\psi(u,v) = 0$ if and only if $u = v = 0$ where $\psi \in \Psi$.

Lemma 1.9. *([14])Suppose (X, d) is a metric space. Let $\{x_n\}$ be a sequence in Xsuch that $d(x_n, x_{n+1}) \to 0$ as $n \to \infty$. If $\{x_n\}$ is not a Cauchy sequence then there exist an $\varepsilon > 0$ and sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ with*

$m(k) > n(k) > k$ such that $d(x_{m(k)}, x_{n(k)}) \geq \varepsilon$, $d(x_{m(k)-1}, x_{n(k)}) < \varepsilon$ and
(i) $\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)+1}) = \varepsilon$;
(ii) $\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \varepsilon$;
(iii).$\lim_{k\to\infty} d(x_{m(k)-1}, x_{n(k)}) = \varepsilon$
We note that also can see $\lim_{k\to\infty} d(x_{m(k)+1}, x_{n(k)+1}) = \varepsilon$ and $\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)-1}) = \varepsilon$

Definition 1.10. *([21])Let (X, d) be a metric space, $\emptyset \neq A, B \subset X$, $\alpha : A \times A \to [0, \infty)$ a function and $f, g : A \to B$ non-self mappings. We say that (f, g) is a triangular α-proximal admissible pair, if for all $p, q, r, t1, t2, s1, s2 \in A$,*

$$T_1 : \begin{cases} \alpha(t_1, t_2) \geq 1 \\ d(s_1, f(t_1)) = d(A, B) \\ d(s_2, g(t_2)) = d(A, B) \end{cases} \Longrightarrow \alpha(s_1, s_2) \geq 1$$

$$T_2 : \begin{cases} \alpha(p, r) \geq 1 \\ \alpha(r, q) \geq 1 \end{cases} \Longrightarrow \alpha(p, q) \geq 1.$$

Let (X, d) be a metric space, $\emptyset \neq A, B \subset X$. We define

$$\begin{array}{l} A_0 = \{x \in A : d(x, y) = d(A, B) \text{ for some } y \in B\} \\ B_0 = \{y \in B : d(x, y) = d(A, B) \text{ for some } x \in A\} \end{array} \tag{1}$$

Definition 1.11. *([21])Let Let (X, d) be a metric space, $\emptyset \neq A, B \subset X$, and $f, g : A \to B$ non-self mappings. We say that (f, g) is a generalized proximal C-contraction pair if, for all $s, t, p, q \in A$,*

$$\left.\begin{array}{l} d(s, f(p)) = d(A, B) \\ d(t, g(q)) = d(A, B) \end{array}\right\} \Longrightarrow d(s, t) \leq \frac{1}{2}\Big(d(p, t) + d(q, s)\Big) - \psi\Big(d(p, t), d(q, s)\Big), \tag{2}$$

in which $\psi \in \Psi$.

Definition 1.12. *([21])Let (X, d) be a metric space, $\emptyset \neq A, B \subset X$, $\alpha : A \times A \to [0, \infty)$ a function and $f, g : A \to B$ non-self functions. If, for all $s, t, p, q \in A$,*

$$\left.\begin{array}{l} d(s, f(p)) = d(A, B) \\ d(t, g(q)) = d(A, B) \end{array}\right\} \Longrightarrow \alpha(p, q) d(s, t) \leq \frac{1}{2}\Big(d(p, t) + d(q, s)\Big) - \psi\Big(d(p, t), d(q, s)\Big), \tag{3}$$

then (f, g) is said to be an α-proximal C_1-contraction pair.

If in the definition above, we replace (2) by

$$(\alpha(p, q) + l)^{d(s,t)} \leq (l + 1)^{\frac{1}{2}\left(d(p,t)+d(q,s)\right)-\psi\left(d(p,t),d(q,s)\right)}, \tag{4}$$

where $l > 0$, then (f, g) is said to be an α-proximal C_2-contraction pair.

In this paper, we generalize some results of Parvaneh et al. [[21]] to obtain some new common best proximity point theorems. Next, by an example and some fixed point results, we support our main result.

2. Main Results

Definition 2.1. *Let A and B are two nonempty subsets of a metric space, (X, d). Let $\mu : A \times A \to [0, \infty)$ a function and $f, g : A \to B$ non-self mappings. We say that (f, g) is a triangular μ − subproximal admissible pair, if for all $p, q, r, s, t_1, t_2, s_1, s_2 \in A$,*

$$T_1 : \begin{cases} \mu(t_1, t_2) \le 1, \\ d(s_1, f(t_1)) = d(A, B), \\ d(s_2, f(t_2)) = d(A, B) \end{cases} \Longrightarrow \mu(s_1, s_2) \le 1$$

$$T_2 : \begin{cases} \mu(p, r) \le 1, \\ \mu(r, q) \le 1 \end{cases} \Longrightarrow \mu(p, q) \le 1$$

Definition 2.2. *Let Let (X, d) be a metric space, $\emptyset \ne A, B \subset X$, and $f, g : A \to B$ non-self mappings. We say that (f, g) is a generalized proximal C-contraction pair of type C-class if, for all $s, t, p, q \in A$,*

$$\left.\begin{array}{l} d(s, f(p)) = d(A, B) \\ d(t, g(q)) = d(A, B) \end{array}\right\} \Longrightarrow d(s, t) \le F\Big(\frac{1}{2}\big(d(p, t) + d(q, s)\big), \psi\big(d(p, t), d(q, s)\big)\Big), \tag{5}$$

in which $\psi \in \Psi_u$.

Definition 2.3. *Let (X, d) be a metric space, $\emptyset \ne A, B \subset X$, $\alpha : A \times A \to [0, \infty)$ a function and $f, g : A \to B$ non-self functions. If, for all $s, t, p, q \in A$,*

$$\left.\begin{array}{l} d(s, f(p)) = d(A, B) \\ d(t, g(q)) = d(A, B) \end{array}\right\} \Longrightarrow \mathrm{h}(\alpha(p, q), d(s, t)) \le \mathcal{F}\Big(\mu(p, q), F\Big(\frac{1}{2}\big(d(p, t) + d(q, s)\big), \psi\big(d(p, t), d(q, s)\big)\Big)\Big), \tag{6}$$

then (f, g) is said to be an α, μ-proximal C-contraction pair of type C-class.

Theorem 2.4. *Let A and B are two nonempty subsets of a metric space, (X, d). Let A be complete and A_0 be nonempty. Moreover, assume that the non-self functions $f, g : A \to B$ satisfy;*
(i). f, g are continuous,
(ii). $f(A_0) \subset B_0$ and $g(A_0) \subset B_0$,
(iii). (f, g) is a generalised proximal C−contraction pair of type C-class ,
Then, the functions f and g have a unique common best proximity point.

Proof. Choose, $s_0 \in A_0$ be arbitrary. Since $f(A_0) \subset B_0$, there exists $s_1 \in A_0$ such that

$$d(s_1, f(s_0)) = d(A, B).$$

Since $g(A_0) \subset B_0$, there exists $s_2 \in A_0$ such that $d(s_2, g(s_1)) = d(A, B)$. Now as $f(A_0) \subset B_0$, there exists $s_3 \in A_0$ such that $d(s_3, f(s_2)) = d(A, B)$.
We continue this process and construct a sequence $\{s_n\}$ such that

$$\begin{cases} d(s_{2n+1}, f(s_{2n})) = d(A, B), \\ d(s_{2n+2}, g(s_{2n+1})) = d(A, B). \end{cases} \tag{7}$$

for each $n \in \mathbb{N}$
Claim(1).

$$\lim_{n\to\infty} d(s_n, s_{n+1}) = 0 \tag{8}$$

From (5) we get,

$$\begin{aligned} d(s_{2n+1}, s_{2n+2}) &\leq F\Big(\frac{1}{2}\Big(d(s_{2n}, s_{2n+2}) + d(s_{2n+1}, s_{2n+1})\Big), \Psi\Big(d(s_{2n}, s_{2n+2}), d(s_{2n+1}, s_{2n+1})\Big)\Big) \\ &= F\Big(\frac{1}{2}d(s_{2n}, s_{2n+2}), \Psi\Big(d(s_{2n}, s_{2n+2}), 0\Big)\Big) \\ &\leq \frac{1}{2}d(s_{2n}, s_{2n+2}) \\ &\leq \frac{1}{2}\Big[d(s_{2n}, s_{2n+1}) + d(s_{2n+1}, s_{2n+2})\Big] \end{aligned} \tag{9}$$

which implies $d(s_{2n+1}, s_{2n+2}) \leq d(s_{2n}, s_{2n+1})$. Therfore, $\{d(s_{2n}, s_{2n+1})\}$ is an non-negative decreasing sequence and so converges to $d > 0$. Now, as $n \to \infty$ in (9), we getget

$$d \leq \frac{1}{2}d(s_{2n}, s_{2n+1}) \leq \frac{1}{2}(d + d) = d$$

that is,

$$\lim_{n\to\infty} d(s_{2n}, s_{2n+1}) = 2d. \tag{10}$$

Again, taking $n \to \infty$ in (9), and using (10)we get

$$F(d, \Psi(2d, 0)) = d$$

So, $d = 0$, or , $\Psi(2d, 0) = 0$ and hence $d = 0$.

Claim(2).$\{s_n\}$ is cauchy.

By, (8) it is enough to show that subsequence $\{s_{2n}\}$ is cauchy. Suppose, to the contrary, that $\{s_{2n}\}$ is not a Cauchy sequence.By lemma (1.9) there exists $\epsilon > 0$ for which we can find subsequences $\{s_{2n_k}\}$and $\{s_{2m_k}\}$ of $\{s_{2n}\}$with $2n_k > 2m_k > 2k$ such that

$$\begin{aligned} \epsilon &= \lim_{k\to\infty} d(s_{2m(k)}, s_{2n(k)}) = \lim_{k\to\infty} d(s_{2m(k)}, s_{2n(k)+1}) \\ &= \lim_{k\to\infty} d(s_{2m(k)+1}, s_{2n(k)}) = \lim_{k\to\infty} d(s_{2m(k)+1}, s_{2n(k)+1}) \end{aligned} \tag{11}$$

From (5) we have

$$d(s_{2n_{k+1}}, s_{2m_k}) \leq F\Big(\frac{1}{2}\Big(d(s_{2m_k}, s_{2n_k}) + s_{2n_{k+1}}, s_{2m_{k-1}}\Big), \Psi\Big(d(s_{2m_k}, s_{2n_k}), s_{2n_{k+1}}, s_{2m_{k-1}}\Big)\Big) \tag{12}$$

Taking $k \to \infty$ in the above inequality and using (11), and the continuity of F,Ψ, we would obtain

$$F\Big(\frac{1}{2}(\epsilon + \epsilon), \Psi(\epsilon, \epsilon)\Big) = \epsilon$$

and therefore, $\epsilon = 0$, or , $\Psi(\epsilon, \epsilon) = 0$, which would imply $\epsilon = 0$, a contradiction. Thus, $\{s_n\}$ is a cauchy sequence. Since A is complete, there is a $z \in A$ such that $s_n \to z$. Now, from

$d(s_{2n+1}, f(s_{2n})) = d(A, B)$, $\qquad d(s_{2n+2}, g(s_{2n+1})) = d(A, B)$

By continuity of f ang g, taking $n \to \infty$ we have $d(z, f(z)) = d(z, g(z)) = d(A, B)$. So, z is a common best proximity point of the mappings f and g. Let, w is also a common best proximity point of mappings f and g. From (1) we have

$$\begin{aligned} d(z, w) &\leq F\Big(\frac{1}{2}\Big(d(z, w) + d(w, z)\Big), -\Psi\Big(d(z, w), d(w, z)\Big)\Big) \\ &= F\Big(d(z, w), \Psi\Big(d(z, w), d(w, z)\Big)\Big) \end{aligned} \tag{13}$$

So, $d(z, w) = 0$, or , $\Psi(d(z, w), d(z, w)) = 0$,Hence $d(z, w) = 0$, and therefore $z = w$. □

Theorem 2.5. *Let A and B are two nonempty subsets of a metric space, (X,d). Let A be complete and A_0 be nonempty. Moreover, assume that the non-self functions $f,g : A \to B$ satisfy;*
(i). f,g are continuous,
(ii). $f(A_0) \subset B_0$ and $g(A_0) \subset B_0$,
(iii). (f,g) is an α,μ-proximal C-contraction pair of type C-class ,
(iv). (f,g) is a triangular $\alpha-$proximal admissible pair and a triangular $\mu-$ subproximal admissible pair,
(v). there exist $s_0,s_1 \in A_0$ such that $d(s_1,f(s_0)) = d(A,B)$, $\alpha(s_1,s_0) \geq 1, \mu(s_1,s_0) \leq 1$. Then, the functions f and g have a common best proximity point.Furthermore, if $z,w \in X$ are common best proximity points and $\alpha(z,w) \geq 1$,$\mu(z,w) \leq 1$, then common best proximity point is unique.

Proof. By (iv), we can find $s_0,s_1 \in A_0$ such that

$$d(s_1,f(s_0)) = d(A,B), \qquad \alpha(s_1,s_0) \geq 1, \mu(s_1,s_0) \leq 1.$$

Define the sequence $\{s_n\}$ as in (7) of the theorem(2.4). Since, (f,g) is triangular $\alpha-$proximal admissible and triangular $\mu-subproximal$ admissible, we have $\alpha(s_n,s_{n+1}) \geq 1$, $\mu(s_n,s_{n+1}) \leq 1$. Then

$$\begin{cases} \alpha(s_n,s_{n+1}) \geq 1, \\ d(s_{2n+1},f(s_{2n})) = d(A,B) \\ d(s_{2n+2},g(s_{2n+1})) = d(A,B). \end{cases} \tag{14}$$

and

$$\begin{cases} \mu(s_n,s_{n+1}) \leq 1, \\ d(s_{2n+1},f(s_{2n})) = d(A,B) \\ d(s_{2n+2},g(s_{2n+1})) = d(A,B). \end{cases} \tag{15}$$

If $s = s_{2n+1}$, $t = s_{2n+2}$, $p = s_{2n}$, $q = s_{2n+1}$, and (f,g) is an α,μ-proximal C-contraction pair of type C-class. Then,

$$\begin{aligned} \mathrm{h}\Big(1,d(s_{2n+1},s_{2n+2})\Big) &\leq \mathrm{h}\Big(\alpha(s_{2n},s_{2n+1}),d(s_{2n+1},s_{2n+2})\Big) \\ &\leq \mathcal{F}\Big[\mu(s_{2n},s_{2n+1}),F\Big(\frac{1}{2}\Big(d(s_{2n},s_{2n+2})+d(s_{2n+1},s_{2n+1})\Big),\psi\Big(d(s_{2n},s_{2n+2}),d(s_{2n+1},s_{2n+1})\Big)\Big)\Big], \\ &\leq \mathcal{F}\Big[1,F\Big(\frac{1}{2}\Big(d(s_{2n},s_{2n+2})+d(s_{2n+1},s_{2n+1})\Big),\psi\Big(d(s_{2n},s_{2n+2}),d(s_{2n+1},s_{2n+1})\Big)\Big)\Big], \end{aligned}$$

so,

$$\begin{aligned} d(s_{2n+1},s_{2n+2}) &\leq F\Big(\frac{1}{2}\Big(d(s_{2n},s_{2n+2})+d(s_{2n+1},s_{2n+1})\Big),\Psi\Big(d(s_{2n},s_{2n+2}),d(s_{2n+1},s_{2n+1})\Big)\Big) \\ &= F\Big(\frac{1}{2}d(s_{2n},s_{2n+2}),\Psi\Big(d(s_{2n},s_{2n+2}),0\Big)\Big) \\ &\leq \frac{1}{2}d(s_{2n},s_{2n+2}) \\ &\leq \frac{1}{2}\Big[d(s_{2n},s_{2n+1})+d(s_{2n+1},s_{2n+2})\Big] \end{aligned} \tag{16}$$

which implies $d(s_{2n+1},s_{2n+2}) \leq d(s_{2n},s_{2n+1})$. Therfore, $\{d(s_{2n},s_{2n+1})\}$ is an non-negative decreasing sequence and so converges to $d > 0$. Now, as $n \to \infty$ in (16), we get

$$d \leq \frac{1}{2}d(s_{2n},s_{2n+1}) \leq \frac{1}{2}(d+d) = d$$

that is,

$$\lim_{n\to\infty} d(s_{2n}, s_{2n+1}) = 2d. \tag{17}$$

Again, taking $n \to \infty$ in (9), and using (17) we get

$$F(d, \Psi(2d, 0)) = d$$

So, $d = 0$, or , $\Psi(2d, 0) = 0$ and hence $d = 0$. Therefore,

$$\lim_{n\to\infty} d(s_n, s_{n+1}) = 0 \tag{18}$$

Now we prove that

$$\alpha(s_{2m_k-1}, s_{2n_k}) \geq 1, \mu(s_{2m_k-1}, s_{2n_k}) \leq 1, \qquad n_k > m_k > k. \tag{19}$$

Since (f, g) is triangular $\alpha-$ proximal admissible and triangular $\mu - sub$proximal admissible and

$$\begin{cases} \alpha(s_{2m_k-1}, s_{2m_k}) \geq 1 \\ \alpha(s_{2m_k}, s_{2m_k+1}) \geq 1 \end{cases}$$

$$\begin{cases} \mu(s_{2m_k-1}, s_{2m_k}) \leq 1 \\ \mu(s_{2m_k}, s_{2m_k+1}) \leq 1 \end{cases}$$

From (T_2) of definition(1.10) and definition(2.1) we have

$$\begin{aligned} \alpha(s_{2m_k-1}, s_{2m_k+1}) &\geq 1 \\ \mu(s_{2m_k-1}, s_{2m_k+1}) &\leq 1. \end{aligned}$$

Again, since (f, g) is triangular $\alpha-$proximal admissible and triangular $\mu - sub$proximal admissible and

$$\begin{cases} \alpha(s_{2m_k-1}, s_{2m_k+1}) \geq 1 \\ \alpha(s_{2m_k+1}, s_{2m_k+2}) \geq 1 \end{cases}$$

$$\begin{cases} \mu(s_{2m_k-1}, s_{2m_k+1}) \leq 1 \\ \mu(s_{2m_k+1}, s_{2m_k+2}) \leq 1 \end{cases}$$

From (T_2) of definition(1.10) and definition(2.1) again, we have

$$\begin{aligned} \alpha(s_{2m_k-1}, s_{2m_k+2}) &\geq 1 \\ \mu(s_{2m_k-1}, s_{2m_k+2}) &\leq 1. \end{aligned}$$

By continuing this process, we get (19).

Now, we prove that $\{s_n\}$ is cauchy.

By, (18) it is enough to show that subsequence $\{s_{2n}\}$ is cauchy. Suppose, to the contrary, that $\{s_{2n}\}$ is not a Cauchy sequence.By lemma (1.9) there exists $\epsilon > 0$ for which we can find subsequences $\{s_{2n_k}\}$and $\{s_{2m_k}\}$ of $\{s_{2n}\}$with $2n_k > 2m_k > 2k$ such that

$$\begin{aligned} \epsilon &= \lim_{k\to\infty} d(s_{2m(k)}, s_{2n(k)}) = \lim_{k\to\infty} d(s_{2m(k)}, s_{2n(k)+1}) \\ &= \lim_{k\to\infty} d(s_{2m(k)+1}, s_{2n(k)}) = \lim_{k\to\infty} d(s_{2m(k)+1}, s_{2n(k)+1}) \end{aligned} \tag{20}$$

Now if let $s = s_{2n_k+1}$, $t = s_{2m_k}$, $p = s_{2n_k}$, $q = s_{2m_k} - 1$, then

$$\begin{aligned} \mathrm{h}\Big(1, d(s_{2n_k+1}, s_{2m_k})\Big) &\leq \mathrm{h}\Big(\alpha(s_{2n_k}, s_{2m_k-1}), d(s_{2n_k+1}, s_{2m_k})\Big) \\ &\leq \mathcal{F}\Big[\mu(s_{2n_k}, s_{2m_k-1}), F\Big(\frac{1}{2}\Big(d(s_{2n_k}, s_{2m_k}) + d(s_{2m_k-1}, s_{2n_k+1})\Big), \psi\Big(d(s_{2n_k}, s_{2m_k}), d(s_{2m_k-1}, s_{2n_k+1})\Big)\Big)\Big] \\ &\leq \mathcal{F}\Big[1, F\Big(\frac{1}{2}\Big(d(s_{2n_k}, s_{2m_k}) + d(s_{2m_k-1}, s_{2n_k+1})\Big), \psi\Big(d(s_{2n_k}, s_{2m_k}), d(s_{2m_k-1}, s_{2n_k+1})\Big)\Big)\Big] \end{aligned}$$

Therefore,

$$d(s_{2n_{k+1}}, s_{2m_k}) \leq F\Big(\frac{1}{2}\Big(d(s_{2m_k}, s_{2n_k}) + s_{2n_{k+1}}, s_{2m_{k-1}}\Big), \Psi\Big(d(s_{2m_k}, s_{2n_k}), d(s_{2n_{k+1}}, s_{2m_{k-1}})\Big)\Big) \tag{21}$$

Taking $k \to \infty$ in the above inequality and using (20), and the continuity of F, Ψ, we would obtain

$$F\Big(\frac{1}{2}(\epsilon + \epsilon), \Psi(\epsilon, \epsilon)\Big) = \epsilon$$

and therefore, $\epsilon = 0$, or , $\Psi(\epsilon, \epsilon) = 0$, which would imply $\epsilon = 0$, a contradiction. Thus, $\{s_n\}$ is a cauchy sequence. Since A is complete, there is a $z \in A$ such that $s_n \to z$. Now, from
$d(s_{2n+1}, f(s_{2n})) = d(A, B)$, $\quad$ $d(s_{2n+2}, g(s_{2n+1})) = d(A, B)$
By continuity of f and g, taking $n \to \infty$ we have $d(z, f(z)) = d(z, g(z)) = d(A, B)$. So, z is a common best proximity point of the mappings f and g. Let, w is also a common best proximity point of mappings f and g. Since $\alpha(z, w) \geq 1$, $\mu(z, w) \leq 1$from (6) we have

$$\begin{aligned} \mathrm{h}\Big(1, d(z, w)\Big) &\leq \mathrm{h}\Big(\alpha(z, w), d(z, w)\Big) \\ &\leq \mathcal{F}\Big[\mu(z, w), F\Big(\frac{1}{2}\Big(d(z, w) + d(w, z)\Big), \psi\Big(d(z, w), d(w, z)\Big)\Big)\Big], \\ &\leq \mathcal{F}\Big[1, F\Big(\frac{1}{2}\Big(d(z, w) + d(w, z)\Big), \psi\Big(d(z, w), d(w, z)\Big)\Big)\Big), \end{aligned}$$

therefore ,

$$\begin{aligned} d(z, w) &\leq F\Big(\frac{1}{2}\Big(d(z, w) + d(w, z)\Big), -\Phi\Big(d(z, w), d(w, z)\Big)\Big) \\ &= F\Big(d(z, w), \Phi\Big(d(z, w), d(w, z)\Big)\Big) \end{aligned}$$

So, $d(z, w) = 0$, or , $\Psi(d(z, w), d(z, w)) = 0$,Hence $d(z, w) = 0$, and therefore $z = w$. □

Definition 2.6. *([21])Let $\alpha : X \times X \to \mathbb{R}$ be a function and $f, g : X \to X$ self-mappings and $p, q, r \in X$ be any three elements. We say that (f, g) is a triangular $\alpha-$admissible pair if*

$$(i)\alpha(p, q) \geq 1 \quad \Longrightarrow \quad \alpha(f(p), g(q)) \geq 1 \ or \ \alpha(g(p), f(q)) \geq 1,$$

$$(ii) \begin{cases} \alpha(p, r) \geq 1 \\ \alpha(r, q) \geq 1 \end{cases} \quad \Longrightarrow \quad \alpha(p, q) \geq 1$$

Definition 2.7. *Let $\mu : X \times X \to \mathbb{R}$ be a function and $f, g : X \to X$ self-mappings and $p, q, r \in X$ be any three elements. We say that (f, g) is a triangular $\mu -$ subadmissible pair if*

$$(i)\mu(p, q) \leq 1 \quad \Longrightarrow \quad \mu(f(p), g(q)) \leq 1 \ or \ \mu(g(p), f(q)) \leq 1,$$

$$(ii) \begin{cases} \mu(p, r) \leq 1 \\ \mu(r, q) \leq 1 \end{cases} \quad \Longrightarrow \quad \mu(p, q) \leq 1$$

The corollary is an consequence of the last theorem.

Corollary 2.8. *Let* (X,d) *be a complete metric space and* $f,g : X \to X$. *Moreover, let the self functions* f *and* g *satisfy:*
(i) f *and* g *are continuous,*
(ii)*there exists* $s_0 \in X$ *such that* $\alpha(s_0, f(s_0)) \geq 1$,
(iii) (f,g) *is a triangular* $\alpha-$*admissible pair and triangular* $\mu-$ *subadmissible pair ,*
(iv)*for all* $p,q \in X$,

$$\alpha(p,q)d(f(p),g(q)) \leq \tfrac{1}{2}\mu(p,q)\Big(d(p,g(q)) + d(q,f(p))\Big) - \Psi\Big(d(p,g(q)), d(q,f(p)\Big)$$

(or)

$$\Big(\alpha(p,q)+l\Big)^{d(f(p),g(q))} \leq (l+1)^{\frac{1}{2}\mu(p,q)\Big(d(p,g(q))+d(q,f(p))\Big)-\Psi\Big(d(p,g(q)),d(q,f(p)\Big)}$$

Then f *and* g *have common fixed point. Moreover, if* $x,y \in X$ *are common fixed points and* $\alpha(x,y) \geq 1, \mu(x,y) \leq 1$, *then the common fixed point of* f *and* g *is unique, that is* $x = y$.

Now, we remove the continuity hypothesis of f and g and get the following theorem.

Theorem 2.9. *Let* A *and* B *are two nonempty subsets of a metric space,* (X,d). *Let* A *be complete, the pair* (A,B) *have the* $V-$*property, and* A_0 *be nonempty. Moreover, assume that the non-self functions* $f,g : A \to B$ *satisfy;*
(i). $f(A_0) \subset B_0$ *and* $g(A_0) \subset B_0$,
(ii). (f,g) *is a generalised proximal* $C-$*contraction pairof type C-class,*
Then, the functions f *and* g *have a unique common best proximity point.*

Proof. *By Theorem*(2.4), *there is a cauchy sequence* $\{s_n\} \subset A$ *and* $z \in A_0$ *such that* (7) *holds and* $s_n \to z$. *Moreover, we have*

$$\begin{aligned} d(z,B) &\leq d(z, f(s_{2n})) \\ &\leq d(z, s_{2n+1}) + d(s_{2n+1}, f(s_{2n})) \\ &\leq d(z, s_{2n+1}) + d(A,B). \end{aligned}$$

we take $n \to \infty$ *in the above inequality, and we get*

$$\lim_{n\to\infty} d(z, f(s_n)) = d(z,B) = d(A,B). \tag{22}$$

Since the pair (A,B) *has the* $V-$*property, there is a* $p \in B$ *such that* $d(z,p) = d(A,B)$ *and so* $z \in A_0$. *Moreover, Since* $f(A_0) \subset B_0$, *there is a* $q \in A$ *such that*

$$d(q, f(z)) = d(A,B). \tag{23}$$

Furthermore $d(s_{2n+2}, g(s_{2n+1})) = d(A,B)$ *for every* $n \in \mathbb{N}$.
Since (f,g) *is a generalised proximal* $C-$*contraction pair, we have*

$$d(q, s_{2n+2}) \leq F\Big(\frac{1}{2}\Big(d(z, s_{2n+2}) + d(s_{2n+1}, q)\Big), \Psi\Big(d(z, s_{2n+2}), d(s_{2n+1}, q)\Big)\Big)$$

Letting $n \to \infty$ *in the above inequality, we have*

$$d(q,z) \leq F\Big(\frac{1}{2}\Big(d(z,q)\Big), \Psi\Big(d(z,q), 0\Big)\Big)$$

So, $d(q,z) = 0$, *or ,* $\Psi(d(q,z), 0) = 0$,*Thus* $d(z,q) = 0$, *which implies that* $z = q$. *Then, by* (23), z *is a best proximity point of* f.
Similiarly, it is easy to prove that z *is a best proximity point of* g. *Then,* z *is a common best proximity point of* f *and* g. *By the proof of Theorem*(2.4), *we conclude that* f *and* g *have unique common best proximity point.* □

Theorem 2.10. *Let A and B be two nonempty subsets of complete metric space (X, d). Let A be complete, the pair (A, B) have $V-$property and A_0 is non-empty. Moreover, suppose that the non-self functions $f, g : A \to B$ satisfy:*
(i)$f(A_0) \subset B_0$ and $g(A_0) \subset B_0$,
(ii)(f, g) is an α, μ-proximal C-contraction pair of type C-class,
(iii)(f, g) is a triangular $\alpha-$proximal admissible pair, and a triangular $\mu-$ subproximal admissible pair
(iv)there exist $s_0, s_1 \in A_0$ such that $d(s_1, f(s_0)) = d(A, B)$, $\alpha(s_1, s_0) \geq 1, \mu(s_1, s_0) \leq 1$.
(v)if $\{s_n\}$ is a sequence in A such that $\alpha(s_n, s_{n+1}) \geq 1$,$\mu(s_n, s_{n+1}) \leq 1$ and $s_n \to s_0$ as $n \to \infty$, then $\alpha(s_n, s_0) \geq 1, \mu(s_n, s_0) \leq 1$ for all $n \in \mathbb{N} \bigcup\{0\}$.
Then f and g have a common best proximity point. Furthermore, if $z, w \in X$ are common best proximity points and $\alpha(z, w) \geq 1, \mu(z, w) \leq 1$, then common best proximity point is unique.

Proof. As similiar to the proof of Theorem (2.5) that there exist a sequence $\{s_n\}$ and z in A such that $s_n \to z$ and $\alpha(s_n, s_{n+1}) \geq 1, \mu(s_n, s_{n+1}) \leq 1$. Now, we have

$$\begin{aligned} d(z, B) &\leq d(z, f(s_{2n})) \\ &\leq d(z, s_{2n+1}) + d(s_{2n+1}, f(s_{2n})) \\ &\leq d(z, s_{2n+1}) + d(A, B). \end{aligned}$$

we take $n \to \infty$ in the above inequality, and we get

$$\lim_{n\to\infty} d(z, f(s_n)) = d(z, B) = d(A, B). \tag{24}$$

Since the pair (A, B) has the $V-$property, there is a $p \in B$ such that $d(z, p) = d(A, B)$ and so $z \in A_0$. Moreover, Since $f(A_0) \subset B_0$, there is a $q \in A$ such that

$$d(q, f(z)) = d(A, B). \tag{25}$$

Furthermore $d(s_{2n+2}, g(s_{2n+1})) = d(A, B)$ for every $n \in \mathbb{N}$. Also, by (v), $\alpha(s_n, z) \geq 1$, $\mu(s_n, z) \leq 1$ for every $n \in \mathbb{N} \bigcup\{0\}$. By (f, g) is an α, μ-proximal C-contraction pair of type C-class,we have

$$\begin{aligned} & \mathrm{h}\Big(1, d(q, s_{2n+2}))\Big) \\ \leq\ & \mathrm{h}\Big(\alpha(z, s_{2n+1}), d(q, s_{2n+2})\Big) \\ \leq\ & \mathcal{F}\Big[\mu(z, s_{2n+1}), F\Big(\frac{1}{2}\Big(d(z, s_{2n+2}) + d(s_{2n+1}, q)\Big) - \Psi\Big(d(z, s_{2n+2}), d(s_{2n+1}, q)\Big)\Big)\Big] \\ \leq\ & \mathcal{F}\Big[1, F\Big(\frac{1}{2}\Big(d(z, s_{2n+2}) + d(s_{2n+1}, q)\Big), \psi\Big(d(z, s_{2n+2}), d(s_{2n+1}, q)\Big)\Big)\Big] \end{aligned}$$

Therefore

$$d(q, s_{2n+2})) \leq F\Big(\frac{1}{2}\Big(d(z, s_{2n+2}) + d(s_{2n+1}, q)\Big), \psi\Big(d(z, s_{2n+2}), d(s_{2n+1}, q)\Big)\Big)$$

Letting $n \to \infty$ in the above inequality, we have

$$d(q, z)) \leq F\Big(\frac{1}{2}\Big(d(z, q)\Big), \psi\Big(0, d(z, q)\Big)\Big)$$

So, $d(q, z) = 0$, or , $\Psi(0, d(q, z)) = 0$,thus $d(z, q) = 0$, which implies that $z = q$. Then, by (25), z is a best proximity point of f. Similiarly, we can prove z is a best proximity point of g. Therefore, z is an common best proximity point of f and g. If $z, w \in X$ are common best proximity points and $\alpha(z, w) \geq 1, \mu(z, w) \leq 1$, then we get

$$\begin{aligned} d(z, w) &\leq F\Big(\frac{1}{2}\Big(d(z, w) + d(w, z)\Big), \Phi\Big(d(z, w), d(w, z)\Big)\Big) \\ &= F\Big(d(z, w), \Phi\Big(d(z, w), d(w, z)\Big)\Big) \\ &\leq d(z, w) \end{aligned}$$

So, $d(z, w) = 0$, or , $\Psi(d(z, w), d(z, w)) = 0$,Therefore, $d(z, w) = 0$ and hence $z = w$. □

The following corollary is an immediate consequence of the main theorem of this section.

Corollary 2.11. *Let (X, d) be a complete metric space and $f, g : X \to X$. Moreover, let the self functions f and g satisfy:*
(i)there exists $s_0 \in X$ such that $\alpha(s_0, f(s_0)) \geq 1$,
(ii) (f, g) is a triangular $\alpha-$admissible pair,
(iii)for all $p, q \in X$,

$$\alpha(p, q)d(f(p), g(q)) \leq \tfrac{1}{2}\mu(p, q)\Big(d(p, g(q)) + d(q, f(p))\Big) - \Psi\Big(d(p, g(q)), d(q, f(p)\Big)$$

(or)

$$\Big(\alpha(p, q) + l\Big)^{d(f(p),g(q))} \leq (l + 1)^{\frac{1}{2}\mu(p,q)\Big(d(p,g(q))+d(q,f(p))\Big)-\Psi\Big(d(p,g(q)),d(q,f(p)\Big)}$$

(iv)if $\{sn\}$ is a sequence in A such that $\alpha(s_n, s_{n+1}) \geq 1$ and $s_n \to s_0$ as $n \to \infty$, then $\alpha(s_n, s_0) \geq 1$ for all $n \in \mathbb{N} \bigcup \{0\}$. Then f and g have common fixed point. Moreover, if $x, y \in X$ are common fixed points and $\alpha(x, y) \geq 1$, then the common fixed point of f and g is unique, that is $x = y$.

Example 2.12. *Consider $X = R$ with the usual metric, $A = \{-8, 0, 8\}$ and $B = \{-4, -2, 4\}$. Then, A and B are nonempty closed subsets of X with $d(A, B) = 2$, $A_0 = \{0\}$ and $B_0 = \{-2\}$. Define $f, g : A \to B$ by*
$f(0) = -2, \quad f(8) = 4, \quad f(-8) = -4$ and $g(x) = -2$ for all $x \in A$.
and $\Psi : [0, \infty) \times [0, \infty) \to [0, \infty)$ by $\Psi(s, t) = \sqrt{st}$ also $F(s, t) = s - t$. If,

$$\begin{cases} d(u, f(p)) = d(A, B) = 2 \\ d(v, f(q)) = d(A, B) = 2 \end{cases}$$

then, $u = v = p = 0$ and $q \in A$. Hence all the conditions of Theorem(2.4) hold for this example and clearly 0 is the unique best proximity point of f and g.

Example 2.13. *Let $X = [0, 2] \times [0, 2]$ and d be the Euclidean metric. Let*
$A = \Big\{(0, m) : 0 \leq m \leq 2)\Big\}$ $\qquad B = \Big\{(2, m) : 0 \leq m \leq 2)\Big\}$
Then, $d(A, B) = 2$, $A_0 = A$ and $B_0 = B$. Define $f, g : A \to B$ by $f(0, m) = (2, m)$ and $g(0, m) = (2, 2)$. Also define $\alpha, \mu : A \times A \to [0, \infty)$ by $\mu(p, q) = 1$ and

$$\alpha(p, q) = \begin{cases} \frac{10}{9} & if\ p, q \in (0, 2) \times \{(0, 0), (0, 2)\}, \\ 0 & otherwise \end{cases}$$

and $\Psi : [0, \infty) \times [0, \infty) \to [0, \infty)$ by

$$\Psi(s, t) = 2 \quad for\ all\ s, t \in X$$

also $F(s,t) = \frac{s}{1+t}$, $h(x,y) = xy$, *and* $\mathcal{F}(s,t) = st$. *Assume that*

$$\begin{cases} d(u, f(p)) = d(A,B) = 2 \\ d(v, f(q)) = d(A,B) = 2 \end{cases}$$

Hence, $u = p$ *and* $v = (0,2)$, *then* $u = v$ *and* (2) *holds. If* $p \neq (0,2)$, *then* $\alpha(p,q) = 0$ *and* (2) *holds, which implies that* (f,g) *is an* $\alpha-$*proximal C-contraction pair of type C-class. Hence, all the hypothesis of the Theorem*$(*)$ *are satisfied. Moreover, if* $\{s_n\}$ *is a sequence such that* $\alpha(s_n, s_{n+1}) \geq 1$ *for every* $n \in \mathbf{N} \cup \{0\}$ *and* $s_n \to s_0$, *then* $s_n = (0,2)$ *for all* $n \in \mathbf{N} \cup \{0\}$ *and hence* $s_0 = (0,2)$. *Then* $\alpha(s_n, s_0) \geq 1$ *for every* $n \in \mathbf{N} \cup \{0\}$. *Clearly,* (A,B) *has the* $V-$ *property and then all the conclusions of Theorem(2.10) hold. Clearly,* $(0,2)$ *is the unique common best proximity point of* f *and* g.

Example 2.14. *Let* $X = [0,3] \times [0,3]$ *and* d *be the Euclidean metric. Let*
$A = \Big\{(0,m) : 0 \leq m \leq 3)\Big\}$ $B = \Big\{(3,m) : 0 \leq m \leq 3)\Big\}$
Then, $d(A,B) = 3$, $A_0 = A$ *and* $B_0 = B$. *Define* $f, g : A \to B$ *by*

$$f(0,m) = \begin{cases} (3,3) & m = \frac{3}{2} \\ (3, \frac{m}{2}) & m \neq \frac{3}{2} \end{cases}$$

and $g(0,m) = (3,3)$. *Also define* $\alpha, \mu : A \times A \to [0,\infty)$ *by* $\mu(p,q) = 1$ *and*

$$\alpha(p,q) = \begin{cases} 3 & if\ p,q \in (0, \frac{3}{2}) \times A, \\ 0 & otherwise \end{cases}$$

and $\Psi : [0,\infty) \times [0,\infty) \to [0,\infty)$ *by*

$$\Psi(s,t) = \frac{1}{2}(s+t) \qquad for\ all\ s,t \in X$$

also $F(s,t) = s - t$, $h(x,y) = xy$, *and* $\mathcal{F}(s,t) = st$.
It is easy to see that all required hypothesis of Theorem(2.10) are satisfied unless (iii). *Clearly* f *and* g *have no common best proximity point.It is worth noting that pair* (f,g) *does not have the triangular* $\alpha-$*proximal admissible property.*

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