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Altering Distances and a Common Fixed Point Theorem in Menger Probabilistic Metric Spaces

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Abstract. This paper presents a common fixed point theorem for two compatible self-mappings satisfying nonlinear contractive type condition defined using a Φ-function. This result extends previous results due to B. S. Choudhury, K. Das, A new contraction principle in Menger spaces, Acta Mathematica Sinica 24 (2008) 1379–1386, and the result due to D. Miheţ, Altering distances in probabilistic Menger spaces, Nonlinear Analysis 71 (2009) 2734–2738.

1. Introduction and Preliminaries

The definition of statistical metric spaces was introduced by K. Menger [12], Schweizer and Sklar [15] and they gave some basic results on these spaces. Following A. N. Šerstnev [20], H. Sherwood gave a notion of probabilistic metric spaces in [17]. Also, in the same paper Sherwood proved a theorem on a characterization of a nested, closed sequence of nonempty sets in a complete probabilistic metric space.

In the standard notation, let D^+ be the set of all distribution functions $F : \mathbb{R} \to [0,1]$, such that F is a nondecreasing, left-continuous mapping, satisfying $F(0) = 0$ and $\sup_{x\in\mathbb{R}} F(x) = 1$. The space D^+ is partially ordered by the usual point-wise ordering of functions, i.e., $F \leq G$ if and only if $F(t) \leq G(t)$ for all $t \in \mathbb{R}$. The maximal element for D^+ in this order is the distribution function ε_0 given by

$$\varepsilon_0(t) = \begin{cases} 0, & t \leq 0, \\ 1, & t > 0. \end{cases}$$

Definition 1.1. *[15] A binary operation $T : [0,1]\times[0,1] \to [0,1]$ is a continuous t-norm if T satisfies the following conditions:*

(a) T is commutative and associative;
(b) T is continuous;
(c) $T(a,1) = a$ for all $a \in [0,1]$;
(d) $T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$, and $a,b,c,d \in [0,1]$.

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Examples of t-norm are $T_M(a,b) = \min\{a,b\}$ and $T_p(a,b) = ab$.

The t-norms are defined recursively by $T^1 = T$ and

$$T^n(x_1,\dots,x_{n+1}) = T(T^{n-1}(x_1,\dots,x_n), x_{n+1}).$$

for $n \geq 2$ and $x_i \in [0,1]$ for all $i \in \{1,\dots,n+1\}$.

Definition 1.2. *A* Menger probabilistic metric space *(briefly, Menger PM-space) is a triple $(X,\mathcal{F},T)$ where X is a nonempty set, T is a continuous t-norm, and $\mathcal{F}$ is a mapping from $X \times X$ into D^+ such that, if $F_{x,y}$ denotes the value of $\mathcal{F}$ at the pair (x,y), the following conditions hold:*

(PM1) $F_{x,y}(t) = \varepsilon_0(t)$ if and only if $x = y$;
(PM2) $F_{x,y}(t) = F_{y,x}(t)$;
(PM3) $F_{x,z}(t+s) \geq T(F_{x,y}(t), F_{y,z}(s))$ for all $x,y,z \in X$ and $s,t \geq 0$.

Remark 1.3. *[16] Every metric space is a PM-space. Let (X,d) be a metric space and $T_M(a,b) = \min\{a,b\}$ is a continuous t-norm. Define $F_{x,y}(t) = \varepsilon_0(t - d(x,y))$ for all $x,y \in X$ and $t > 0$. The triple $(X,\mathcal{F},T)$ is a PM-space induced by the metric d.*

Definition 1.4. *Let $(X,\mathscr{F},T)$ be a Menger PM-space.*

(1) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is said to be convergent to x in X if, for every $\varepsilon > 0$ and $\lambda > 0$ there exists positive integer N such that $F_{x_n,x}(\varepsilon) > 1 - \lambda$ whenever $n \geq N$.

(2) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is called Cauchy sequence if, for every $\varepsilon > 0$ and $\lambda > 0$ there exists positive integer N such that $F_{x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $n,m \geq N$.

(3) A Menger PM-space is said to be complete if every Cauchy sequence in X is convergent to a point in X.

The (ε,λ)-topology ([15]) in a Menger PM-space $(X,\mathcal{F},T)$ is introduced by the family of neighborhoods $\mathcal{N}_x$ of a point $x \in X$ given by

$$\mathcal{N}_x = \{N_x(\varepsilon,\lambda) : \ \varepsilon > 0, \lambda \in (0,1)\}$$

where

$$N_x(\varepsilon,\lambda) = \{y \in X : \ F_{x,y}(\varepsilon) > 1 - \lambda\}.$$

The (ε,λ)-topology is a Hausdorff topology. In this topology the function f is continuous in $x_0 \in X$ if and only if for every sequence $x_n \to x_0$ we have that $f(x_n) \to f(x_0)$.

The following lemma was proved by B. Schweizer and A. Sklar.

Lemma 1.5. *[15] Let $(X,\mathcal{F},T)$ be a Menger PM-space. Then the function $\mathcal{F}$ is lower semi-continuous for every fixed $t > 0$, i.e. for every fixed $t > 0$ and every two convergent sequences $\{x_n\},\{y_n\} \subseteq X$ such that $x_n \to x$, $y_n \to y$ it follows that*

$$\liminf_{n\to\infty} F_{x_n,y_n}(t) = F_{x,y}(t).$$

Definition 1.6. *Let $(X,\mathcal{F},T)$ be a Menger PM-space and $A \subseteq X$. The closure of the set A is the smallest closed set containing A, and is denoted by $\overline{A}$.*

Obviously, having in mind the Hausdorff topology, and the definition of converging sequences we have the next remark.

Remark 1.7. *$x \in \overline{A}$ if and only if there exists a sequence $\{x_n\}$ in A such that $x_n \to x$.*

The concept of probabilistic boundness was introduced by H. Sherwood in [17]. We give a definition of probabilistic bounded sets.

Definition 1.8. *[5] Let $(X, \mathcal{F}, T)$ be a Menger PM-space and $A \subseteq X$. The probabilistic diameter of set A is given by*

$$\delta_A(t) = \inf_{x,y\in A} \sup_{\varepsilon<t} F_{x,y}(\varepsilon).$$

The diameter of the set A is defined by

$$\delta_A = \sup_{t>0} \inf_{x,y\in A} \sup_{\varepsilon<t} F_{x,y}(\varepsilon).$$

If there exists $\lambda \in (0, 1)$ such that $\delta_A = 1 - \lambda$ the set A will be called probabilistic semi-bounded. If $\delta_A = 1$ the set A will be called probabilistic bounded.

Lemma 1.9. *Let $(X, \mathcal{F}, T)$ be a Menger PM-space. A set $A \subseteq X$ is probabilistic bounded if and only if for each $\lambda \in (0, 1)$ there exists $t > 0$ such that $F_{x,y}(t) > 1 - \lambda$ for all $x, y \in A$.*

Proof: The proof follows from the definitions of $\sup A$ and $\inf A$ of non-empty sets.

It is not difficult to see that every metrically bounded set is also probabilistic bounded if it is considered in the induced PM-space.

H. Sherwood proved the following theorem.

Theorem 1.10. *[17] Let $(X, \mathcal{F}, T)$ be a Menger PM-space and $\{F_n\}$ a nested sequence of nonempty, closed subsets of X such that $\delta_{F_n} \to \varepsilon_0$ as $n \to \infty$. Then there is exactly one point $x_0 \in F_n$, for every $n \in \mathbb{N}$.*

It is easy to show that the following lemma is satisfied.

Lemma 1.11. *Let $(X, \mathcal{F}, T)$ be a Menger PM-space. A collection $\{F_n\}_{n\in\mathbb{N}}$ is said to have probabilistic diameter zero i.e. for each $r \in (0, 1)$ and each $t > 0$ there exists $n^* \in \mathbb{N}$ such that $F_{x,y}(t) > 1 - r$ for all $x, y \in F_{n^*}$ if and only if $\delta_{F_n} \to \varepsilon_0$ as $n \to \infty$.*

Fixed point results for mappings defined on PM-spaces were obtained by several authors ([2], [6], [8], [16], [19]). Many of results in fixed point theory on probabilistic metric spaces are proved for spaces with t-norm T satisfying $T(a, a) \geq a$ (see [19]). In this paper we will prove a common fixed point theorem without a restriction on the t-norm that defines the PM-space. Also in proving common fixed point results one needs to consider some notion of commutativity. The concept of compatible mappings was introduced by G. Jungck [11] and S.N. Mishra [14]. Compatible mappings defined on spaces with non-deterministic distances were considered by B. Singh and M.S. Chauhan [18], S.N. Ješić et al. [7] and N.A. Babačev [1].

Khan et al. in [9] introduced the concept of altering distance functions that alter the distance between two points in metric spaces.

Definition 1.12. *[9] A function $h : [0, \infty) \to [0, \infty)$ is an altering distance function if*

(i) h is monotone increasing and continuous and
(ii) $h(t) = 0$ if and only if $t = 0$.

B.S. Choudhury and K. Das [4] extended the concept of altering distance functions to Menger PM-spaces.

Definition 1.13. *[4] A function $\phi : [0, \infty) \to [0, \infty)$ is said to be a Φ-function if the following conditions hold:*

(i) $\phi(t) = 0$ if and only if $t = 0$;
(ii) ϕ is strictly increasing and $\phi(t) \to \infty$ as $t \to \infty$;
(iii) ϕ is left-continuous in $(0, \infty)$;
(iv) ϕ is continuous at 0.

The class of all Φ-functions will be denoted by Φ.

B.S. Choudhury and K. Das [4] proved the following result.

Theorem 1.14. *[4] Let $(X, \mathcal{F}, T_M)$ be a complete Menger PM-space, with continuous t-norm T_M given by $T_M(a,b) = \min\{a,b\}$ and let f be a continuous self-mapping on X such that for every $x, y \in X$, and all $t > 0$*

$$F_{fx,fy}(\phi(t)) \geq F_{x,y}(\phi(t/c)) \tag{1}$$

where ϕ is a Φ-function and $0 < c < 1$. Then f has a unique fixed point.

The previous result was generalized and improved by D. Miheţ in [13].

Theorem 1.15. *[13] Let $(X, \mathcal{F}, T)$ be a complete Menger PM-space with continuous t-norm T and let f be a self-mapping on X such that for every $x, y \in X$, and all $t > 0$*

$$F_{fx,fy}(\phi(t)) \geq F_{x,y}(\phi(t/c)) \tag{2}$$

where ϕ is a Φ-function and $0 < c < 1$. If there exists $x \in X$ such that the orbit of f in x, $O(f,x) = \{f^m x, m \in \mathbb{N}\backslash\{0\}\}$ is probabilistic bounded, then f has a unique fixed point.

The concept of compatible mappings was introduced by G. Jungck in [11]. Following Jungck, the definition of compatible mappings defined on Menger PM-spacesx is as follows.

Definition 1.16. *[14] Let $(X, \mathcal{F}, T)$ be a Menger PM-space and f and g self-mappings on X. We say that the mappings f and g are compatible if*

$$\liminf_{n\to\infty} F_{f(g(x_n)),g(f(x_n))}(t) = 1 \text{ for every } t > 0, \tag{3}$$

holds whenever $(x_n)_{n\in\mathbb{N}}$ is a sequence in X such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = z \in X$ holds.

It is easy to see that the class of compatible mappings is broader than the class of commuting mappings. Indeed, every pair of commuting mappings is also compatible, while the converse is not true ([19]).

2. Main Results

Theorem 2.1. *Let $(X, \mathcal{F}, T)$ be a complete Menger probabilistic metric space. Let f and g be compatible self-mappings on X and f continuous such that $g(X) \subseteq f(X)$. Assume that for all $x, y \in X$ and every $t > 0$*

$$F_{g(x),g(y)}(\phi(t)) \geq F_{f(x),f(y)}(\phi(t/c)) \tag{4}$$

holds, where ϕ is a Φ-function and $0 < c < 1$. If there exists a point $u_0 \in X$ and $n_0 \in \mathbb{N}$ such that the set

$$S = \{g(u_{n_0-1}), g(u_{n_0}), \ldots\}, \tag{5}$$

where $f(u_i) = g(u_{i-1})(i \in \mathbb{N})$, is a probabilistic bounded set, then the mappings g and f have a unique common fixed point in X.

Proof. First, we will prove that the mapping g is continuous. We will prove that for the sequence $y_n \to y$ we have that $g(y_n) \to g(y)$ as $n \to \infty$. Let $t > 0$ be arbitrary. There exists $s > 0$ such that $t > \phi(s)$. Since f is continuous we have that $f(y_n) \to f(y)$ i.e. for all $t > 0$ it holds that $F_{f(y_n),f(y)}(t) \to 1$ as $n \to \infty$. Since distribution functions are nondecreasing, applying (4) we have that $F_{g(y_n),g(y)}(t) \geq F_{g(y_n),g(y)}(\phi(s)) \geq F_{f(y_n),f(y)}(\phi(s/c)) \to 1$ as $n \to \infty$, i.e. $F_{g(y_n),g(y)}(t) \to 1$ as $n \to \infty$ for all $t > 0$, i.e. $g(y_n) \to g(y)$ as $n \to \infty$.

For $u_0 \in X$ from $g(X) \subseteq f(X)$ it follows that there exists a point $u_1 \in X$ such that $g(u_0) = f(u_1)$. By induction, a sequence $\{u_n\}$ can be chosen such that $g(u_{n-1}) = f(u_n)$ and the set $S = \{g(u_{n_0-1}), g(u_{n_0}), \ldots\}$ is probabilistic bounded.

Let us consider the nested sequence of nonempty closed sets defined by

$$F_n = \overline{\{g(u_{n-1}), g(u_n), \ldots\}} \left(= \overline{\{f(u_n), f(u_{n+1}), \ldots\}}\right), \ n \geq n_0.$$

We shall prove that the family $\{F_n\}_{n\geq n_0}$ has probabilistic diameter zero.

Let $r \in (0,1)$ and let $t > 0$ be arbitrary. We will prove that there exists $n^* \in \mathbb{N}$ such that $F_{x,y}(t) > 1 - r$ for all $x, y \in F_{n^*}$. For arbitrary $k \geq n_0$, $k \in \mathbb{N}$, from $F_k \subseteq \overline{S}$ it follows that F_k is a probabilistic bounded set i.e. there exists $t_0 > 0$ such that

$$F_{x,y}(t_0) > 1 - r \quad \text{for} \quad \text{all} \quad x, y \in F_k. \tag{6}$$

There exists $s > 0$ such that $t > \phi(s)$. Since ϕ is a Φ-function it follows that there exists $l \in \mathbb{N}$ such that $\phi(s/c^l) > t_0$. Let $n^* = l + k$ and $x, y \in F_{n^*}$ be arbitrary. There exist sequences $\left\{g(u_{n(i)-1})\right\}$, $\left\{g(u_{n(j)-1})\right\}$ in F_{n^*} $(n(i), n(j) \geq n^*,\ i, j \in \mathbb{N})$ such that $\lim\limits_{i\to\infty} g(u_{n(i)-1}) = x$ and $\lim\limits_{j\to\infty} g(u_{n(j)-1}) = y$.

Since $t > \phi(s)$ from (4) we have

$$\begin{aligned} F_{g(u_{n(i)-1}),g(u_{n(j)-1})}(t) &\geq F_{g(u_{n(i)-1}),g(u_{n(j)-1})}(\phi(s)) \\ &\geq F_{f(u_{n(i)-1}),f(u_{n(j)-1})}(\phi(s/c)) \\ &= F_{g(u_{n(i)-2}),g(u_{n(j)-2})}(\phi(s/c)). \end{aligned}$$

Thus, by induction we get

$$F_{g(u_{n(i)-1}),g(u_{n(j)-1})}(t) \geq F_{g(u_{n(i)-l-1}),g(u_{n(j)-l-1})}(\phi(s/c^l)).$$

Since $\phi(s/c^l) > t_0$ and because $F_{x,y}(\cdot)$ is a nondecreasing function, from the previous inequalities it follows that

$$F_{g(u_{n(i)-1}),g(u_{n(j)-1})}(t) \geq F_{g(u_{n(i)-l-1}),g(u_{n(j)-l-1})}(\phi(s/c^l)) \geq F_{g(u_{n(i)-l-1}),g(u_{n(j)-l-1})}(t_0). \tag{7}$$

As $\{g(u_{n(i)-l-1})\}$, $\{g(u_{n(j)-l-1})\}$ are sequences in F_k, from (6) and (7) it follows that, for all $i, j \in \mathbb{N}$ we have

$$F_{g(u_{n(i)-1}),g(u_{n(j)-1})}(t) > 1 - r.$$

Taking the lim inf as $i, j \to \infty$, and applying Lemma 1.5 we get that $F_{x,y}(t) > 1 - r$ for all $x, y \in F_{n^*}$ i.e. the family $\{F_n\}_{n\geq n_0}$ has probabilistic diameter zero.

Applying Lemma 1.11 and Theorem 1.10 we conclude that this family has nonempty intersection, which consists of exactly one point z. Since the family $\{F_n\}_{n\geq n_0}$ has probabilistic diameter zero and $z \in F_n$ for all $n \geq n_0$, we get that for each $r \in (0,1)$ and each $t > 0$ there exists $k_0 \geq n_0$ such that for all $n \geq k_0$

$$F_{g(u_{n-1}),z}(t) > 1 - r.$$

From the last inequality it follows that for each $r \in (0,1)$ and each $t > 0$ we have that

$$\liminf_{n\to\infty} F_{g(u_{n-1}),z}(t) > 1 - r.$$

Taking $r \to 0$ we get that for each $t > 0$

$$\liminf_{n\to\infty} F_{g(u_{n-1}),z}(t) = 1$$

holds, i.e. $\lim\limits_{n\to\infty} g(u_{n-1}) = z$. Since $f(u_n) = g(u_{n-1})$ we have that $\lim\limits_{n\to\infty} f(u_n) = z$.

Since f and g are compatible, from $\lim\limits_{n\to\infty} g(u_n) = \lim\limits_{n\to\infty} f(u_n) = z$ we have that

$$\liminf_{n\to\infty} F_{f(g(u_n)),g(f(u_n))}(t) = 1$$

holds for every $t > 0$. It follows that for each $t > 0$ $F_{f(z),g(z)}(t) = 1$ holds, i.e.

$$g(z) = f(z). \tag{8}$$

Let us prove that $g(z) = z$. Let $t > 0$ be arbitrary. There exists $s > 0$ such that $t > \phi(s)$. Thus

$$F_{gz,g(u_{n-1})}(t) \geq F_{gz,g(u_{n-1})}(\phi(s)) \geq F_{fz,f(u_{n-1})}(\phi(s/c)) = F_{gz,g(u_{n-2})}(\phi(s/c))$$
$$\geq F_{fz,f(u_{n-2})}(\phi(s/c^2)) = F_{gz,g(u_{n-3})}(\phi(s/c^2)) \geq \ldots \geq F_{fz,f(u_0)}(\phi(s/c^{n-1})).$$

Taking the lim inf as $n \to \infty$, it follows that $F_{gz,z}(t) \geq 1$, i.e. $g(z) = z$, and consequently $f(z) = g(z) = z$.

Let us prove that z is a unique common fixed point of f and g. Let y be another common fixed point of f and g. Let $t > 0$ be arbitrary. There exists $s > 0$ such that $t > \phi(s)$. Thus

$$F_{z,y}(t) = F_{gz,gy}(t) \geq F_{gz,gy}(\phi(s)) \geq F_{fz,fy}(\phi(s/c)) = F_{gz,gy}(\phi(s/c))$$
$$\geq F_{fz,fy}(\phi(s/c^2)) = F_{gz,gy}(\phi(s/c^2)) \geq \ldots \geq F_{fz,fy}(\phi(s/c^n)).$$

Taking the lim inf as $n \to \infty$, it follows that $F_{z,y}(t) \geq 1$, i.e. $z = y$, which shows that the common fixed point of f and g is unique. □

If we take f to be identity mapping in the statement of Theorem 2.1, then since the identity mapping commutes with f and commuting mappings are compatible, we get that the following corollary holds.

Corollary 2.2. *Theorem 1.15 is a consequence of Theorem 2.1.*

Since every metric space is a Menger PM-space, as is shown in previous remarks, we see that Theorem 2.1 is an improvement of the main results of G. Jungck, proved in [10], and D.W. Boyd, J.S.W. Wong proved in [3].

Example 2.3. *Let $(X, \mathcal{F}, T)$ be a complete Menger probabilistic metric space induced by the metric $d(x, y) = |x - y|$ on $X = [0, +\infty) \subset \mathbb{R}$ given in Remark 1.3. Let*

$$f(x) = 2x, \qquad g(x) = \frac{2x}{2+x}, \qquad g(X) = [0, 2) \subset X = f(X), \qquad \phi(t) = t, \qquad c = \frac{1}{2}.$$

We now prove that all the conditions of Theorem 2.1 are satisfied. Since $g(f(x)) = \frac{2x}{1+x}$ and $f(g(x)) = \frac{4x}{2+x}$ we conclude that $f(x)$ and $g(x)$ are not commuting mappings, but they are compatible.
Note that

$$F_{f(g(x)),g(f(x))}(t) = \varepsilon_0\left(t - \frac{2x^2}{(1+x)(2+x)}\right) \text{ and } F_{f(x),g(x)}(t) = \varepsilon_0\left(t - \frac{2x^2+2x}{2+x}\right)$$

Since $\frac{2x^2}{(1+x)(2+x)} \leq \frac{2x^2+2x}{2+x}$ holds for all $x \geq 0$, we get

$$F_{f(g(x)),g(f(x))}(t) \geq F_{f(x),g(x)}(t)$$

for all $x, t \geq 0$. For a sequence $\{x_n\}$ in $[0, +\infty)$ such that $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = z$, from the previous inequality it follows that $\liminf_{n\to\infty} F_{f(g(x_n)),g(f(x_n))}(t) = 1$, i.e. mappings f and g are compatible.

We shall prove that the condition (4) is satisfied, too. Since $\frac{4|x-y|}{(2+x)(2+y)} \leq |x - y|$, for all $x, y \geq 0$, we have

$$F_{g(x),g(y)}\left(\phi(t)\right) = \varepsilon_0\left(t - \frac{4|x-y|}{(2+x)(2+y)}\right)$$
$$\geq \varepsilon_0\left(t - |x-y|\right) = \varepsilon_0\left(2t - |2x - 2y|\right) = F_{f(x),f(y)}(\phi(2t)) = F_{f(x),f(y)}(\phi(t/c)).$$

Since all the conditions of Theorem 2.1 are satisfied, we have that $f(x)$ and $g(x)$ have a unique common fixed point. It is easy to see that this point is $x = 0$.

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