Filomat 31:9 (2017), 2727–2748
https://doi.org/10.2298/FIL1709727Y

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Asymptotic Behavior of Second-Order Impulsive Partial Stochastic Functional Neutral Integrodifferential Equations with Infinite Delay

Zuomao Yan[a], **Xiumei Jia**[a]

[a]*Department of Mathematics, Hexi University, Zhangye, Gansu 734000, P.R. China*

Abstract. In this paper, the existence and asymptotic stability in p-th moment of mild solutions to a class of second-order impulsive partial stochastic functional neutral integrodifferential equations with infinite delay in Hilbert spaces is considered. By using Hölder's inequality, stochastic analysis, fixed point strategy and the theory of strongly continuous cosine families with the Hausdorff measure of noncompactness, a new set of sufficient conditions is formulated which guarantees the asymptotic behavior of the nonlinear second-order stochastic system. These conditions do not require the the nonlinear terms are assumed to be Lipschitz continuous. An example is also discussed to illustrate the efficiency of the obtained results.

1. Introduction

The theory of stochastic partial differential equations has attracted much attention since it plays a vital role in many important areas such as insurance, finance, population dynamics (see [6, 7, 18, 19, 21, 24, 34, 36]). Neutral stochastic partial differential equations arise in many areas of applied mathematics and for this reason these equations have been investigated extensively in the last decades. The existence and uniqueness, and stability for first-order neutral stochastic partial differential equations with delays and without delays have been extensively studied by many authors; see [8, 12, 16, 22] and the references therein.

In many cases, it is advantageous to treat the second-order stochastic differential equations directly rather than to convert them to first-order systems. The second-order stochastic partial differential equations are the right model in continuous time to account for integrated processes that can be made stationary. We know that it is useful for engineers to model mechanical vibrations or charge on a capacitor or condenser subjected to white noise excitation through a second-order stochastic differential equations. Further, many authors investigated the the qualitative analysis of solutions for those equations in Hilbert spaces by using different techniques. Among them, Balasubramaniam and Muthukumar [4] established the approximate controllability of second-order neutral stochastic evolution differential equations by using the Sadovskii fixed-point theorem. Ren and Sun [25] discussed the existence and uniqueness of mild solutions for second-order neutral stochastic evolution equations with infinite delay under Carathéodory conditions by means of the successive approximation. Using the Banach contraction mapping principle, Mahmudov and

2010 *Mathematics Subject Classification*. Primary 34A37, 34F05, 35R60; Secondary 60H15, 35B35.
Keywords. Asymptotic stability, Impulsive partial stochastic functional neutral integrodifferential equations, Infinite delay, Strongly continuous cosine families, Fixed point theorem.
Received: 03 September 2015; Accepted: 18 April 2016
Communicated by Svetlana Janković
Research supported by the National Natural Science Foundation of China (11461019), the President Fund of Scientific Research Innovation and Application of Hexi University (xz2013-10).
Email addresses: yanzuomao@163.com (Zuomao Yan), jiaxiumei04@163.com (Xiumei Jia)

McKibben [23] derived the approximate controllability of abstract second-order neutral stochastic evolution equations. Sakthivel et al. in [27, 31] studied the asymptotic stability of second-order stochastic partial differential equations. Chen [9] also considered the exponential stability and the asymptotic stability for mild solution to second-order neutral stochastic partial differential equations with infinite delay by using Picard approximations and the elementary inequality.

However, in addition to stochastic effects, impulsive effects likewise exist in real systems. A lot of dynamical systems have variable structures subject to stochastic abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc. Therefore, it is necessary and important to consider the existence, uniqueness and other quantitative and qualitative properties of solutions to stochastic systems with impulsive effects [33, 35, 37]. Recently, based on the above stochastic analysis method, the existence, uniqueness and stability of mild solutions for various first-order impulsive stochastic partial differential equations and integrodifferential equations have been extensively studied. For example, Sakthivel and Luo [29, 30], Anguraj and Vinodkumar [2], He and Xu [17], Chen et al. [10, 11], Long et al. [20]. Ren and Sun [26] investigated the existence, uniqueness and stability of solution to second-order neutral impulsive stochastic evolution equations with delay by using the successive approximation. Using the Banach contraction mapping principle and the cosine function theory, Sakthivel et al. in [32] discussed the asymptotic stability of second-order impulsive stochastic differential equations but the result is only in connection without delay. Arthi et al. [3] proved the exponential stability of mild solution for the second-order neutral stochastic partial differential equations with impulses.

In this paper we consider the existence and asymptotic stability of mild solutions for second-order impulsive neutral partial stochastic functional integrodifferential equations with infinite delay in Hilbert spaces of the form

$$d[x'(t) - g(t, x(t-\rho_1(t)))] = \Big[Ax(t) + h\Big(t, x(t-\rho_2(t)), \int_0^t a(t,s,x(s-\rho_3(s)))ds\Big)\Big]dt + f\Big(t, x(t-\rho_4(t)), \int_0^t b(t,s,x(s-\rho_5(s)))ds\Big)dw(t), \quad t \geq 0, t \neq t_k, \tag{1}$$

$$\Delta x(t_k) = I_k(x(t_k^-)), \quad t = t_k, k = 1, \dots, m, \tag{2}$$

$$\Delta x'(t_k) = J_k(x(t_k^-)), \quad t = t_k, k = 1, \dots, m, \tag{3}$$

$$x_0(\cdot) = \varphi \in \mathfrak{B}_{\mathcal{F}_0}([\tilde{m}(0), 0], H), \quad x'(0) = \phi, \tag{4}$$

where the state $x(\cdot)$ takes values in a separable real Hilbert space H with inner product $\langle \cdot, \cdot \rangle_H$ and norm $\| \cdot \|_H$. The operator $A : D(A) \to H$ is the infinitesimal generator of a strongly continuous cosine family on H. Let K be another separable Hilbert space with inner product $\langle \cdot, \cdot \rangle_K$ and norm $\| \cdot \|_K$. Suppose $\{w(t) : t \geq 0\}$ is a given K-valued Wiener process with a covariance operator $Q > 0$ defined on a complete probability space $(\Omega, \mathcal{F}, P)$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$, which is generated by the Wiener process w; and $g : [0,\infty) \times H \to H, h : [0,\infty)\times H\times H \to H, a, b : [0,\infty)\times[0,\infty)\times H \to H, f : [0,\infty)\times H\times H \to L(K,H)$, are all Borel measurable, where $L(K, H)$ denotes the space of all bounded linear operators from K into H; $I_k, J_k : H \to H (k = 1, \dots, m)$, are given functions. Moreover, the fixed moments of time t_k satisfies $0 < t_1 < \cdots < t_m < \lim_{k\to\infty} t_k = \infty$, $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively; $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, represents the jump in the state x at time t_k with I_k, J_k determining the size of the jump; let $\rho_i(t) \in C(\mathbb{R}^+, \mathbb{R}^+) (i = 1, 2, 3, 4, 5)$ satisfy $t - \rho_i(t) \to \infty$ as $t \to \infty$, and $\tilde{m}(0) = \max\{\inf_{s\geq 0}(s - \rho_i(s)), i = 1, 2, 3, 4, 5\}$. Here $\mathfrak{B}_{\mathcal{F}_0}([\tilde{m}(0), 0], H)$ denote the family of all almost surely bounded, $\mathcal{F}_0$- measurable, continuous random variables $\varphi(t) : [\tilde{m}(0), 0] \to H$ with norm $\| \varphi \|_{\mathfrak{B}} = \sup_{\tilde{m}(0)\leq t\leq 0} E \| \varphi(t) \|_H$.

To the best of the authors' knowledge, no results about the existence and asymptotic stability of mild solutions for second-order impulsive neutral partial stochastic functional integrodifferential equations with infinite delay, which is expressed in the form (1)-(4). Although the papers [29, 30, 32] studied the asymptotic stability for nonlinear impulsive stochastic differential and functional differential equations with delay and with infinite delay, besides the fact that [29, 30, 32] applies to the asymptotic stability of systems under the Lipschitz conditions, the class of nonlinear impulsive stochastic systems is also different from the one studied here. From a practical viewpoint, the second-order impulsive stochastic partial differential and neutral functional differential equations with infinite delay deserve a study because they describe a kind of system present in the real world. Hence, for a more realistic abstract model of the equation and for studying the asymptotic stability property, this can be considered by introducing stochastic systems. Motivated by the above consideration, we study this interesting problem, which is natural generalizations of the concepts for impulsive equations well known in the theory of infinite dimensional systems.

The most common and easily verified conditions to guarantee the existence and stability of mild solutions are the impulsive stochastic systems with the nonlinear function is a Lipschitz function. In this paper, we discuss this problem by introducing a more appropriate concept for mild solutions. Then, using Hölder's inequality, stochastic analysis, the Darbo fixed point theorem and the theory of strongly continuous cosine families combined with techniques of the Hausdorff measure of noncompactness, we get the existence and asymptotic stability of mild solutions for system (1)-(4). Especially, as compared to the case for the previous results, we no longer require the Lipschitz continuity of f, h and the compactness assumption on associated operators. In fact, we assume that the nonlinear items f, h are continuous functions while the neutral item g satisfies the generally Lipschitz continuity condition, and some suitable conditions on the above-defined functions, which can make the solution operator satisfies all conditions of the Darbo fixed point theorem. Though the results are not must be limited to apply this theorem under our assumptions, for example, we can choose the Krasnoselskii-Schaefer type fixed point theorem. However, the Darbo fixed point theorem is first used to consider the stability for second-order impulsive partial stochastic functional differential equations. The known results appeared in [9, 29, 30, 32] are generalized to the impulsive neutral stochastic functional integrodifferential systems settings and the case of infinite delay.

The rest of this paper is organized as follows. In Section 2, we introduce some notations and necessary preliminaries. In Section 3, we give our main results. In Section 4, an example is given to illustrate our results. Finally, concluding remarks are given in Section 5.

2. Preliminaries

Let K and H be two real separable Hilbert spaces with inner product $\langle\cdot,\cdot\rangle_K$ and $\langle\cdot,\cdot\rangle_H$, their inner products and by $\|\cdot\|_K, \|\cdot\|_H$ their vector norms, respectively.

Let $(\Omega, \mathcal{F}, P; \mathbb{F})(\mathbb{F} = \{\mathcal{F}\}_{t\geq 0})$ be a complete probability space satisfying that $\mathcal{F}_0$ contains all P-null sets. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued Brownian motion with a trace class operator Q, denote $\mathrm{Tr}(Q) = \sum_{i=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfies that $Qe_i = \lambda_i e_i$. So, actually, $w(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i} w_i(t) e_i$, where $\{w_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Brownian motions. Then, the above K-valued stochastic process $w(t)$ is called a Q-Wiener process. Let $L(K, H)$ be the space of bounded linear operators mapping K into H equipped with the usual norm $\|\cdot\|_H$ and $L(H)$ denotes the Hilbert space of bounded linear operators from H to H. For $\tilde{\psi} \in L(K, H)$ we define

$$\| \tilde{\psi} \|_{L_2^0}^2 = \mathrm{Tr}(\tilde{\psi} Q \tilde{\psi}^*) = \sum_{i=1}^{\infty} \| \sqrt{\lambda_i} \tilde{\psi} e_i \|^2 .$$

If $\| \tilde{\psi} \|_{L_2^0}^2 < \infty$, then ς is called a Q-Hilbert–Schmidt operator, and let $L_2^0(K, H)$ denote the space of all Q-Hilber–Schmidt operators $\tilde{\psi} : K \to H$.

Let Υ be the space of all $\mathcal{F}_0$-adapted process $\psi(t, \tilde{w}) : [\tilde{m}(0), \infty) \times \Omega \to \mathbb{R}$ which is almost certainly continuous in t for fixed $\tilde{w} \in \Omega$. Moreover $\psi(s, \tilde{w}) = \varphi(s)$ for $s \in [\tilde{m}(0), 0]$ and $E \| \psi(t, \tilde{w}) \|_H^p \to 0$ as $t \to \infty$. Also Υ is a Banach space when it is equipped with a norm defined by

$$\| \psi \|_{\Upsilon}^p = \sup_{t\geq 0} E \| \psi(t) \|_H^p .$$

The notation $B_r(x, H)$ stands for the closed ball with center at x and radius $r > 0$ in H.

Definition 2.1. *([15]) The one parameter cosine family* $\{C(t) : t \in \mathbb{R}\} \subset L(H)$ *satisfying*

(i) $C(0) = I$;

(ii) $C(t)x$ *is in continuous in* t *on* $\mathbb{R}$ *for all* $x \in \mathbb{R}$;

(iii) $C(t+s) + C(t-s) = 2C(t)C(s)$ *for all* $t, s \in \mathbb{R}$

is called a strongly continuous cosine family.

The corresponding strongly continuous sine family $\{S(t) : t \in \mathbb{R}\} \subset L(H)$ is defined by $S(t)x = \int_0^t C(s)xds, t \in \mathbb{R}, x \in H$. The generator $A : H \to H$ of $\{C(t) : t \in \mathbb{R}\}$ is given by $Ax = (d^2/dt^2)C(t)x|_{t=0}$ for all $x \in D(A) = \{x \in H : C(\cdot)x \in C^2(\mathbb{R}, H)\}$. It is well known that the infinitesimal generator A is a closed, densely defined operator on H. Such cosine and the corresponding sine families and their generators satisfy the following properties.

Lemma 2.2. *([14]) Suppose that* A *is the infinitesimal generator of a cosine family of operators* $\{C(t) : t \in \mathbb{R}\}$ *Then, the following holds:*

(a) There exists $M_1 \geq 1$ *and* $\alpha \geq 0$ *such that* $\| C(t) \|_H \leq M_1 e^{\alpha t}$ *and hence* $\| S(t) \|_H \leq M_1 e^{\alpha t}$.

(b) $A \int_s^r S(u)xdu = [C(r) - C(u)]x$ *for all* $0 \leq s \leq r < \infty$.

(c) There exists $M_2 \geq 1$ *such that* $\| S(s) - S(r) \|_H \leq M_2 |\int_r^s e^{\alpha|\theta|} d\theta|$ *for all* $0 \leq r \leq s < \infty$.

Definition 2.3. *A stochastic process* $\{x(t), t \in [0, T]\} (0 \leq T < \infty)$ *is called a mild solution of Eqs. (1)-(4) if*

(i) $x(t)$ *is adapted to* $\mathcal{F}_t, t \geq 0$.

(ii) $x(t) \in H$ *has càdlàg paths on* $t \in [0, T]$ *a.s and for each* $t \in [0, T]$, $x(t)$ *satisfies the integral equation*

$$\begin{aligned} x(t) &= C(t)\varphi(0) + S(t)[\phi - g(0, \varphi(-\rho_1(0)))] + \int_0^t C(t-s)g(s, x(s-\rho_1(s)))ds \\ &+ \int_0^t S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s, \tau, x(\tau - \rho_3(\tau)))d\tau\Big)ds \\ &+ \int_0^t S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s, \tau, x(\tau - \rho_2(\tau)))d\tau\Big)dw(s) \\ &+ \sum_{0<t_k<t} C(t-t_k)I_k(x(t_k^-)) + \sum_{0<t_k<t} S(t-t_k)J_k(x(t_k^-)), \end{aligned} \tag{5}$$

and

$$x_0(\cdot) = \varphi \in \mathfrak{B}_{\mathcal{F}_0}([\tilde{m}(0), 0], H), \quad x'(0) = \phi.$$

Definition 2.4. *Let* $p \geq 2$ *be an integer. Eq. (5) is said to be stable in p-th moment if for arbitrarily given* $\varepsilon > 0$ *there exists a* $\tilde{\delta} > 0$ *such that* $\| \varphi \|_{\mathfrak{B}} < \tilde{\delta}$ *guarantees that guarantees that*

$$E\Big[\sup_{t \geq 0} \| x(t) \|_H^p\Big] < \varepsilon.$$

Definition 2.5. *Let* $p \geq 2$ *be an integer. Eq. (5) is said to be asymptotically stable in p-th moment if it stable in p-th moment and for any* $\varphi \in \mathfrak{B}_{\mathcal{F}_0}([\tilde{m}(0), 0], H)$,

$$\lim_{T \to +\infty} E\Big[\sup_{t \geq T} \| x(t) \|_H^p\Big] = 0.$$

Now, we introduce the Hausdorff measure of noncompactness χ_Y defined by

$$\chi_Y(B) = \inf\{\varepsilon > 0; B \text{ has a finite } \varepsilon - \text{net in } H\},$$

for bounded set B in any Hilbert space Y. Some basic properties of $\chi_Y(\cdot)$ are given in the following lemma.

Lemma 2.6. *([5]) Let Y be a real Hilbert space and $B, C \subseteq Y$ be bounded, the following properties are satisfied:*

(1) B is pre-compact if and only if $\chi_Y(B) = 0$;

(2) $\chi_Y(B) = \chi_Y(\overline{B}) = \chi_Y(convB)$, where $\overline{B}$ and $convB$ are the closure and the convex hull of B respectively;

(3) $\chi_Y(B) \leq \chi_Y(C)$ when $B \subseteq C$;

(4) $\chi_Y(B + C) \leq \chi_Y(B) + \chi_Y(C)$ where $B + C = \{x + y : x \in B, y \in C\}$;

(5) $\chi_Y(B \cup C) = \max\{\chi_Y(B), \chi_Y(C)\}$;

(6) $\chi_Y(\lambda B) \leq |\lambda|\chi_Y(B)$ for any $\lambda \in \mathbb{R}$;

(7) If the map $\Phi : D(\Phi) \subseteq Y \to Z$ is Lipschitz continuous with constant κ then $\chi_Z(\Phi B) \leq \kappa\chi_Y(B)$ for any bounded subset $B \subseteq D(\Phi)$, where Z is a Banach space;

Definition 2.7. *([28]) The map $\Phi : V \subseteq Y \to Y$ is said to be a χ_Y-contraction if there exists a positive constant $\kappa < 1$ such that $\chi_Y(\Phi(B)) \leq \kappa\chi_Y(B)$ for any bounded close subset $B \subseteq V$ where Y is a Banach space.*

In this paper we denote by χ_C the Hausdorff's measure of noncompactness of $C([0, b], H)$ and by χ_{Υ} the Hausdorff's measure of noncompactness of Υ.

Lemma 2.8. *([13]) For any $p \geq 1$ and for arbitrary $L_2^0(K, H)$-valued predictable process $\phi(\cdot)$ such that*

$$\sup_{s\in[0,t]} E\left\| \int_0^s \phi(v)dw(v)\right\|_H^{2p} \leq (p(2p-1))^p\left(\int_0^t (E \parallel \phi(s) \parallel_{L_2^0}^{2p})^{1/p}ds\right)^p, \ t \in [0, \infty).$$

In the rest of this paper, we denote by $C_p = (p(p-1)/2)^{p/2}$.

Lemma 2.9. *([1] Darbo). If $V \subseteq Y$ is closed and convex and $0 \in V$, the continuous map $\Phi : V \to V$ is a χ_Y-contraction, if the set $\{x \in V : x = \lambda\Phi x\}$ is bounded for $0 < \lambda < 1$, then the map Φ has at least one fixed point in V.*

3. Main Results

In this section we present our main results on the existence and asymptotic stability in the p-th moment of mild solutions of system (1)-(4). To do this, we make the following hypotheses:

(H1) A is the infinitesimal generator of a strongly continuous cosine family $\{C(t) : t \geq 0\}$ on H and the corresponding sine family $\{S(t) : t \geq 0\}$ satisfy the conditions $\parallel C(t) \parallel_H \leq Me^{-\alpha t}$ and $\parallel S(t) \parallel_H \leq Me^{-\beta t}, t \geq 0$ for some constants $M \geq 1, \alpha > 0$ and $\beta > 0$.

(H2) The function $g : [0, \infty) \times H \to H$ is continuous and there exists $L_g > 0$ such that

$$E \parallel g(t, \psi_1) - g(t, \omega_2) \parallel_H^p \leq L_g E \parallel \psi_1 - \psi_2 \parallel_H^p, \quad t \geq 0, \psi_1, \psi_2 \in H,$$

and

$$E \parallel g(t, \psi) \parallel_H^p \leq L_g E \parallel \psi \parallel_H^p, \quad t \geq 0, \psi \in H$$

with $M^p L_g \alpha^{-p} < 1$.

(H3) There exists a function continuous function $m_a : [0,\infty) \to [0,\infty)$ such that

$$E\left\| \int_0^t a(t,s,\psi)ds \right\|_H^p \leq m_a(t)\Theta_a(E \parallel \psi \parallel_H^p)$$

for a.e.$t \geq 0$ and all $\psi \in H$, where $\Theta_a : [0,\infty) \to (0,\infty)$ is a continuous and nondecreasing function.

(H4) The function $h : [0,\infty) \times H \times H \to H$ satisfies the following conditions:

(i) The function $h : [0,\infty) \times H \times H \to H$ is continuous.

(ii) There exist a continuous function $m_h : [0,\infty) \to [0,\infty)$ and a continuous nondecreasing function $\Theta_h : [0,\infty) \to (0,\infty)$ such that

$$E \parallel h(t,\psi,x) \parallel_H^p \leq m_h(t)\Theta_h(E \parallel \psi \parallel_H^p) + E \parallel x \parallel_H^p,\ t \geq 0, \psi, x \in H.$$

(iii) The set $\{S(t-s)h(s,\psi,\int_0^s a(s,\tau,\psi)d\tau) : t,s \in [0,b], \psi \in B_r(0,H)\}$ is relatively compact in H.

(H5) There exists a function continuous function $m_b : [0,\infty) \to [0,\infty)$ such that

$$E\left\| \int_0^t b(t,s,\psi)ds \right\|_H^p \leq m_b(t)\Theta_b(E \parallel \psi \parallel_H^p)$$

for a.e.$t \geq 0$ and all $\psi \in H$, where $\Theta_b : [0,\infty) \to (0,\infty)$ is a continuous and nondecreasing function.

(H6) The function $f : [0,\infty) \times H \times H \to L(K,H)$ satisfies the following conditions:

(i) The function $f : [0,\infty) \times H \times H \to L(K,H)$ is continuous.

(ii) There exist a continuous function $m_f : [0,\infty) \to [0,\infty)$ and a continuous nondecreasing function $\Theta_f : [0,\infty) \to (0,\infty)$ such that

$$E \parallel f(t,\psi,x) \parallel_H^p \leq m_f(t)\Theta_f(E \parallel \psi \parallel_H^p) + E \parallel x \parallel_H^p,\ t \geq 0, \psi, x \in H.$$

(iii) The set $\{S(t-s)f(s,\psi,\int_0^s b(s,\tau,\psi)d\tau) : t,s \in [0,b], \psi \in B_r(0,H)\}$ is relatively compact in H.

(H7) The functions $I_k, J_k : H \to H$ are completely continuous and that there are constants $d_k^{(j)}, k = 1,2,\ldots m, j = 1,2,3,4$ such that $E \parallel I_k(x) \parallel_H^p \leq d_k^{(1)} E \parallel x \parallel_H^p + d_k^{(2)}, E \parallel J_k(x) \parallel_H^p \leq d_k^{(3)} E \parallel x \parallel_H^p + d_k^{(4)}$ for every $x \in H$.

In the proof of the main results, we need the following lemmas.

Lemma 3.1. *Assume that conditions (H1), (H2) hold. Let Φ_1 be the operator defined by: for each $x \in \Upsilon$,*

$$(\Phi_1 x)(t) = \int_0^t C(t-s)g(s,x(s-\rho_1(s)))ds. \tag{6}$$

Then Φ_1 is continuous on $[0,\infty)$ in p-th mean and maps Υ into itself.

Proof. We first prove that Φ_1 is continuous in p-th moment on $[0,\infty)$. Let $x \in \Upsilon$, $\tilde{t} \geq 0$ and $|\xi|$ be sufficiently small, we have

$$\begin{aligned}
&E \parallel (\Phi_1 x)(\tilde{t}+\xi) - (\Phi_1 x)(\tilde{t}) \parallel_H^p \\
&\leq 2^{p-1} E \Big\| \int_0^{\tilde{t}} [C(\tilde{t}+\xi-s) - C(\tilde{t}-s)] g(s, x(s-\rho_1(s))) ds \Big\|_H^p \\
&\quad + 2^{p-1} E \Big\| \int_{\tilde{t}}^{\tilde{t}+\xi} C(\tilde{t}+\xi-s) g(s, x(s-\rho_1(s))) ds \Big\|_H^p \\
&\leq 2^{p-1} E \Big[\int_0^{\tilde{t}} \parallel [C(\tilde{t}+\xi-s) - C(\tilde{t}-s)] g(s, x(s-\rho_1(s))) \parallel_H ds \Big]^p \\
&\quad + 2^{p-1} M^p E \Big[\int_{\tilde{t}}^{\tilde{t}+\xi} e^{-\alpha(\tilde{t}+\xi-s)} \parallel g(s, x(s-\rho_1(s))) \parallel_H ds \Big]^p \\
&\leq 2^{p-1} \Big[\int_0^{\tilde{t}} \parallel C(\tilde{t}+\xi-s) - C(\tilde{t}-s) \parallel_H^{(p/p-1)} ds \Big]^{p-1} \int_0^{\tilde{t}} E \parallel g(s, x(s-\rho_1(s))) \parallel_H^p ds \\
&\quad + 2^{p-1} M^p \Big[\int_{\tilde{t}}^{\tilde{t}+\xi} e^{-(p\alpha/p-1)(\tilde{t}+\xi-s)} ds \Big]^{p-1} \int_{\tilde{t}}^{\tilde{t}+\xi} E \parallel g(s, x(s-\rho_1(s))) \parallel_H^p ds \to 0 \quad \text{as } \xi \to \infty.
\end{aligned}$$

Thus Φ_1 is continuous in p-th moment on $[0,\infty)$.

Next we show that $\Phi_1(\Upsilon) \subset \Upsilon$. By (H1) and (H2), from the equation (6), we have for $t \in [\tilde{m}(0), \infty)$,

$$\begin{aligned}
E \parallel (\Phi_1 x)(t) \parallel_H^p &\leq E \Big[\int_0^t \parallel C(t-s) g(s, x(s-\rho_1(s))) \parallel_H ds \Big]^p \\
&\leq M^p E \Big[\int_0^t e^{-\alpha(t-s)} \parallel g(s, x(s-\rho_1(s))) \parallel_H ds \Big]^p \\
&\leq M^p \Big[\int_0^t e^{-\alpha(t-s)} ds \Big]^{p-1} \int_0^t e^{-\alpha(t-s)} E \parallel g(s, x(s-\rho_1(s))) \parallel_H^p ds \\
&\leq M^p \alpha^{1-p} \int_0^t e^{-\alpha(t-s)} E \parallel x(s-\rho_2(s)) \parallel_H^p ds \\
&= K_1 \int_0^t e^{-\alpha(t-s)} E \parallel x(s-\rho_2(s)) \parallel_H^p ds.
\end{aligned}$$

However, for any any $\varepsilon > 0$ there exists a $\tilde{t}_1 > 0$ such that $E \parallel x(s-\rho_1(s)) \parallel_H^p < \varepsilon$ for $t \geq \tilde{t}_1$. Thus, we obtain

$$\begin{aligned}
E \parallel (\Phi_1 x)(t) \parallel_H^p &\leq K_1 \int_0^t e^{-\alpha(t-s)} E \parallel x(s-\rho_1(s)) \parallel_H^p ds \\
&\leq K_1 e^{-\alpha t} \int_0^{\tilde{t}_1} e^{\alpha s} E \parallel x(s-\rho_1(s)) \parallel_H^p ds + K_1 \alpha^{-1} \varepsilon.
\end{aligned}$$

As $e^{-\alpha t} \to 0$ as $t \to \infty$ and, there exists $\tilde{t}_2 \geq \tilde{t}_1$ such that for any $t \geq \tilde{t}_2$ we have

$$K_1 e^{-\alpha t} \int_0^{\tilde{t}_1} e^{\alpha s} E \parallel x(s-\rho_1(s)) \parallel_H^p ds < \varepsilon - K_1 \alpha^{-1} \varepsilon.$$

From the above inequality, for any $t \geq \tilde{t}_2$, we obtain $E \parallel (\Phi_1 x)(t) \parallel_H^p < \varepsilon$. That is to say $E \parallel (\Phi_1 x)(t) \parallel_H^p \to 0$ as $t \to \infty$. So we conclude that $\Phi_1(\Upsilon) \subset \Upsilon$. □

Lemma 3.2. *Assume that conditions (H1), (H3), (H4)(i)-(ii) hold. Let Φ_2 be the operator defined by: for each $x \in \Upsilon$,*

$$(\Phi_2 x)(t) = \int_0^t S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds. \tag{7}$$

Then Φ_2 is continuous and maps Υ into itself.

Proof. We first prove that Φ_2 is continuous in p-th moment on $[0,\infty)$. Let $x \in \Upsilon$, $\tilde{t} \geq 0$ and $|\xi|$ be sufficiently small, we have

$$\begin{aligned}
&E \parallel (\Phi_2 x)(\tilde{t}+\xi) - (\Phi_2 x)(\tilde{t}) \parallel_H^p \\
&\leq 2^{p-1} E\Big\| \int_0^{\tilde{t}} [S(\tilde{t}+\xi-s) - S(\tilde{t}-s)]h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds\Big\|_H^p \\
&\quad + 2^{p-1} E\Big\| \int_{\tilde{t}}^{\tilde{t}+\xi} S(\tilde{t}+\xi-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds\Big\|_H^p \\
&\leq 2^{p-1} E\Big[\int_0^{\tilde{t}} \Big\|[S(\tilde{t}+\xi-s) - S(\tilde{t}-s)]h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p \\
&\quad + 2^{p-1} M^p E\Big[\int_{\tilde{t}}^{\tilde{t}+\xi} e^{-\beta(\tilde{t}+\xi-s)} \Big\|h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p \\
&\leq 2^{p-1} \Big[\int_0^{\tilde{t}} \Big\|S(\tilde{t}+\xi-s) - S(\tilde{t}-s)\Big\|_H^{(p/p-1)} ds\Big]^{p-1} \int_0^{\tilde{t}} E \parallel h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p ds \\
&\quad + 2^{p-1} M^p \Big[\int_{\tilde{t}}^{\tilde{t}+\xi} e^{-(p\beta/p-1)(\tilde{t}+\xi-s)} ds\Big]^{p-1} \\
&\quad \times \int_{\tilde{t}}^{\tilde{t}+\xi} E\Big\|h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p ds \to 0 \quad \text{as } \xi \to \infty.
\end{aligned}$$

Thus Φ_2 is continuous in p-th moment on $[0,\infty)$.

Next we show that $\Phi_2(\Upsilon) \subset \Upsilon$. By (H1), (H3) and (H4)(i)-(ii), from the equation (7), we have for $t \in [\tilde{m}(0), \infty)$,

$$\begin{aligned}
&E \parallel (\Phi_2 x)(t) \parallel_H^p \\
&\leq E\Big[\int_0^t \Big\|S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p \\
&\leq M^p E\Big[\int_0^t e^{-\beta(t-s)} \Big\|h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p \\
&\leq M^p \Big[\int_0^t e^{-\beta(t-s)} ds\Big]^{p-1} \int_0^t e^{-\beta(t-s)} E\Big\|h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p ds \\
&\leq M^p \beta^{1-p} \int_0^t e^{-\beta(t-s)} [m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p) + m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds \\
&= K_2 \int_0^t e^{-\beta(t-s)} [m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p) + m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds.
\end{aligned}$$

However, for any any $\varepsilon > 0$ there exists a $\tilde{\tau}_1 > 0$ such that $E \parallel x(s-\rho_2(s)) \parallel_H^p < \varepsilon$ and $E \parallel x(s-\rho_3(s)) \parallel_H^p < \varepsilon$ for $t \geq \tilde{\tau}_1$. Thus, we obtain

$$\begin{aligned}
E \parallel (\Phi_2 x)(t) \parallel_H^p &\leq K_2 \int_0^t e^{-\beta(t-s)} [m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p) + m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds \\
&\leq K_2 e^{-\beta t} \int_0^{\tilde{\tau}_1} e^{\beta s} [m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p)
\end{aligned}$$

$$+ m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds + K_2L_{h,a}[\Theta_h(\varepsilon) + \Theta_a(\varepsilon)],$$

where $L_{h,a} = \sup_{t\geq 0}\int_{\tilde{\tau}_1}^t e^{-\beta(t-s)}[m_h(s) + m_a(s)]ds$. As $e^{-\beta t} \to 0$ as $t \to \infty$ and, there exists $\tilde{\tau}_2 \geq \tilde{\tau}_1$ such that for any $t \geq \tilde{\tau}_2$ we have

$$K_2e^{-\beta t}\int_0^{\tilde{\tau}_1} e^{\beta s}[m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p) + m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds < \varepsilon - K_2L_{h,a}[\Theta_h(\varepsilon) + \Theta_a(\varepsilon)].$$

From the above inequality, for any $t \geq \tilde{\tau}_2$, we obtain $E \parallel (\Phi_2x)(t) \parallel_H^p < \varepsilon$. That is to say $E \parallel (\Phi_2x)(t) \parallel_H^p \to 0$ as $t \to \infty$. So we conclude that $\Phi_2(\Upsilon) \subset \Upsilon$. □

Lemma 3.3. *Assume that conditions (H1), (H5), (H6)(i)-(ii) hold. Let Φ_3 be the operator defined by: for each $x \in \Upsilon$,*

$$(\Phi_3x)(t) = \int_0^t S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s). \tag{8}$$

Then Φ_3 is continuous and maps Υ into itself.

Proof. We first prove that Φ_3 is continuous in p-th moment on $[0,\infty)$. Let $x \in \Upsilon$, $\tilde{t} \geq 0$ and $|\xi|$ be sufficiently small, we have

$$\begin{aligned}
&E \parallel (\Phi_3x)(\tilde{t}+\xi) - (\Phi_3x)(\tilde{t}) \parallel_H^p \\
&\leq 2^{p-1}E\Big\|\int_0^{\tilde{t}}[S(\tilde{t}+\xi-s) - S(\tilde{t}-s)]f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)\Big\|_H^p \\
&\quad + 2^{p-1}E\Big\|\int_{\tilde{t}}^{\tilde{t}+\xi} S(\tilde{t}+\xi-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)\Big\|_H^p \\
&\leq 2^{p-1}C_p\Big[\int_0^{\tilde{t}}\Big(E\Big\|[S(\tilde{t}+\xi-s) - S(\tilde{t}-s)]f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big)^{2/p}ds\Big]^{p/2} \\
&\quad + 2^{p-1}C_p\Big[\int_{\tilde{t}}^{\tilde{t}+\xi}\Big(E\Big\|S(\tilde{t}+\xi-s) \\
&\quad \times f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big)^{2/p}ds\Big]^{p/2} \to 0 \quad \text{as } \xi \to \infty.
\end{aligned}$$

Thus Φ_3 is continuous in p-th moment on $[0,\infty)$.

Next we show that $\Phi_3(\Upsilon) \subset \Upsilon$. By (H1),(H5) and (H6)(i)-(ii), from the equation (8), we have for $t \in [\tilde{m}(0),\infty)$,

$$\begin{aligned}
&E \parallel (\Phi_3x)(t) \parallel_H^p \\
&\leq C_p\Big[\int_0^t\Big(E\Big\|S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big)^{2/p}ds\Big]^{p/2} \\
&\leq C_pM^p\Big[\int_0^t\Big[e^{-p\beta(t-s)}E\Big\|f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big]^{2/p}ds\Big]^{p/2} \\
&\leq C_pM^p\Big[\int_0^t\Big[e^{-p\beta(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]\Big]^{2/p}ds\Big]^{p/2} \\
&\leq C_pM^p\Big[\int_0^t e^{-[\frac{2(p-1)}{p-2}]\beta(t-s)}ds\Big]^{p/2-1}\int_0^t e^{-\beta(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds \\
&\leq C_pM^p\Big[\frac{2\beta(p-1)}{p-2}\Big]^{1-p/2}\int_0^t e^{-\beta(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds \\
&= K_3\int_0^t e^{-\beta(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds.
\end{aligned}$$

However, for any any $\varepsilon > 0$ there exists a $\tilde{\theta}_1 > 0$ such that $E \parallel x(s-\rho_4(s)) \parallel_H^p < \varepsilon$ and $E \parallel x(s-\rho_5(s)) \parallel_H^p < \varepsilon$ for $t \geq \tilde{\theta}_1$. Thus, we obtain

$$\begin{aligned}E \parallel (\Phi_3 x)(t) \parallel_H^p &\leq K_3 \int_0^t e^{-\beta(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds \\ &\leq K_3 \int_0^{\tilde{\theta}_1} e^{\beta s}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) \\ &\quad + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds + K_3 L_{f,b}[\Theta_f(\varepsilon) + \Theta_b(\varepsilon)],\end{aligned}$$

where $L_{f,b} = \sup_{t\geq 0}\int_{\tilde{\theta}_1}^t e^{-\beta(t-s)}[m_f(s) + m_b(s)]ds$. As $e^{-\beta t} \to 0$ as $t \to \infty$ and, there exists $\tilde{\theta}_2 \geq \tilde{\theta}_1$ such that for any $t \geq \tilde{\theta}_2$ we have

$$K_3 e^{-\delta t}\int_0^{t_1} e^{\delta s}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds < \varepsilon - K_3 L_{f,b}[\Theta_f(\varepsilon) + \Theta_b(\varepsilon)].$$

From the above inequality, for any $t \geq \tilde{\theta}_2$, we obtain $E \parallel (\Phi_3 x)(t) \parallel_H^p < \varepsilon$. That is to say $E \parallel (\Phi_3 x)(t) \parallel_H^p \to 0$ as $t \to \infty$. So we conclude that $\Phi_3(\Upsilon) \subset \Upsilon$. □

Now, we are ready to present our main result.

Theorem 3.4. *Assume the conditions (H1)-(H7) hold. Let $p \geq 2$ be an integer. Then the impulsive stochastic differential equations (1)-(4) is asymptotically stable in p-th moment, provided that $(14m)^{p-1}M^p\sum_{k=1}^m (d_k^{(1)} + d_k^{(3)}) < 1$, and*

$$\int_1^\infty \frac{1}{s + \Theta_h(s) + \Theta_a(s) + \Theta_f(s) + \Theta_b(s)} ds = \infty. \tag{9}$$

Proof. We define the nonlinear operator $\Psi : \Upsilon \to \Upsilon$ as $(\Psi x)(t) = \varphi(t)$ for $t \in [\tilde{m}(0), 0]$ and for $t \geq 0$,

$$\begin{aligned}(\Psi x)(t) &= C(t)\varphi(0) + S(t)[\phi - g(0, \varphi(-\rho_1(0)))] + \int_0^t C(t-s)g(s, x(s-\rho_1(s)))ds \\ &\quad + \int_0^t S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds \\ &\quad + \int_0^t S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s) \\ &\quad + \sum_{0<t_k<t} C(t-t_k)I_k(x(t_k^-)) + \sum_{0<t_k<t} S(t-t_k)J_k(x(t_k^-)).\end{aligned}$$

Using (H1)-(H7), and the proof of the Lemmas 3.1-3.3, it is clear that the nonlinear operator Ψ is well defined and continuous. Moreover, for each $t \geq 0$ we have

$$\begin{aligned}E \parallel (\Psi x)(t) \parallel_H^p &\leq 7^{p-1}E \parallel C(t)\varphi(0) \parallel_H^p + 7^{p-1}E \parallel S(t)[\phi - g(0,\varphi(-\rho_1(0)))] \parallel_H^p \\ &\quad + 7^{p-1}E\Big\| \int_0^t C(t-s)g(s, x(s-\rho_1(s)))ds \Big\|_H^p \\ &\quad + 7^{p-1}E\Big\| \int_0^t S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds \Big\|_H^p \\ &\quad + 7^{p-1}E\Big\| \int_0^t S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s) \Big\|_H^p \\ &\quad + 7^{p-1}E\Big\| \sum_{0<t_k<t} C(t-t_k)I_k(x(t_k^-)) \Big\|_H^p + 7^{p-1}E\Big\| \sum_{0<t_k<t} S(t-t_k)J_k(x(t_k^-)) \Big\|_H^p.\end{aligned} \tag{10}$$

Using (H1) and (H2), we have

$$7^{p-1}E \parallel C(t)\varphi(0) \parallel_H^p \quad \leq M^p e^{-p\alpha t} E \parallel \varphi(0) \parallel_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty,$$

$$7^{p-1}E \parallel S(t)[\phi - g(0, \varphi(-\rho_1(0)))] \parallel_H^p \leq 14^{p-1} M^p e^{-p\beta t}[E \parallel \phi \parallel_H^p + L_g E \parallel \varphi(-\rho_1(0)) \parallel_H^p] \rightarrow 0 \quad \text{as } t \rightarrow \infty.$$

By (H1)-(H7) and the proof of the Lemmas 3.1-3.3 again, we obtain

$$7^{p-1}E \left\| \int_0^t C(t-s) g(s, x(s-\rho_1(s))) ds \right\|_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty,$$

$$7^{p-1}E \left\| \int_0^t S(t-s) h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau, x(\tau-\rho_3(\tau))) d\tau\Big) ds \right\|_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty,$$

$$7^{p-1}E \left\| \int_0^t U(t,s) f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau, x(\tau-\rho_5(\tau))) d\tau\Big) dw(s) \right\|_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty,$$

and

$$\begin{aligned} 7^{p-1}E \left\| \sum_{0<t_k<t} S(t-t_k) I_k(x(t_k^-)) \right\|_H^p &\leq 7^{p-1} \sum_{0<t_k<t} E \parallel S(t-t_k) I_k(x(t_k^-)) \parallel_H^p \\ &\leq 7^{p-1} M^p e^{-p\alpha t} E \parallel I_k(x(t_k^-)) \parallel_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty, \end{aligned}$$

$$\begin{aligned} 7^{p-1}E \left\| \sum_{0<t_k<t} S(t-t_k) J_k(x(t_k^-)) \right\|_H^p &\leq 7^{p-1} \sum_{0<t_k<t} E \parallel S(t-t_k) J_k(x(t_k^-)) \parallel_H^p \\ &\leq 7^{p-1} M^p e^{-p\beta t} E \parallel J_k(x(t_k^-)) \parallel_H^p \rightarrow 0 \quad \text{as } t \rightarrow \infty. \end{aligned}$$

So Ψ maps Υ into itself.

Next we prove that the operator Ψ has a fixed point, which is a mild solution of the problem (1)-(4). We shall employ Lemma 2.9. For better readability, we break the proof into a sequence of steps.

Step 1. For $0 < \lambda < 1$, set $\{x \in \Upsilon : x = \lambda \Psi x\}$ is bounded.

Let $x \in \Upsilon$ be a possible solution of $x = \lambda \Psi(x)$ for some $0 < \lambda < 1$. Then, by (H1)-(H7), we have for each $t \in [0, T]$

$$\begin{aligned} &E \parallel x(t) \parallel_H^p \\ &\quad \leq 7^{p-1}E \parallel C(t)\varphi(0) \parallel_H^p + 7^{p-1}E \parallel S(t)[\phi - g(0, \varphi(-\rho_1(0)))] \parallel_H^p \\ &\qquad + 7^{p-1}E \left\| \int_0^t C(t-s) g(s, x(s-\rho_1(s))) ds \right\|_H^p \\ &\qquad + 7^{p-1}E \left\| \int_0^t S(t-s) h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau, x(\tau-\rho_3(\tau))) d\tau\Big) ds \right\|_H^p \\ &\qquad + 7^{p-1}E \left\| \int_0^t S(t-s) f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau, x(\tau-\rho_5(\tau))) d\tau\Big) dw(s) \right\|_H^p \\ &\qquad + 7^{p-1}E \left\| \sum_{0<t_k<t} C(t-t_k) I_k(x(t_k^-)) \right\|_H^p + 7^{p-1}E \left\| \sum_{0<t_k<t} S(t-t_k) J_k(x(t_k^-)) \right\|_H^p \\ &\quad \leq 7^{p-1} e^{-\alpha p t} M^p E \parallel \varphi(0) \parallel_H^p + 14^{p-1} e^{-\beta p t} M^p [E \parallel \phi \parallel_H^p + L_g E \parallel \varphi(-\rho_1(0)) \parallel_H^p] \end{aligned}$$

$$
\begin{aligned}
&+ 7^{p-1}M^pE\Big[\int_0^t e^{-\alpha(t-s)} \parallel g(s, x(s-\rho_1(s)) \parallel_H ds\Big]^p \\
&+ 7^{p-1}M^pE\Big[\int_0^t e^{-\beta(t-s)} \Big\| h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p \\
&+ 7^{p-1}C_p\Big[\int_0^t \Big[e^{-p\beta(t-s)}E\Big\| f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big]^{2/p} ds\Big]^{p/2} \\
&+ (7m)^{p-1}M^p\sum_{k=1}^m e^{-\alpha p(t-t_k)}E \parallel I_k(x(t_k^-)) \parallel_H^p + (7m)^{p-1}M^p\sum_{k=1}^m e^{-\beta p(t-t_k)}E \parallel J_k(x(t_k^-)) \parallel_H^p \\
\le\ & 7^{p-1}e^{-\alpha pt}M^pE \parallel \varphi(0) \parallel_H^p + 14^{p-1}e^{-\beta pt}M^p[E \parallel \phi \parallel_H^p + L_gE \parallel \varphi(-\rho_1(0)) \parallel_H^p] \\
&+ 7^{p-1}M^pT^{p-1}L_g\int_0^t e^{-\alpha p(t-s)}E \parallel x(s-\rho_1(s)) \parallel_H^p ds \\
&+ 7^{p-1}M^pT^{p-1}\int_0^t e^{-\beta p(t-s)}[m_h(s)\Theta_h(E \parallel x(s-\rho_2(s)) \parallel_H^p) + m_a(s)\Theta_a(E \parallel x(s-\rho_3(s)) \parallel_H^p)]ds \\
&+ 7^{p-1}C_pM^pT^{p/2-1}\int_0^t e^{-\beta p(t-s)}[m_f(s)\Theta_f(E \parallel x(s-\rho_4(s)) \parallel_H^p) + m_b(s)\Theta_b(E \parallel x(s-\rho_5(s)) \parallel_H^p)]ds \\
&+ (7m)^{p-1}M^p\sum_{k=1}^m e^{-\alpha p(t-t_k)}[d_k^{(1)}E \parallel x(t_k^-) \parallel_H^p + d_k^{(2)}] + (7m)^{p-1}M^p\sum_{k=1}^m e^{-\beta p(t-t_k)}[d_k^{(3)}E \parallel x(t_k^-) \parallel_H^p + d_k^{(4)}].
\end{aligned}
$$

By the definition of Υ, it follows that

$$E \parallel x(s-\rho_i(s)) \parallel_H^p \le 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^p + 2^{p-1}\sup_{s\in[0,t]} \parallel x(s) \parallel_H^p, i = 1,2,3,4,5.$$

If $\mu(t) = 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^p + 2^{p-1}\sup_{s\in[0,t]} \parallel x(s) \parallel_H^p$, we obtain that

$$
\begin{aligned}
\mu(t) \le\ & 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^p + 14^{p-1}e^{-\alpha pt}M^pE \parallel \varphi(0) \parallel_H^p + 28^{p-1}e^{-\beta pt}M^p[E \parallel \phi \parallel_H^p + L_gE \parallel \varphi(-\rho_1(0)) \parallel_H^p] \\
&+ 14^{p-1}M^pT^{p-1}L_ge^{-\alpha pt}\int_0^t e^{\alpha ps}\mu(s)ds \\
&+ 14^{p-1}M^pT^{p-1}e^{-\beta pt}\int_0^t e^{\beta ps}[m_h(s)\Theta_h(\mu(s)) + m_a(s)\Theta_a(\mu(s))]ds \\
&+ 14^{p-1}C_pM^pT^{p/2-1}e^{-\beta pt}\int_0^t e^{\beta ps}[m_f(s)\Theta_f(\mu(s)) + m_b(s)\Theta_b(\mu(s))]ds \\
&+ (14m)^{p-1}M^pe^{-\alpha pt}\sum_{k=1}^m e^{\alpha pt_k}[d_k^{(1)}\mu(t) + d_k^{(2)}] + (14m)^{p-1}M^pe^{-\beta pt}\sum_{k=1}^m e^{\beta pt_k}[d_k^{(3)}\mu(t) + d_k^{(4)}].
\end{aligned}
$$

Since $\widetilde{L} = (14m)^{p-1}M^p\sum_{k=1}^m(d_k^{(1)} + d_k^{(3)}) < 1$, we obtain

$$
\begin{aligned}
e^{\gamma pt}\mu(t) \le \frac{1}{1-\widetilde{L}}\Big[&\widetilde{M} + 14^{p-1}M^pT^{p-1}L_g\int_0^t e^{\gamma ps}\mu(s)ds \\
&+ 14^{p-1}M^pT^{p-1}\int_0^t e^{\gamma ps}[m_h(s)\Theta_h(\mu(s)) + m_a(s)\Theta_a(\mu(s))]ds \\
&+ 14^{p-1}C_pM^pT^{p/2-1}\int_0^t e^{\gamma ps}[m_f(s)\Theta_f(\mu(s)) + m_b(s)\Theta_b(\mu(s))]ds\Big],
\end{aligned}
$$

where

$$\widetilde{M} = 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^{p} + 14^{p-1} M^p E \parallel \varphi(0) \parallel_H^p + 28^{p-1} M^p [E \parallel \phi \parallel_H^p + L_g E \parallel \varphi(-\rho_1(0)) \parallel_H^p]$$
$$+ (14m)^{p-1} M^p \sum_{k=1}^{m} d_k^{(2)} + (14m)^{p-1} M^p \sum_{k=1}^{m} d_k^{(4)},\ \gamma = \min\{\alpha, \beta\}.$$

Denoting by $\zeta(t)$ the right-hand side of the above inequality, we have

$$e^{\gamma p t}\mu(t) \le \zeta(t) \ \text{ for all } t \in [0, T],$$

and $\zeta(0) = \frac{1}{1-\widetilde{L}}\widetilde{M}$,

$$\zeta'(t) = \frac{1}{1-\widetilde{L}}\Big[14^{p-1} M^p T^{p-1} L_g e^{\gamma p t}\mu(t) + 14^{p-1} M^p T^{p-1} e^{\gamma p t}[m_h(t)\Theta_h(\mu(t)) + m_a(t)\Theta_a(\mu(t))]$$
$$+ 14^{p-1} C_p M^p T^{p/2-1} e^{\gamma p t}[m_f(t)\Theta_f(\mu(t)) + m_b(t)\Theta_b(\mu(t))]\Big]$$
$$\le \frac{1}{1-\widetilde{L}}\Big[14^{p-1} M^p T^{p-1} L_g \zeta(t) + 14^{p-1} M^p T^{p-1} e^{\gamma p t}[m_h(t)\Theta_h(e^{-\gamma p t}\zeta(t)) + m_a(t)\Theta_a(e^{-\gamma p t}\zeta(t))]$$
$$+ 14^{p-1} C_p M^p T^{p/2-1} e^{\gamma p t}[m_f(t)\Theta_f(e^{-\gamma p t}\zeta(t)) + m_b(t)\Theta_b(e^{-\gamma p t}\zeta(t))]\Big].$$

If $\xi(t) = e^{-\gamma p t}\zeta(t)$, then $\xi(0) = \zeta(0)$, $\zeta(t) \le \xi(t)$, and

$$\zeta'(t) \le \frac{1}{1-\widetilde{L}}\Big[14^{p-1} M^p T^{p-1} L_g \xi(t) + 14^{p-1} M^p T^{p-1} e^{\gamma p t}[m_h(t)\Theta_h(\xi(t)) + m_a(t)\Theta_a(\xi(t))]$$
$$+ 14^{p-1} C_p M^p T^{p/2-1} e^{\gamma p t}[m_f(t)\Theta_f(\xi(t)) + m_b(t)\Theta_b(\xi(t))]\Big],$$

and we have

$$\xi'(t) = (-\gamma p)e^{-\gamma p t}\zeta(t) + e^{-\gamma p t}\zeta'(t)$$
$$\le (-\gamma p)\xi(t) + \frac{1}{1-\widetilde{L}}\Big[14^{p-1} M^p T^{p-1} L_g e^{-\gamma p t}\xi(t) + 14^{p-1} M^p T^{p-1}[m_h(t)\Theta_h(\xi(t)) + m_a(t)\Theta_a(\xi(t))]$$
$$+ 14^{p-1} C_p M^p T^{p/2-1}[m_f(t)\Theta_f(\xi(t)) + m_b(t)\Theta_b(\xi(t))]\Big]$$
$$\le m^*(t)[\xi(t) + \Theta_h(\xi(t)) + \Theta_a(\xi(t)) + \Theta_f(\xi(t)) + \Theta_b(\xi(t))],$$

where

$$m^*(t) = \max\Big\{(-\gamma p) + \frac{1}{1-\widetilde{L}} 14^{p-1} M^p T^{p-1} L_g e^{-\gamma p t}, \frac{1}{1-\widetilde{L}} 14^{p-1} M^p T^{p-1} m_h(t), \frac{1}{1-\widetilde{L}} 14^{p-1} M^p T^{p-1} m_a(t),$$
$$\frac{1}{1-\widetilde{L}} 14^{p-1} C_p M^p T^{p/2-1} m_f(t), \frac{1}{1-\widetilde{L}} 14^{p-1} C_p M^p T^{p/2-1} m_b(t)\Big\}.$$

This implies for each $t \in [0, T]$ that

$$\int_{\xi(0)}^{\xi(t)} \frac{du}{u + \Theta_h(u) + \Theta_a(u) + \Theta_f(u) + \Theta_b(u)} \le \int_0^T m^*(s)ds < \infty.$$

This inequality shows that there is a constant $\widetilde{K}$ such that $\xi(t) \le \widetilde{K}, t \in [0, T]$, and hence $\parallel x \parallel_{\Upsilon}^{p} \le \mu(t) \le \widetilde{K}$, where $\widetilde{K}$ depends only on p, γ, M, T and on the functions $m_h(\cdot), \Theta_a(\cdot), m_f(\cdot), \Theta_b(\cdot)$. This indicates that $x(\cdot)$ are bounded on $[0, T]$.

Step 2. $\Psi : \Upsilon \to \Upsilon$ is continuous.

Let $\{x_n(t)\}_{n=0}^{\infty} \subseteq \Upsilon$ with $x_n \to x (n \to \infty)$ in Υ. Then there is a number $r > 0$ such that $E \parallel x_n(t) \parallel_H^p \leq r$ for all n and a.e. $t \in [0, T]$, so $x_n \in B_r(0, \Upsilon) = \{x \in \Upsilon : \parallel x \parallel_{\Upsilon}^p \leq r\}$ and $x \in B_r(0, \Upsilon)$. By the assumptions (H3)-(H7), we have

$$E\Big\| h\Big(s, x_n(s-\rho_2(s)), \int_0^s a(s,\tau,x_n(\tau-\rho_3(\tau)))d\tau\Big) - h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p \to 0 \quad \text{as } n \to \infty,$$

$$E\Big\| f\Big(s, x_n(s-\rho_4(s)), \int_0^s b(s,\tau,x_n(\tau-\rho_5(\tau)))d\tau\Big) - f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p \to 0 \quad \text{as } n \to \infty$$

for each $s \in [0, t]$, and since

$$E\Big\| h\Big(s, x_n(s-\rho_2(s)), \int_0^s a(s,\tau,x_n(\tau-\rho_3(\tau)))d\tau\Big) - h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p$$
$$\leq 2\max\{\Theta_h(r^*), \Theta_a(r^*)\}[m_h(s) + m_h(s)],$$

$$E\Big\| f\Big(s, x_n(s-\rho_4(s)), \int_0^s b(s,\tau,x_n(\tau-\rho_5(\tau)))d\tau\Big) - f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p$$
$$\leq 2\max\{\Theta_f(r^*), \Theta_b(r^*)\}[m_f(s) + m_b(s)].$$

Then by the dominated convergence theorem and $I_k, J_k, k = 1, 2, \ldots, m$, are completely continuous, we have for $t \in [0, T]$,

$$\begin{aligned}
&E \parallel (\Psi x_n)(t) - (\Psi x)(t) \parallel_H^p \\
&\leq 5^{p-1}E\Big\| \int_0^t C(t-s)[g(s, x_n(s-\rho_1(s)) - g(s, x(s-\rho_1(s))]ds\Big\|_H^p \\
&\quad + 5^{p-1}E\Big\| \int_0^t S(t-s)\Big[h\Big(s, x_n(s-\rho_2(s)), \int_0^s a(s,\tau,x_n(\tau-\rho_3(\tau)))d\tau\Big) \\
&\quad - h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big]ds\Big\|_H^p \\
&\quad + 5^{p-1}E\Big\| \int_0^t S(t-s)\Big[f\Big(s, x_n(s-\rho_4(s)), \int_0^s b(s,\tau,x_n(\tau-\rho_5(\tau)))d\tau\Big) \\
&\quad - f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big]dw(s)\Big\|_H^p \\
&\quad + 5^{p-1}E\Big\| \sum_{0<t_k<t} C(t-t_k)[I_k(x_n(t_k^-)) - I_k(x(t_k^-))]\Big\|_H^p \\
&\quad + 5^{p-1}E\Big\| \sum_{0<t_k<t} S(t-t_k)[J_k(x_n(t_k^-)) - J_k(x(t_k^-))]\Big\|_H^p \\
&\leq 5^{p-1}T^{p-1}\int_0^t e^{-\alpha p(t-s)}E \parallel g(s, x_n(s-\rho_1(s)) - g(s, x(s-\rho_1(s)) \parallel_H^p ds \\
&\quad + 5^{p-1}T^{p-1}\int_0^t e^{-\beta p(t-s)}E\Big\| h\Big(s, x_n(s-\rho_2(s)), \int_0^s a(s,\tau,x_n(\tau-\rho_3(\tau)))d\tau\Big) \\
&\quad - h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p ds
\end{aligned}$$

$$+ 5^{p-1}M^pC_pT^{p/2-1}\int_0^t e^{-\beta p(t-s)}E\Big\|f\Big(s,x_n(s-\rho_4(s)),\int_0^s b(s,\tau,x_n(\tau-\rho_5(\tau)))d\tau\Big)$$
$$- f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p ds$$
$$+ (5m)^{p-1}\sum_{k=1}^m e^{-\alpha(t-t_k)}E\parallel I_k(x_n(t_k^-)) - I_k(x(t_k^-))\parallel_H^p$$
$$+ (5m)^{p-1}\sum_{k=1}^m e^{-\beta(t-t_k)}E\parallel J_k(x_n(t_k^-)) - J_k(x(t_k^-))\parallel_H^p \to 0 \quad \text{as } n\to\infty.$$

Then, we have for all $t\in[0,T]$,

$$\parallel \Psi x_n - \Psi x\parallel_{\Upsilon}^p \to 0 \quad \text{as } n\to\infty.$$

Therefore, Ψ is continuous.

Step 3. Ψ is χ-contraction.

To see this, we decompose Ψ as $\Psi_1+\Psi_2$ for $t\in[0,T]$, where

$$(\Psi_1x)(t) = C(t)\varphi(0) + S(t)[\phi - g(0,\varphi(-\rho_1(0)))] + \int_0^t C(t-s)g(s,x(s-\rho_1(s)))ds,$$

and

$$(\Psi_2x)(t) = \int_0^t S(t-s)h\Big(s,x(s-\rho_2(s)),\int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds$$
$$+ \int_0^t S(t-s)f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)$$
$$+ \sum_{0<t_k<t} C(t-t_k)I_k(x(t_k^-)) + \sum_{0<t_k<t} S(t-t_k)J_k(x(t_k^-)).$$

(1) Ψ_1 is a contraction on Υ.

Let $t\in[0,T]$ and $x,y\in\Upsilon$. From (H1) and (H2), we have

$$E\parallel(\Psi_1x)(t) - (\Psi_1y)(t)\parallel_H^p$$
$$= E\Big\|\int_0^t C(t-s)[g(s,x(s-\rho_1(s))) - g(s,y(s-\rho_1(s)))]ds\Big\|_H^p$$
$$\le M^pE\Big[\int_0^t e^{-\alpha(t-s)}\parallel g(s,x(s-\rho_1(s))) - g(s,y(s-\rho_1(s)))\parallel_H ds\Big]^p$$
$$\le M^pL_g\Big[\int_0^t e^{-\alpha(t-s)}ds\Big]^{p-1}\int_0^t e^{-\alpha(t-s)}E\parallel x(s-\rho_1(s)) - y(s-\rho_1(s))\parallel_H^p ds$$
$$\le M^pL_g\alpha^{1-p}\int_0^t e^{-\alpha(t-s)}ds\sup_{s\in[0,T]}E\parallel x(s)-y(s)\parallel_H^p$$
$$\le M^pL_g\alpha^{-p}\parallel x-y\parallel_{\Upsilon}^p.$$

Taking supremum over t

$$\parallel \Psi_1x - \Psi_1y\parallel_{\Upsilon}^p \le L_0\parallel x-y\parallel_{\Upsilon}^p,$$

where $L_0 = M^pL_g\alpha^{-p} < 1$. Hence, Ψ_1 is a contraction on Υ.

(2) Ψ_2 is a compact operator.

For this purpose, we decompose Ψ_2 by $\Psi_2 = \Upsilon_1 + \Upsilon_2$, where

$$(\Upsilon_1 x)(t) = \int_0^t S(t-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds + \int_0^t S(t-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s),$$

and

$$(\Upsilon_2 x)(t) = \sum_{0<t_k<t} C(t-t_k)I_k(x(t_k^-)) + \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)).$$

(i) Υ_1 is a compact operator.

We now prove that $\Upsilon_1(B_r(0,\Upsilon))(t) = \{(\Upsilon_1 x)(t) : x \in B_r(0,\Upsilon)\}$ is relatively compact for every $t \in [0,T]$. If $x \in B_r(0,\Upsilon)$, from the definition of Υ, it follows that

$$\begin{aligned} E \parallel x(s-\rho_i(s)) \parallel_H^p &\le 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^p + 2^{p-1} \sup_{s\in[0,T]} E \parallel x(s) \parallel_H^p \\ &\le 2^{p-1} \parallel \varphi \parallel_{\mathfrak{B}}^p + 2^{p-1} r := r^*, i = 1,2,3,4,5. \end{aligned}$$

It follows from conditions (H4)(iii) and (H6)(iii) that the sets $\{S(t-s)h(s,\psi,\int_0^s a(s,\tau,\psi)d\tau) : t,s \in [0,T], \parallel \psi \parallel_H^p \le r^*\}$ and $\{S(t-s)f(s,\psi,\int_0^s b(s,\tau,\psi)d\tau) : t,s \in [0,T], \parallel \psi \parallel_H^p \le r^*\}$ are relatively compact in H. Moreover, for $x \in B_r(0,\Upsilon)$, from the mean value theorem for the Bochner integral, we can infer that

$$\begin{aligned} (\Upsilon_1 x)(t) \in\ & t\overline{\mathrm{conv}}\Big\{S(t-s)h\Big(s,\psi,\int_0^s a(s,\tau,\psi)d\tau\Big) : t,s \in [0,T], \parallel \psi \parallel_H^p \le r^*\Big\} \\ &+ t^{\frac{1}{2}}\overline{\mathrm{conv}}\Big\{S(t-s)f\Big(s,\psi,\int_0^s b(s,\tau,\psi)d\tau\Big) : t,s \in [0,T], \parallel \psi \parallel_H^p \le r^*\Big\} \end{aligned}$$

for all $t \in [0,T]$, and $\overline{\mathrm{conv}}$ denotes the convex hull. As a result we conclude that the set $\{(\Upsilon_1 x)(t) : x \in B_r(0,\Upsilon)$ is relatively compact in H for every $t \in [0,T]$.

Next we show that Υ_1 maps bounded sets into equicontinuous sets of Υ. Let $0 < \varepsilon < t < T$. From $(\Upsilon_1 B_r(0,\Upsilon))(t)$ is relatively compact for each t and by the strong continuity of $S(t)$, we can choose $0 < \delta < T - t$ with

$$\parallel S(t+\xi)x - S(t)x \parallel_H \le \varepsilon$$

for $x \in (\Phi_2 B_r(0,\Upsilon))(t)$ when $0 < \xi < \delta$. For any $x \in B_r(0,\Upsilon)$. Using (H1)-(H5) and Hölder's inequality, it follows that

$$\begin{aligned} &E \parallel (\Upsilon_1 x)(t+\xi) - (\Upsilon_1 x)(t) \parallel_H^p \\ &\le 6^{p-1}E\Big\| \int_0^{t-\varepsilon} [S(t+\xi-s) - S(t-s)]h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds\Big\|_H^p \\ &+ 6^{p-1}E\Big\| \int_{t-\varepsilon}^{t} [S(t+\xi-s) - S(t-s)]h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds\Big\|_H^p \\ &+ 6^{p-1}E\Big\| \int_t^{t+\xi} S(t+\xi-s)h\Big(s, x(s-\rho_2(s)), \int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)ds\Big\|_H^p \\ &+ 6^{p-1}E\Big\| \int_0^{t-\varepsilon} [S(t+\xi-s) - S(t-s)]f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)\Big\|_H^p \\ &+ 6^{p-1}E\Big\| \int_{t-\varepsilon}^{t} [S(t+\xi-s) - S(t-s)]f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)\Big\|_H^p \\ &+ 6^{p-1}E\Big\| \int_t^{t+\xi} S(t+\xi-s)f\Big(s, x(s-\rho_4(s)), \int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)dw(s)\Big\|_H^p \end{aligned}$$

$$
\begin{aligned}
&\le 6^{p-1}(t-\varepsilon)^{p-1}\int_0^{t-\varepsilon} E\Big\|[S(t+\xi-s)-S(t-s)]h\Big(s,x(s-\rho_2(s)),\int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H^p ds\\
&+6^{p-1}E\Big[\int_{t-\varepsilon}^{t}\| S(t+\xi-s)-S(t-s)\|_H \Big\|h\Big(s,x(s-\rho_2(s)),\int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p\\
&+6^{p-1}E\Big[\int_{t}^{t+\xi}\| S(t+\xi-s)\|_H \Big\|h\Big(s,x(s-\rho_2(s)),\int_0^s a(s,\tau,x(\tau-\rho_3(\tau)))d\tau\Big)\Big\|_H ds\Big]^p\\
&+6^{p-1}C_p\Big[\int_0^{t-\varepsilon}\Big[E\Big\|[S(t+\xi-s)-S(t-s)]f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big]^{2/p}ds\Big]^{p/2}\\
&+6^{p-1}C_p\Big[\int_{t-\varepsilon}^{t}\Big[\| S(t+\xi-s)-S(t-s)\|_H^p E\Big\|f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big]^{2/p}ds\Big]^{p/2}\\
&+6^{p-1}C_p\Big[\int_{t}^{t+\xi}\Big[\| S(t+\xi-s)\|_H^p E\Big\|f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p\Big]^{2/p}ds\Big]^{p/2}\\
&\le 6^{p-1}(t-\varepsilon)^p\varepsilon^p+12^{p-1}M^p\Big[\int_{t-\varepsilon}^{t}e^{-\beta(t-s)}ds\Big]^{p-1}\int_{t-\varepsilon}^{t}e^{-\beta(t-s)}[m_h(s)\Theta_h(E\| x(s-\rho_1(s))\|_H^p)\\
&+m_a(s)\Theta_a(E\| x(s-\rho_2(s))\|_H^p)]ds\\
&+6^{p-1}M^p\Big[\int_{t}^{t+\xi}e^{-\beta(t+\xi-s)}ds\Big]^{p-1}\int_{t}^{t+\xi}e^{-\beta(t+\xi-s)}[m_h(s)\Theta_h(E\| x(s-\rho_1(s))\|_H^p)\\
&+m_a(s)\Theta_a(E\| x(s-\rho_2(s))\|_H^p)]ds\\
&+6^{p-1}C_p(t-\varepsilon)^{p/2-1}\int_0^{t-\varepsilon}E\Big\|S(t+\xi-s)-S(t-s)]f\Big(s,x(s-\rho_4(s)),\int_0^s b(s,\tau,x(\tau-\rho_5(\tau)))d\tau\Big)\Big\|_H^p ds\\
&+12^{p-1}C_pM^p\Big[\int_{t-\varepsilon}^{t}[e^{-p\beta(t-s)}[m_f(s)\Theta_f(E\| x(s-\rho_4(s))\|_H^p)+m_b(s)\Theta_b(E\| x(s-\rho_5(s))\|_H^p)]]^{2/p}ds\Big]^{p/2}\\
&+6^{p-1}C_pM^p\Big[\int_{t}^{t+\xi}[e^{-p\beta(t+\xi-s)}[m_f(s)\Theta_f(E\| x(s-\rho_4(s))\|_H^p)+m_b(s)\Theta_b(E\| x(s-\rho_5(s))\|_H^p)]]^{2/p}ds\Big]^{p/2}\\
&\le 6^{p-1}(t-\varepsilon)^p\varepsilon^p+12^{p-1}M^p\max\{\Theta_h(r^*),\Theta_a(r^*)\}\beta^{1-p}\int_{t-\varepsilon}^{t}e^{-\beta(t-s)}[m_h(s)+m_a(s)]ds\\
&+6^{p-1}M^p\max\{\Theta_h(r^*),\Theta_a(r^*)\}\beta^{1-p}\int_{t}^{t+\xi}e^{-\beta(\tau_2-s)}[m_h(s)+m_a(s)]ds\\
&+6^{p-1}C_p(t-\varepsilon)^{p/2}\varepsilon^p\\
&+12^{p-1}C_pM^p\max\{\Theta_f(r^*),\Theta_b(r^*)\}\Big[\frac{2\beta(p-1)}{p-2}\Big]^{1-p/2}\int_{t-\varepsilon}^{t}e^{-\beta(t-s)}[m_f(s)+m_b(s)]ds\\
&+6^{p-1}C_pM^p\max\{\Theta_f(r^*),\Theta_b(r^*)\}\Big[\frac{2\beta(p-1)}{p-2}\Big]^{1-p/2}\int_{t}^{t+\xi}e^{-\beta(t+\xi-s)}[m_f(s)+m_b(s)]ds.
\end{aligned}
$$

Then the right-hand side of the above inequality is independent of $x \in B_r$ and tends to zero as $\xi \to 0$ and sufficiently small positive number ε. Thus, the set $\{\Upsilon_1 x : x \in B_r(0, \Upsilon)\}$ is equicontinuous.

(ii) Υ_2 is a compact operator.

To prove the compactness of Υ_2, note that

$$
\begin{aligned}
(\Upsilon_2 x)(t) &= \sum_{0<t_k<t} S(t-t_k)I_k(x(t_k^-)) + \sum_{0<t_k<t} S(t-t_k)J_k(x(t_k^-))\\
&= \begin{cases} 0, & t\in[0,t_1],\\ C(t-t_1)I_1(x(t_1^-))+S(t-t_1)J_1(x(t_1^-)), & t\in(t_1,t_2],\\ \cdots & \\ \sum_{k=1}^m C(t-t_k)I_k(x(t_k^-))+\sum_{k=1}^m S(t-t_k)J_k(x(t_k^-)), & t\in(t_m,T],\end{cases}
\end{aligned}
$$

and that the interval $[0, T]$ is divided into finite subintervals by t_k, $k = 1, 2, \ldots, m$, so that we only need to prove that

$$W = \{C(t - t_1)I_1(x(t_1^-)) + S(t - t_1)J_1(x(t_1^-)), \quad t \in [t_1, t_2],\ x \in B_r(0, \Upsilon)\}$$

is relatively compact in $C([t_1, t_2], H)$, as the cases for other subintervals are the same. In fact, from (H1) and (H7), it follows that the set $\{C(t-t_1)I_1(x(t_1^-)) + S(t-t_1)J_1(x(t_1^-)),\ x \in B_r(0, \Upsilon)\}$ is relatively compact in H for all $t \in [t_1, t_2]$.

Next, using the semigroup property, we have for $t_1 \leq s < t \leq t_2$

$$\begin{aligned} & E \parallel [C(t - t_1) - C(s - t_1)]I_1(x(t_1^-)) + [S(t - t_1) - S(s - t_1)]J_1(x(t_1^-)) \parallel_H^p \\ & \leq 2^{p-1}E \parallel [C(t - t_1) - C(s - t_1)]I_1(x(t_1^-)) \parallel_H^p + 2^{p-1}E \parallel [S(t - t_1) - S(s - t_1)]J_1(x(t_1^-)) \parallel_H^p . \end{aligned}$$

Thus, we see that the functions in W are equicontinuous due to the compactness of I_1, J_1 and the strong continuity of the operator $C(t), S(t)$ for all $t \in [0, T]$. Now an application of the Arzelá-Ascoli theorem justifies the relatively compactness of W. Therefore, we conclude that operator Υ_2 is also a compact map.

Let arbitrary bounded subset $V \subset \Upsilon$. Since the mapping Ψ_2 is a compact operator, we get that $\chi_\Upsilon(\Psi_2 V) = 0$. Consequently

$$\chi_\Upsilon(\Psi V) = \chi_\Upsilon(\Psi_1 V + \Psi_2 V) \leq \chi_\Upsilon(\Psi_1 V) + \chi_\Upsilon(\Psi_2 V) \leq L_0 \chi_\Upsilon(V) < \chi_\Upsilon(V).$$

Therefore, Ψ is χ−contraction. In view of Lemma 2.9, we conclude that Ψ has at least one fixed point $x^* \in V \subset \Upsilon$. Then, x is a fixed point of the operator Ψ, which is a mild solution of the system (1)-(4) with $x(s) = \varphi(s)$ on $[\tilde{m}(0), 0]$ and $E \parallel x(t) \parallel_H^p \to 0$ as $t \to \infty$. This shows that the asymptotic stability of the mild solution of (1)-(4). In fact, let $\varepsilon > 0$ be given and choose $\tilde{\gamma} > 0$ such that $\tilde{\gamma} < \varepsilon$ and satisfies $[14^{p-1}M^p + 14^{p-1}(K_2 L_{h,a} + K_3 L_{f,b}]\tilde{\gamma} + (14^{p-1}K_1\alpha^{-1} + \widetilde{L})\varepsilon < \varepsilon$. If $x(t) = x(t, \varphi)$ is mild solution of (1)-(4), with $\parallel \varphi \parallel_{\mathcal{B}}^p + E \parallel \phi \parallel_H^p + L_g E \parallel \varphi(-\rho_1(0)) \parallel_H^p < \tilde{\gamma}$, then $(\Psi x)(t) = x(t)$ and satisfies $E \parallel x(t) \parallel_H^p < \varepsilon$ for every $t \geq 0$. Notice that $E \parallel x(t) \parallel_H^p < \varepsilon$ on $t \in [\tilde{m}(0), 0]$. If there exists $\tilde{t}$ such that $E \parallel x(\tilde{t}) \parallel_H^p = \varepsilon$ and $E \parallel x(s) \parallel_H^p < \varepsilon$ for $s \in [\tilde{m}(0), \tilde{t}]$. Then (10) show that

$$E \parallel x(t) \parallel_H^p \leq [14^{p-1}M^p e^{-p\gamma\tilde{t}} + 14^{p-1}(K_2 L_{h,a} + K_3 L_{f,b})]\tilde{\gamma} + (14^{p-1}K_1\alpha^{-1} + \widetilde{L})\varepsilon < \varepsilon,$$

which contradicts the definition of $\tilde{t}$. Therefore, the mild solution of (1)-(4) is asymptotically stable in p-th moment. □

Remark 3.5. *In [9, 32], the authors get the asymptotic stability results under the Lipschitz continuity of the nonlinear items when A generates a strongly continuous cosine family, respectively. Here, the Darbo fixed point theorem is effectively used to study the asymptotic stability of the system (1)-(4). The results are obtained by using the mixed Lipschitz and continuous conditions, and some appropriate assumptions without the Lipschitz assumption on f, h. Also, the results can generalize and improve the existing ones.*

4. Example

Consider the following second-order impulsive partial stochastic neutral differential equations of the form

$$\begin{aligned} d\Big[\frac{\partial}{\partial t}z(t,x) - \vartheta(t, z(t-\rho(t), x))\Big] = & \frac{\partial^2}{\partial^2 t}z(t,x)dt + \varsigma\Big(t, z(t-\rho(t),x), \int_0^t \varsigma_1(t,s,z(s-\rho(s),x)ds\Big)dt \\ & + \varpi\Big(t, z(t-\rho(t),x), \int_0^t \varpi_1(t,s,z(s-\rho(s),x)\Big)dw(t), \\ & t \geq 0, 0 \leq x \leq \pi, t \neq t_k, \end{aligned} \tag{11}$$

$$z(t,0) = z(t,\pi) = 0, \quad t \geq 0, \tag{12}$$

$$z(t,x) = \varphi(t,x), \quad \frac{\partial}{\partial t}z(0,x) = \phi(x), \quad t \le 0, 0 \le x \le \pi, \tag{13}$$

$$\triangle z(t_k,x) = \int_0^{t_k} \eta_k(t_k - s)z(s,x)ds, \quad \triangle z'(t_k,x) = \int_0^{t_k} \tilde{\eta}_k(t_k - s)z(s,x)ds, \tag{14}$$

where $(t_k)_k \in \mathbb{N}$ is a strictly increasing sequence of positive numbers, $\rho(t) \in C(\mathbb{R}^+,\mathbb{R}^+)$, and $\eta_k, \tilde{\eta}_k \in C(\mathbb{R}^+,\mathbb{R}^+), k = 1,2,\ldots,m$. $w(t)$ denotes a one-dimensional standard Wiener process in H defined on a stochastic space $(\Omega,\mathcal{F},P)$.

Let $H = L^2([0,\pi])$ with the norm $\|\cdot\|$ and define the operator A by $A\omega = \omega''$ with the domain

$$D(A) := \{\omega(\cdot) \in H : \omega, \omega' \text{ are absolutely continuous}, \omega'' \in H, \omega(0) = \omega(\pi) = 0\}.$$

It is well-known that A is the infinitesimal generator of a strongly continuous cosine family $\{C(t) : t \in \mathbb{R}\}$ in H. Furthermore, A has a discrete spectrum with eigenvalues of the form $-n^2, n \in \mathbb{N}$, and corresponding normalized eigenfunctions given by $e_n(x) = \sqrt{2/\pi}\sin(nx)$. Then the following properties hold:

(a) The set $\{e_n : n \in \mathbb{N}\}$ is an orthonormal basis of H and $A\omega = -\sum_{n=1}^{\infty} n^2\langle\omega, e_n\rangle e_n$ for every $x \in D(A)$.

(b) For $\omega \in H, C(t)\omega = \sum_{n=1}^{\infty}\cos(nt)\langle\omega, e_n\rangle e_n$, and the associated sine family is $S(t)\omega = \sum_{n=1}^{\infty}\frac{1}{n}\sin(nt)\langle\omega, e_n\rangle e_n$. Consequently, $\| C(t) \| \le e^{-\pi^2 t}, \| S(t) \| \le e^{-\pi^2 t}$ for all $t \in \mathbb{R}$ and $S(t)$ is compact for every $t \in \mathbb{R}$.

(c) If Φ is the group of translations on H defined by $\Phi(t)\omega(\xi) = \tilde{\omega}(\xi + t)$, where $\tilde{\omega}$ is the extension of ω with period 2π, then $C(t) = \frac{1}{2}(\Phi(t) + \Phi(-t))$ and $A = B^2$, where B is the generator of Φ and $D = \{\omega \in H^1 : \omega(0) = \omega(\pi)\}$(see [17] for details). In particular, we observe that the inclusion $\iota : D \to H$ is compact.

Additionally, we will assume that

(i) The function $\vartheta : [0,\infty) \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive constant L_ϑ such that $\vartheta(t,0) = 0$, and

$$|\vartheta(t,u) - \vartheta(t,v)| \le L_\vartheta|u - v|, \ t \ge 0, u, v \in \mathbb{R}.$$

(ii) The function $\varsigma : [0,\infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive continuous function $m_\varsigma(\cdot) : [0,\infty) \to \mathbb{R}$ such that

$$|\varsigma(t,u,v)| \le m_\varsigma(t)|u| + 2^{1-p}|v|, \ t \ge 0, u, v \in \mathbb{R}.$$

(iii) The function $\varsigma_1 : [0,\infty) \times [0,\infty) \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive continuous function $m_{\varsigma_1}(\cdot) : [0,\infty) \to \mathbb{R}$ such that

$$\left|\int_0^t \varsigma_1(t,s,u)ds\right| \le m_{\varsigma_1}(t)|u|, \ t \ge 0, u \in \mathbb{R}.$$

(iv) The function $\vartheta : [0,\infty) \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive continuous function $m_\varpi(\cdot) : [0,\infty) \to \mathbb{R}$ such that

$$|\varpi(t,u,v)| \le m_\varpi(t)|u| + 2^{1-p}|v|, \ t \ge 0, u, v \in \mathbb{R}.$$

(v) The function $\vartheta_1 : [0,\infty) \times \mathbb{R} \to \mathbb{R}$ is continuous and there exists a positive continuous function $m_{\varpi_1}(\cdot) : [0,\infty) \to \mathbb{R}$ such that

$$\left|\int_0^t \varpi_1(t,s,u)ds\right| \le m_{\varpi_1}(t)|u|, \ t \ge 0, u \in \mathbb{R}.$$

Let $z(s)(x) = z(s,x)$. We can define respectively $g : [0,\infty)\times H \to H, h : [0,\infty)\times H\times H \to H, f : [0,\infty)\times H\times H \to L(K,H)$ and $I_k, J_k : H \to H$ by

$$g(t,z(t-\rho(t))(x) = \vartheta(t,z(t-\rho(t),x)),$$

$$h\Big(t,z(t-\rho(t)),\int_0^t a(t,s,z(s-\rho(s))ds\Big)(x) = \varsigma\Big(t,z(t-\rho(t),x),\int_0^t \varsigma_1(t,s,z(s-\rho(s),x))ds\Big),$$

$$f\Big(t,z(t-\rho(t)),\int_0^t b(t,s,z(s-\rho(s))ds\Big)(x) = \varpi\Big(t,z(t-\rho(t),x),\int_0^t \varpi_1(t,s,z(s-\rho(s),x))ds\Big),$$

$$I_k(z)(x) = \int_0^\pi \eta_k(s)z(s,x)ds,\quad J_k(z)(x) = \int_0^\pi \tilde{\eta}_k(s)z(s,x)ds.$$

Then the problem (11)-(14) can be written as (1)-(4). Moreover, using (i) we can prove that

$$\begin{aligned}E\parallel g(t,z_1)-g(t,z_2)\parallel^p &= E\Big[\Big(\int_0^\pi |\vartheta(t,z_1(x))-\vartheta(t,z_2(x))|^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le E\Big[\Big(\int_0^\pi L_\vartheta|z_1(x)-z_2(x)|^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le L_\vartheta^p E\parallel z_1-z_2\parallel^p\end{aligned}$$

for all $(t,z_j)\in[0,+\infty)\times H, j=1,2$, and $E\parallel g(t,z)\parallel_H^p\le L_\vartheta^p\parallel z\parallel^p$ for all $(t,z)\in[0,+\infty)\times H$. By assumptions (ii) and (iii) we have

$$\begin{aligned}E\parallel h(t,z,y)\parallel^p &= E\Big[\Big(\int_0^\pi |\varsigma(t,z(x),y(x))|^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le E\Big[\Big(\int_0^\pi [L_\varsigma(t)|z(x)|+2^{1-p}|y(x)|]^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le 2^{p-1}[(L_\varsigma(t))^pE\parallel z\parallel^p + 2^{1-p}E\parallel y\parallel^p]\\ &= m_h(t)E\parallel z\parallel^p + E\parallel y\parallel^p\end{aligned}$$

for all $(t,z,y)\in[0,+\infty)\times H\times H$, and

$$\begin{aligned}E\Big\|\int_0^t a(t,s,z)ds\Big\|^p &= E\Big[\Big(\int_0^\pi\Big|\int_0^t \varsigma_1(s,z(x))ds\Big|^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le E\Big[\Big(\int_0^\pi |m_{\varsigma_1}(t)z(x)|^2dx\Big)^{\frac{1}{2}}\Big]^p\\ &\le m_a(t)E\parallel z\parallel^p\end{aligned}$$

for all $(t,s,z)\in[0,+\infty)\times[0,+\infty)\times H$, where $m_h(t)=m_\varsigma^p(t), m_a(t)=m_{\varsigma_1}^p(t)$. Similarly, by using assumptions (iv) and (v) we have

$$E\parallel f(t,z,y)\parallel^p\le m_f(t)E\parallel z\parallel^p + E\parallel y\parallel^p$$

for all $(t,z,y)\in[0,+\infty)\times H\times H$, and

$$E\Big\|\int_0^t b(t,s,z)ds\Big\|^p\le m_b(t)E\parallel z\parallel^p$$

for all $(t,s,z)\in[0,+\infty)\times[0,+\infty)\times H$, where $m_f(t)=m_\varpi^p(t), m_b(t)=m_{\varpi_1}^p(t)$. Therefore (H1)-(H6) are all satisfied and condition (9) holds with $\Theta_h(s)=\Theta_a(s)=\Theta_f(s)=\Theta_b(s)=s$. It is clear that I_k, J_k are bounded linear maps with

$$E\parallel I_k(z)\parallel^p\le d_kE\parallel z\parallel^p,\quad E\parallel J_k(z)\parallel^p\le \tilde{d}_kE\parallel z\parallel^p,\quad z\in H, k=1,2,\dots,m,$$

where $d_k = (\int_0^\pi |\eta_k(s)|^2 ds)^{p/2}, \tilde{d}_k = (\int_0^\pi |\tilde{\eta}_k(s)|^2 ds)^{p/2}, k = 1, 2, \ldots, m$. Moreover, the map I_k, J_k are completely continuous. Further, suppose that $(14m)^{p-1} \sum_{k=1}^{m}(d_k + \tilde{d}_k) < 1$ holds. Then, from Theorem 3.4, we can conclude that the mild solution of (11)-(14) is asymptotically stable in p-th mean.

5. Conclusion

This paper has investigated the existence and asymptotic stability in p-th moment of mild solutions for a class of second-order impulsive partial stochastic functional neutral integrodidifferential equations with infinite delay in Hilbert spaces. Firstly, we introduce a more appropriate concept for mild solutions. Secondly, the existence and asymptotic stability of mild solutions is investigated by utilizing Hölder's inequality, stochastic analysis, the Darbo fixed point theorem and the theory of strongly continuous cosine families combined with techniques of the Hausdorff measure of noncompactness. Here, the results are obtained under the nonlinear items f, h are continuous functions and not the assumptions of compactness on associated operators. Finally, an example is provided to show the effectiveness of the proposed results.

There are two direct issues which require further study. We will investigate the asymptotic stability for second-order impulsive stochastic partial functional differential equations with not instantaneous impulses. Also, we will be devoted to study the stability of pseudo almost periodic solutions to impulsive stochastic partial functional differential equations.

Acknowledgments

The authors would like to thank the editor and the reviewers for their constructive comments and suggestions.

References

[1] R. Agarwal, M. Meehan and D. O'Regan, Fixed point theory and applications, in: Cambridge Tracts in Mathematics, Cambridge University Press, New York, 2001.
[2] A. Anguraj and A.Vinodkumar, Existence, uniqueness and stability results of impulsive stochastic semilinear neutral functional differential equations with infinite delays, Electron. J. Qual. Theory Differ. Equ. 67 (2009) 1–13.
[3] G. Arthi, J.H. Park and H.Y. Jung, Exponential stability for second-order neutral stochastic differential equations with impulses, Internat. J. Control 88 (2015) 1300–1309.
[4] P. Balasubramaniam and P. Muthukumar, Approximate controllability of second-order stochastic distributed implicit functional differential systems with infinite delay, J. Optim. Theory Appl. 143 (2009) 225–244.
[5] J. Banas and K. Goebel, Measure of Noncompactness in Banach Space, in: Lecture Notes in Pure and Applied Matyenath, Dekker, New York, 1980.
[6] T. Caraballo and K. Liu, Exponential stability of mild solutions of stochastic partial differential equations with delays, Stoch. Anal. Appl. 17 (1999) 743–763.
[7] T. Caraballo and J. Real, Partial differential equations with delayed random perturbations: Existence, uniqueness and stability of solutions, Stoch. Anal. Appl. 11 (1993) 497–511.
[8] T. Caraballo, J. Real and T. Taniguchi, The exponential stability of neutral stochastic delay partial differential equations, Discrete Contin. Dyn. Syst. 18 (2007) 259–313.
[9] H. Chen, The asymptotic behavior for second-order neutral stochastic partial differential equations with infinite delay, Discrete Dyn. Nat. Soc. 2011 (2011) 584510 1–15.
[10] H. Chen, Impulsive-integral inequality and exponential stability for stochastic partial differential equations with delays, Statist. Probab. Lett. 80 (2010) 50–56.
[11] H. Chen, C. Zhu and Y. Zhang, A note on exponential stability for impulsive neutral stochastic partial functional differential equations, Appl. Math. Comput. 227 (2014) 139–147.
[12] J. Cui, L.T. Yan and X.C. Sun, Exponential stability for neutral stochastic partial differential equations with delays and Poisson jumps, Statist. Probab. Lett. 81 (2011) 1970–1977.
[13] G. Da Prato and J. Zabczyk, Stochastic Equations in Infinite Dimensions, Cambridge University Press, Cambridge, 1992.
[14] H. O. Fattorini, Second Order Linear Differential Equations in Banach Spaces, North Holland Mathematics Studies Series No. 108 Elsevier Science, North-Holland, Amsterdam, The Netherlands, 1985.
[15] J.K. Goldstein, Semigroups of Linear Operators and Applications, Oxford University Press, New York, 1985.
[16] T.E. Govindan, Almost sure exponential stability for stochastic neutral partial functional differential equations, Stochastics 77 (2005) 139–154.
[17] D. He and L. Xu, Existence and stability of mild solutions to impulsive stochastic neutral partial functional differential equations, Electron. J. Differential Equations 2013 (2013) 1–15.

[18] K. Liu, Stability of Infinite Dimensional Stochastic Differential Equations with Applications, Chapman Hall, CRC, London, 2006.
[19] K. Liu and A. Truman, A note on almost exponential stability for stochastic partial differential equations, Statist. Probab. Lett. 50 (2000) 273–278.
[20] S. Long, L. Teng and D. Xu, Global attracting set and stability of stochastic neutral partial functional differential equations with impulses, Statist. Probab. Lett. 82 (2012) 1699–1709.
[21] J. Luo, Fixed points and exponential stability of mild solutions of stochastic partial differential equations with delays, J. Math. Anal. Appl. 342 (2008) 753–760.
[22] J. Luo and T. Taniguchi, Fixed points and stability of stochastic neutral partial differential equations with infinite delays, Stoch. Anal. Appl. 27 (2009) 1163–1173.
[23] N. I. Mahmudov and M. A. McKibben, Approximate controllability of second-order neutral stochastic evolution equations, Dyn. Contin. Discrete Impuls. Syst. Ser. B 13 (2006) 619–634.
[24] X. Mao, Stochastic Differential Equations and Applications, Horwood, Chichestic, UK, 1997.
[25] Y. Ren, T. Hou, R. Sakthivel and X. Cheng, A note on the second-order non-autonomous neutral stochastic evolution equations with infinite delay under Caratheodory conditions, Appl. Math. Comput. 232 (2014) 658–665.
[26] Y. Ren and D. Sun, Second-order neutral impulsive stochastic evolution equations with delay, J. Math. Phys. 50 (2009) 102709 1–14.
[27] P. Revathi, R. Sakthivel, D.-Y. Song, Y. Ren and P. Zhang, Existence and stability results for second-order stochastic equations driven by fractional Brownian motion, Transport Theory Statist. Phys. 42 (2013) 299–317.
[28] Y.V. Rogovchenko, Nonlinear impulse evolution systems and applications to population models, J. Math. Anal. Appl. 207 (1997) 300–315.
[29] R. Sakthivel and J. Luo, Asymptotic stability of impulsive stochastic partial differential equations with infinite delays, J. Math. Anal. Appl. 356 (2009) 1–6.
[30] R. Sakthivel and J. Luo, Asymptotic stability of nonlinear impulsive stochastic differential equations, Statist. Probab. Lett. 79 (2009) 1219–1223.
[31] R. Sakthivel and Y. Ren, Exponential stability of second-order stochastic evolution equations with Poisson jumps, Commun. Nonlinear Sci. Numer. Simul. 17 (2012) 4517–4523.
[32] R. Sakthivel, Y. Ren and H. Kim, Asymptotic stability of second-order neutral stochastic differential equations, J. Math. Phys. 51 (2010) 052701 1–9.
[33] R. Sakthivel, Y. Ren and N. I. Mahmudov, Approximate controllability of second-order stochastic differential equations with impulsive effects, Modern Phys. Lett. B 24 (2010) 1559–1572.
[34] R. Sakthivel, P. Revathi and S.M. Anthoni, Existence of pseudo almost automorphic mild solutions to stochastic fractional differential equations, Nonlinear Anal. 75 (2012) 3339–3347.
[35] R. Sakthivel, P. Revathi and Y. Ren, Existence of solutions for nonlinear fractional stochastic differential equations, Nonlinear Anal. 81 (2013) 70–86.
[36] T. Taniguchi and K. Liu, A. Truman, Existence, uniqueness, and asymptotic behavior of mild solution to stochastic functional differential equations in Hilbert spaces, J. Differential Equations 18 (2002) 72–91.
[37] Z. Yan and X. Yan, Existence of solutions for impulsive partial stochastic neutral integrodifferential equations with state-dependent delay, Collect. Math. 64 (2013) 235–250.