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An Application of Quasi-Monotone Sequences to Absolute Matrix Summability

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Abstract. Recently, Bor [5] has obtained two main theorems dealing with $|\bar{N}, p_n|_k$ summability factors of infinite series and Fourier series. In the present paper, we have generalized these theorems for $|A, \theta_n|_k$ summability method by using quasi-monotone sequences.

1. Introduction

A sequence (d_n) is said to be δ-quasi-monotone, if $d_n \to 0$, $d_n > 0$ ultimately, and $\Delta d_n \geq -\delta_n$, where $\Delta d_n = d_n - d_{n+1}$ and δ=(δ_n) is a sequence of positive numbers (see [1]). For any sequence (λ_n) we write that $\Delta\lambda_n = \lambda_n - \lambda_{n+1}$. The sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| < \infty$. Let $\sum a_n$ be a given infinite series with partial sums (s_n). We denote by u_n^α and t_n^α the nth Cesàro means of order α, with $\alpha > -1$, of the sequences (s_n) and (na_n), respectively, that is (see [6]),

$$u_n^\alpha = \frac{1}{A_n^\alpha}\sum_{v=0}^{n} A_{n-v}^{\alpha-1} s_v \quad \text{and} \quad t_n^\alpha = \frac{1}{A_n^\alpha}\sum_{v=1}^{n} A_{n-v}^{\alpha-1} v a_v, \quad ({t_n}^1 = t_n) \tag{1}$$

where

$$A_n^\alpha = \frac{(\alpha+1)(\alpha+2)....(\alpha+n)}{n!} = O(n^\alpha), \quad A_{-n}^\alpha = 0 \quad \texttt{for} \quad n > 0. \tag{2}$$

A series $\sum a_n$ is said to be summable $| C, \alpha |_k$, $k \geq 1$, if (see [8], [10])

$$\sum_{n=1}^{\infty} n^{k-1} \mid u_n^\alpha - u_{n-1}^\alpha \mid^k = \sum_{n=1}^{\infty} \frac{1}{n} \mid t_n^\alpha \mid^k < \infty. \tag{3}$$

If we set α=1, then we have $| C, 1 |_k$ summability. Let (p_n) be a sequence of positive numbers such that

$$P_n = \sum_{v=0}^{\infty} p_v \to \infty \quad as \quad n \to \infty, \quad (P_{-i} = p_{-i} = 0, \ i \geq 1). \tag{4}$$

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The sequence-to-sequence transformation

$$w_n = \frac{1}{P_n}\sum_{v=0}^{n} p_v s_v \tag{5}$$

defines the sequence (w_n) of the Riesz mean or simply the $(\bar{N}, p_n)$ mean of the sequence (s_n), generated by the sequence of coefficients (p_n) (see [9]). The series $\sum a_n$ is said to be summable $|\bar{N}, p_n|_k$, $k \geq 1$, if (see [2])

$$\sum_{n=1}^{\infty}\left(\frac{P_n}{p_n}\right)^{k-1}|w_n - w_{n-1}|^k < \infty. \tag{6}$$

In the special case when $p_n = 1$ for all values of n (respect. $k = 1$), then $|\bar{N}, p_n|_k$ summability is the same as $|C, 1|_k$ (respect. $|\bar{N}, p_n|$) summability. We write $X_n = \sum_{v=1}^{n} \frac{p_v}{P_v}$, then (X_n) is a positive increasing sequence tending to infinity with n. Let $A = (a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $As = (A_n(s))$, where

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v, \quad n = 0, 1, ... \tag{7}$$

Let (θ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $|A, \theta_n|_k$, $k \geq 1$, if (see [11])

$$\sum_{n=1}^{\infty} \theta_n^{k-1} \left|\bar{\Delta} A_n(s)\right|^k < \infty, \tag{8}$$

where

$$\bar{\Delta} A_n(s) = A_n(s) - A_{n-1}(s). \tag{9}$$

If we take $\theta_n = \frac{P_n}{p_n}$, then $|A, \theta_n|_k$ summability, then we have $\left|A, p_n\right|_k$ summability (see [12]), and if we take $\theta_n = n$, then we have $|A|_k$ summability (see [13]). And also if we take $\theta_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$, then we have $|\ \bar{N}, p_n\ |_k$ summability. Furthermore, if we take $\theta_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n, then $|A, \theta_n|_k$ summability reduces to $|C, 1|_k$ summability (see [8]). Finally, if we take $\theta_n = n$ and $a_{nv} = \frac{p_v}{P_n}$, then we obtain $\left|R, p_n\right|_k$ summability (see [3]).

2. Known Results

The following theorem is known dealing with $|\bar{N}, p_n|_k$ summability of infinite series (see [5]).

Theorem 2.1. *Let $\lambda_n \to 0$ as $n \to \infty$ and let (p_n) be a sequence of positive numbers such that*

$$P_n = O(np_n) \quad as\ n \to \infty. \tag{10}$$

Suppose that there exists a sequence of numbers (A_n) which is δ-quasi-monotone with $\sum nX_n\delta_n < \infty$, $\sum A_n X_n$ is convergent, and $|\Delta\lambda_n| \leq |A_n|$ for all n. If

$$\sum_{n=1}^{m} \frac{p_n}{P_n}\frac{|t_n|^k}{{X_n}^{k-1}} = O(X_m) \quad as \quad m \to \infty, \tag{11}$$

satisfies, then the series $\sum a_n\lambda_n$ is summable $|\bar{N}, p_n|_k$, $k \geq 1$.

3. Main Results

The aim of this paper is to generalize Theorem 2.1 for $|A, \theta_n|_k$ summability factors of infinite series, and is to apply this theorem to Fourier series.

Before stating the main theorem, we must first introduce some further notations.

Given a normal matrix $A = (a_{nv})$, we associate two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, \ldots \quad \bar{\Delta} a_{nv} = a_{nv} - a_{n-1,v}, \quad a_{-1,0} = 0 \tag{12}$$

and

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{\Delta}\bar{a}_{nv}, \quad n = 1, 2, \ldots \tag{13}$$

It may be noted that $\bar{A}$ and $\hat{A}$ are the well-known matrices of series-to-sequence and series-to-series transformations, respectively. Then, we have

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v \tag{14}$$

and

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{15}$$

Theorem 3.1. *Let $\lambda_n \to 0$ as $n \to \infty$ and let (p_n) be a sequence of positive numbers satisfying the condition (10). Suppose that there exists a sequence of numbers (A_n) which is δ-quasi-monotone with $\sum nX_n\delta_n < \infty$, $\sum A_n X_n$ is convergent, and $|\Delta\lambda_n| \leq |A_n|$ for all n. Let $(\theta_n a_{nn})$ be a non-increasing sequence. If $A = (a_{nv})$ is a positive normal matrix such that*

$$\bar{a}_{n0} = 1, \ n = 0, 1, \ldots, \tag{16}$$

$$a_{n-1,v} \geq a_{nv}, \ \text{for} \ \ n \geq v + 1, \tag{17}$$

$$\hat{a}_{n,v+1} = O(v|\bar{\Delta} a_{nv}|), \tag{18}$$

$$\sum_{n=1}^{\infty} \theta_n^{k-1} a_{nn}^k \frac{|t_n|^k}{X_n^{k-1}} = O(X_m) \quad as \quad m \to \infty, \tag{19}$$

then the series $\sum a_n \lambda_n$ is summable $|A, \theta_n|_k$, $k \geq 1$.

We need the following lemma for the proof of Theorem 3.1.

Lemma 3.2. ([4]) *Under the conditions of Theorem 2.1, we have that*

$$|\lambda_n| X_n = O(1) \ as \ n \to \infty, \tag{20}$$

$$nX_n |A_n| = O(1) \ as \ n \to \infty, \tag{21}$$

$$\sum_{n=1}^{\infty} nX_n |\Delta A_n| < \infty. \tag{22}$$

4. Proof of Theorem 3.1

Proof. Let (V_n) denotes the A-transform of the series $\sum_{n=1}^{\infty} a_n\lambda_n$. Then, by (14) and (15), we have

$$\bar{\Delta}V_n = \sum_{v=1}^{n} \hat{a}_{nv}a_v\lambda_v.$$

Applying Abel's transformation to this sum, we get that

$$\begin{aligned}\bar{\Delta}V_n &= \sum_{v=1}^{n} \hat{a}_{nv}a_v\lambda_v\frac{v}{v} = \sum_{v=1}^{n-1} \bar{\Delta}(\frac{a_{nv}\lambda_v}{v})\sum_{r=1}^{v} ra_r + \frac{\hat{a}_{nn}\lambda_n}{n}\sum_{r=1}^{n} ra_r = \sum_{v=1}^{n-1} \bar{\Delta}(\frac{a_{nv}\lambda_v}{v})(v+1)t_v + \hat{a}_{nn}\lambda_n\frac{n+1}{n}t_n\\ &= \sum_{v=1}^{n-1} \bar{\Delta}a_{nv}\lambda_v t_v\frac{v+1}{v} + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\Delta\lambda_v t_v\frac{v+1}{v} + \sum_{v=1}^{n-1} \hat{a}_{n,v+1}\lambda_{v+1}\frac{t_v}{v} + a_{nn}\lambda_n t_n\frac{n+1}{n}\\ &= V_{n,1} + V_{n,2} + V_{n,3} + V_{n,4}.\end{aligned}$$

To complete the proof of Theorem 3.1, by Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \theta_n^{k-1} \mid V_{n,r} \mid^k < \infty, \quad \text{for} \quad r = 1,2,3,4. \tag{23}$$

It may be noted that, the following results can be seen by condition (16) and (17), we have

$$\sum_{n=v+1}^{m+1} |\bar{\Delta}a_{nv}| \le a_{vv}, \tag{24}$$

$$\sum_{v=1}^{n-1} |\bar{\Delta}a_{nv}| \le a_{nn}. \tag{25}$$

First, by applying Hölder's inequality with indices k and k', where $k > 1$ and $\frac{1}{k} + \frac{1}{k'} = 1$, and using conditions (24) and (25) for the third sum, and since $(\theta_n a_{nn})$ is a non-increasing sequence, we have that

$$\begin{aligned}\sum_{n=2}^{m+1} \theta_n^{k-1} \mid V_{n,1} \mid^k &\le \sum_{n=2}^{m+1} \theta_n^{k-1} \left\{\sum_{v=1}^{n-1} |\frac{v+1}{v}| \left|\bar{\Delta}a_{nv}\right| |\lambda_v||t_v|\right\}^k\\ &= O(1)\sum_{n=2}^{m+1} \theta_n^{k-1}\sum_{v=1}^{n-1} \left|\bar{\Delta}a_{nv}\right| |\lambda_v|^k|t_v|^k \times \left\{\sum_{v=1}^{n-1} \left|\bar{\Delta}a_{nv}\right|\right\}^{k-1} = O(1)\sum_{n=2}^{m+1} \theta_n^{k-1}a_{nn}^{k-1}\left\{\sum_{v=1}^{n-1} |\bar{\Delta}a_{nv}||\lambda_v|^k|t_v|^k\right\}\\ &= O(1)\sum_{v=1}^{m} |\lambda_v|^{k-1}|\lambda_v||t_v|^k\sum_{n=v+1}^{m+1} (\theta_n a_{nn})^{k-1}|\bar{\Delta}a_{nv}| = O(1)\sum_{v=1}^{m} (\theta_v a_{vv})^{k-1}\frac{1}{X_v^{k-1}}|\lambda_v||t_v|^k a_{vv}\\ &= O(1)\sum_{v=1}^{m-1} \Delta|\lambda_v|\sum_{r=1}^{v} \theta_r^{k-1}a_{rr}^k\frac{|t_r|^k}{X_r^{k-1}} + O(1)|\lambda_m|\sum_{v=1}^{m} \theta_v^{k-1}a_{vv}^k\frac{|t_v|^k}{X_v^{k-1}}\\ &= O(1)\sum_{v=1}^{m-1} |\Delta\lambda_v|X_v + O(1)|\lambda_m|X_m = O(1)\sum_{v=1}^{m-1} |A_v|X_v + O(1)|\lambda_m|X_m\\ &= O(1) \quad \text{as} \quad m \to \infty,\end{aligned}$$

by virtue of the hypotheses of Theorem 3.1 and Lemma 3.2. Now, using Hölder's inequality we have that

$$\begin{aligned}
\sum_{n=2}^{m+1} \theta_n^{k-1} \mid V_{n,2} \mid^k &\leq \sum_{n=2}^{m+1} \theta_n^{k-1} \left\{ \sum_{v=1}^{n-1} |\frac{v+1}{v}||\hat{a}_{n,v+1}||\Delta\lambda_v||t_v| \right\}^k = O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} \left\{ \sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||A_v||t_v| \right\}^k \\
&= O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} \sum_{v=1}^{n-1} (v|A_v|)^k |\bar{\Delta} a_{nv}||t_v|^k \times \left\{ \sum_{v=1}^{n-1} \left| \bar{\Delta} a_{nv} \right| \right\}^{k-1} = O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} a_{nn}^{k-1} \sum_{v=1}^{n-1} (v|A_v|)^k |\bar{\Delta} a_{nv}||t_v|^k \\
&= O(1) \sum_{v=1}^{m} (v|A_v|)^{k-1} (v|A_v|) |t_v|^k \sum_{n=v+1}^{m+1} (\theta_n a_{nn})^{k-1} |\bar{\Delta} a_{nv}| = O(1) \sum_{v=1}^{m} \theta_v^{k-1} a_{vv}^k \frac{1}{X_v^{k-1}} |t_v|^k (v|A_v|) \\
&= O(1) \sum_{v=1}^{m-1} \Delta(v|A_v|) \sum_{r=1}^{v} \theta_r^{k-1} a_{rr}^k \frac{1}{X_r^{k-1}} |t_r|^k + O(1) m|A_m| \sum_{v=1}^{m} \theta_v^{k-1} a_{vv}^k \frac{1}{X_v^{k-1}} |t_v|^k \\
&= O(1) \sum_{v=1}^{m-1} |\Delta(v|A_v|)| X_v + O(1) m|A_m| X_m \\
&= O(1) \sum_{v=1}^{m-1} |(v+1)\Delta|A_v| - |A_v|| X_v + O(1) m|A_m| X_m \\
&= O(1) \sum_{v=1}^{m-1} v X_v |\Delta A_v| + O(1) \sum_{v=1}^{m-1} |A_v| X_v + O(1) m|A_m| X_m \\
&= O(1) \quad \text{as} \quad m \to \infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 3.1 and Lemma 3.2. Again, as in $V_{n,1}$ we have that

$$\begin{aligned}
\sum_{n=2}^{m+1} \theta_n^{k-1} \mid V_{n,3} \mid^k &= \sum_{n=2}^{m+1} \theta_n^{k-1} \left| \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \frac{t_v}{v} \right|^k \leq \sum_{n=2}^{m+1} \theta_n^{k-1} \left\{ \sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\lambda_{v+1}| \frac{|t_v|}{v} \right\}^k \\
&= O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} \left\{ \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_{v+1}||t_v| \right\}^k = O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_{v+1}|^k |t_v|^k \times \left\{ \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}| \right\}^{k-1} \\
&= O(1) \sum_{n=2}^{m+1} \theta_n^{k-1} a_{nn}^{k-1} \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_{v+1}|^k |t_v|^k = O(1) \sum_{v=1}^{m} |\lambda_{v+1}|^k |t_v|^k \sum_{n=v+1}^{m+1} (\theta_n a_{nn})^{k-1} |\bar{\Delta} a_{nv}| \\
&= O(1) \sum_{v=1}^{m} \theta_v^{k-1} a_{vv}^k |t_v|^k |\lambda_{v+1}|^{k-1} |\lambda_{v+1}| = O(1) \sum_{v=1}^{m} \theta_v^{k-1} a_{vv}^k \frac{1}{X_v^{k-1}} |\lambda_{v+1}||t_v|^k \\
&= O(1) \quad \text{as} \quad m \to \infty,
\end{aligned}$$

by virtue of the hypotheses of Theorem 3.1 and Lemma 3.2. Finally, as in $V_{n,1}$, we have that

$$\begin{aligned}
\sum_{n=1}^{m} \theta_n^{k-1} |V_{n,4}|^k &= O(1) \sum_{n=1}^{m} \theta_n^{k-1} a_{nn}^{k-1} a_{nn} |\lambda_n|^k |t_n|^k = O(1) \sum_{n=1}^{m} \theta_n^{k-1} a_{nn}^k |\lambda_n|^{k-1} |\lambda_n||t_n|^k \\
&= O(1) \sum_{n=1}^{m} \theta_n^{k-1} a_{nn}^k \frac{1}{X_n^{k-1}} |\lambda_n||t_n|^k = O(1) \quad \text{as} \quad m \to \infty,
\end{aligned}$$

by virtue of hypotheses of the Theorem 3.1 and Lemma 3.2. This completes the proof of Theorem 3.1. □

4. Applications

4.1. An Application to Trigonometric Fourier Series

Let f be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. We may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

$$\int_{-\pi}^{\pi} f(t)dt = 0$$

and

$$f(t) \sim \sum_{n=1}^{\infty}(a_n cosnt + b_n sinnt) = \sum_{n=1}^{\infty} C_n(t), \tag{26}$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)dt, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)cos(nt)dt, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)sin(nt)dt.$$

We write

$$\phi(t) = \frac{1}{2}\left\{f(x+t) + f(x-t)\right\}, \tag{27}$$

$$\phi_\alpha(t) = \frac{\alpha}{t^\alpha}\int_0^t (t-u)^{\alpha-1}\phi(u)\,du, \quad (\alpha > 0). \tag{28}$$

It is well known that if $\phi(t) \in \mathcal{BV}(0, \pi)$, then $t_n(x) = O(1)$, where $t_n(x)$ is the $(C, 1)$ mean of the sequence $(nC_n(x))$ (see [7]).
Using this fact, the following theorem has been proved.

Theorem 4.1. ([5]) *If $\phi_1(t) \in \mathcal{BV}(0, \pi)$, and the sequences (A_n), (λ_n), and (X_n) satisfy the conditions of Theorem 2.1, then the series $\sum C_n(x)\lambda_n$ is summable $|\bar{N}, p_n|_k$, $k \geq 1$.*

We have generalized Theorem 4.1 for $|A, \theta_n|_k$ summability method in the following form.

Theorem 4.2. *Let A be a normal matrix as in Theorem 3.1. If $\phi_1(t) \in \mathcal{BV}(0, \pi)$, and the sequences (A_n), (λ_n), and (X_n) satisfy the conditions of Theorem 3.1, then the series $\sum C_n(x)\lambda_n$ is summable $|A, \theta_n|_k$, $k \geq 1$.*

In this paper, the concept of absolute matrix summability is investigated. In this investigation, we prove an interesting theorem related to $|A, \theta_n|_k$. We also obtain applications to Fourier series. We can apply Theorem 3.1 and Theorem 4.2 to weighted mean $A = (a_{nv})$ is defined as $a_{nv} = \frac{p_v}{P_n}$ when $0 \leq v \leq n$, where $P_n = p_0 + p_1 + ... + p_n$. We have that,

$$\bar{a}_{nv} = \frac{P_n - P_{v-1}}{P_n} \quad \text{and} \quad \hat{a}_{n,v+1} = \frac{p_n P_v}{P_n P_{n-1}}.$$

The following results can be easily verified.

1. If we take $\theta_n = \frac{P_n}{p_n}$ in Theorem 3.1 and Theorem 4.2, then we have a result dealing with $\left|A, p_n\right|_k$ summability.
2. If we take $\theta_n = n$ in Theorem 3.1 and Theorem 4.2, then we have a result dealing with $|A|_k$ summability.
3. If we take $\theta_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 3.1 and Theorem 4.2, then we have Theorem 2.1 and Theorem 4.1, respectively.
4. If we take $\theta_n = n$, $a_{nv} = \frac{p_v}{P_n}$ and $p_n = 1$ for all values of n in Theorem 3.1 and Theorem 4.2, then we have a new result concerning $|C, 1|_k$ summability.
5. If we take $\theta_n = n$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 3.1 and Theorem 4.2, then we have $\left|R, p_n\right|_k$ summability.

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