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Exponential Stability for Neutral Stochastic Partial Integro-Differential Equations of Second Order with Poisson Jumps

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Abstract. This paper studies the existence, uniqueness and the exponential stability in p-th moment of the mild solution of neutral second order stochastic partial differential equations with infinite delay and Poisson jumps. The existence and uniqueness of the mild solution of neutral second order stochastic differential equation is first established by means of Banach fixed point principle and stochastic analysis. The exponential stability in the p-th moment for the mild solution to impulsive neutral stochastic integro-differential equations with Poisson jump is obtained by establishing an integral inequality.

1. Introduction

The theory of stochastic differential equation is growing as an important field of study in recent decades, empowered by their various applications to the many problems from biology, mechanics, electrical engineering and physics and so on; see [1, 3, 4, 7, 13], wherein, all the time, future state of systems depends on the present state as well as on its past history leading to stochastic functional differential equation instead of stochastic differential equations. Stochastic differential equations involving Poisson jumps have become very popular in modeling the phenomena arising in the many fields. In financial and actuarial modeling and other areas of application, such jump diffusions are frequently used to illustrate the dynamics of various state variables. In finance, these may represent, for instance, asset prices, credit ratings, stock indices, interest rates, exchange rates or commodity prices. The jump component can capture event-driven uncertainties, for example, corporate defaults, operational failures or insured events. For details, see [6, 8, 14, 17, 19, 23–27, 29, 31, 35, 36].

Second order differential equations capture the dynamic behavior of many natural phenomena and have found applications in various fields, for example, mathematical physics, biology and finance. Converting a second-order system into a first-order system may not yield desired results due to the behavior of the semigroup generated by the linear part of the converted first order system. In many cases, it is advantageous to treat the second-order stochastic differential equations directly rather than converting them to first-order systems. A variety of problems arising in, mechanics, elasticity theory, molecular dynamics and quantum mechanics can be described in general by second order nonlinear differential equations, see [2, 5, 9–12, 16, 21, 22, 28, 29, 35, 36]. The second-order differential equations involving randomness are seem

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to be correct model in continuous time to account for integrated processes that can be made stationary. Due to this reason, researchers' interest is focused on second order differential equations. In recent years, existence and stability results for second order stochastic evolution equations have been considered by many researchers [2, 3, 9–12, 28, 29]. Taniguchi and Luo[31] considered a stochastic evolution equation driven by Poisson jumps and studied the almost surely exponentially stability or exponentially ultimate bounded in mean square. Caraballo and Liu [30] derived the sufficient conditions for the pth exponential stability and pathwise exponential stability of mild solutions by using properties of the stochastic convolution. Chen [15] established the pth exponential stability of mild solution for impulsive stochastic partial differential equation with delays by establishing an impulsive-integral inequality, and generalized and improved the results of [30]. Cui et al [19] proved the existence and exponential stability in mean square as well as almost surely exponential stability of mild solutions utilizing Banach fixed point theorem. Chen [16] obtained the exponential stability and asymptotical stability for mild solution to the second-order neutral stochastic partial differential equations with infinite delay. In [17], the mean square exponential stability of the mild solution to neutral stochastic partial differential equations and Poisson jump is discussed by using established integral inequality. Arthi et al [11] obtained a set of sufficient conditions proving exponential stability of the mild solution of second order neutral stochastic differential equation by establishing an impulsive integral inequality. Ren and Chen [34] discussed the existence and uniqueness of the solution to neutral stochastic functional differential equation driven by Poisson jumps with non-Lipschitz coefficients. Ren and Sakthivel[35] considered a second-order neutral stochastic evolution equations with infinite delay and Poisson jumps, and studied existence, uniqueness and stability of the mild solution by means of the successive approximation and Bihari inequality. Sakthivel and Ren [29] discussed the exponential stability problem of second-order nonlinear stochastic evolution equations with Poisson jumps using stochastic analysis theory. By Banach fixed point principle, Diop et al [24] first established the existence, uniqueness and the shown the exponential stability in mean square of mild solution for stochastic neutral partial functional integro-differential equations with delays and Poisson jumps. Yang and Zhu [32] first established the existence and uniqueness mild solution of stochastic partial differential equation with Poisson jumps, and then exponential stability in p-th moment by mean of fixed point theory was proved. They also proved the mild solution is almost surely p-th moment exponentially stable by using Borel-Cantelli Lemma. By using Kunita's first inequality, Chadha and Bora [2] proved the existence, uniqueness and exponential stability of the mild solution of second order impulsive neutral stochastic differential equation involving Poisson jumps with help of established a new integral inequality. In [36], p-th moment asymptotic stability of mild solutions to second-order impulsive partial stochastic functional neutral integro-differential equations involving infinite delay was considered without assuming Lipschitz continuity of nonlinear term.

Motivated by above mentioned work, this work investigates the existence, uniqueness and exponential stability in p-th moment of a mild solution to a class of second order neutral stochastic integro-differential equation involving Poisson jumps and impulsive effects

$$
\begin{aligned}
& d[u'(t)+H(t,u_t,\int_{-\infty}^{0}h(\theta,u(t+\theta))d\theta))] = [Au(t)+F(t,u_t,\int_{-\infty}^{0}f(\theta,u(t+\theta))d\theta))]dt \\
& \quad + G(t,u_t,\int_{-\infty}^{0}g(\theta,u(t+\theta))d\theta)dw(t)+\int_Z \mathcal{K}(t,u_t,y)\widetilde{N}(dt,dy),\ t\geq 0,\ \ t\neq t_n \qquad (1) \\
& \Delta u(t_n) = I_n(u(t_n^-)),\ n=1,2,\cdots, \qquad (2) \\
& \Delta u'(t_n) = J_n(u(t_n^-)),\ n=1,2,\cdots, \qquad (3) \\
& u_0(\cdot)=\phi\in C,\ \ u'(0)=x_1, \qquad (4)
\end{aligned}
$$

where x_1 is an $\mathcal{F}_0$ measurable X-valued random variable independent of the Wiener process $w(t)$, $H, F : [0,\infty)\times C\times X\to X$, $h, f : [0,\infty)\times[0,\infty)\times C\to X$, $G : [0,\infty)\times C\times X\to \mathcal{L}_2^0(K,X)$, $\mathcal{K} : [0,\infty)\times C\times Z\to X$ are appropriate functions satisfying certain conditions. The operator $A : D(A)\subset X\to X$ is infinitesimal generator of a strongly continuous cosine family on X. The impulsive points t_n satisfy the inequality $0<t_1<t_2<\cdots<t_n<\cdots$, and $I_n, J_n : C\to X$, $\Delta u(t)=u(t^+)-u(t^-)$, where $u(t^+)$ and $u(t^-)$ represent the right and left limit of u at t, respectively.

This article has four sections. Section 2 summarizes various important working tools on the Wiener process, Poisson jumps and second order differential equations. Section 3 establishes the existence and uniqueness of the mild solution of second order stochastic differential equation with Poisson jumps by means of Banach space fixed point theorem. Section 4 provides the exponential stability of the mild solution of second order stochastic differential equations. An integral inequality is used to establish some algebraic criteria of p-th exponential stability of the second order impulsive stochastic differential equation with Poisson jumps utilizing Kunita inequality. This work generalizes and improves some previous existing results.

2. Preliminaries

This section presents some basic definitions, theorems and lemmas which will be required establishing main results.

Throughout the article, the notations $(X, \|\cdot\|_X, (\cdot,\cdot)_X)$ and $(K, \|\cdot\|_K, (\cdot,\cdot)_K)$ stand for the separable Hilbert spaces. The notation $C(J, X)$ stands for the Banach space of continuous functions from $J = [a, b](a, b \in \mathbb{R}, b > a)$ to X with supremum norm, i.e., $\|y\|_C = \sup_{t\in J}\|y(t)\|$, $\forall\ y \in C(J, X)$ and $L^1(J, X)$ denotes the Banach space of functions $y : J \to X$ which are Bochner integrable normed by $\|y\|_{L^1} = \int_a^b \|y(t)\|dt$, for all $y \in L^1(J, X)$. A measurable function $y : J \to X$ is Bochner integrable if and only if $\|y\|$ is Lebesgue integrable. The notation $B(X)$ stands for the Banach space of all linear bounded operator from X into itself with norm

$$\|f\|_{B(X)} = \sup\{\|f(y)\| : \ \|y\| \leq 1\}, \ \ \forall \ \ f \in B(X). \tag{5}$$

Let $C((-\infty, 0], X)$ be the space of all bounded and continuous function ζ from $(-\infty, 0]$ to X with the norm $\|\cdot\|_C = \sup_{\theta\in(-\infty,0]}\|\zeta(\theta)\|$, and $\mathcal{C}$ be the space of all $\mathcal{F}_t(t \geq 0)$-measurable and $C((t_j, t_{j+1}], X)(j = 1, 2, \cdots,$ and for $j = 0,\ t_j = -\infty,\ t_{j+1} = 0)$-valued random variables, where $\mathcal{F}_t$ is defined next paragraph.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with a normal filtration $\mathbf{F} = \mathcal{F}_t,\ t \in [0, T]$ that satisfies the usual conditions, i.e. right continuous and $\mathcal{F}_0$ containing all $\mathbb{P}$-null sets. A filtration $\mathbf{F}$ is a sequence of σ-algebra $\{\mathcal{F}_t\}_{t\geq 0}$ with $\mathcal{F}_t \subset \mathcal{F}$ for each t and $t_1 \leq t_2 \Rightarrow \mathcal{F}_{t_1} \subset \mathcal{F}_{t_2}$. An X-valued random variable is an $\mathcal{F}_t$-measurable function $y(t) : \Omega \to X$ and the space $S = \{y(t, \omega) : \Omega \to X : t \in [a, b]\}$ which contains all random variables is called a stochastic process. In addition, we use the notation $y(t)$ instead of $y(t, w)$, where $y(t) : J \to X \in S$. We assume that $\{w(t) : t \geq 0\}$ is a K-valued Wiener process defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P}; \mathbf{F})$ with covariance operator Q, where Q is a positive, self-adjoint, trace class operator on K. Especially, $w(t)$ denotes a K-valued Q-Wiener process with respect to $\{\mathcal{F}_t\}_{t\geq 0}$. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K and $\{\lambda_n\}_{n=1}^{\infty}$ be a bounded sequence of nonnegative real numbers with $Qe_i = \lambda_i e_i$. Further, we consider a sequence β_i of independent Brownian motions with

$$(w(t), e) = \sum_{n=1}^{\infty} \lambda_n(e_n, e)\beta_n(t), \ \ e \in K, \ \ t \in [0, T]$$

and $\mathcal{F}_t = \mathcal{F}_t^W$ is the σ-algebra generated by $\{w(s) : 0 \leq s \leq t\}$. The symbol $L(K, X)$ stands for the space of all bounded linear operators from K into X with the usual norm $\|\cdot\|_{L(K,X)}$ and $L(X)$ when $K = X$.

Let $p(t),\ t \geq 0$ be a σ-finite stationary $\mathcal{F}_t$-adapted Poisson point process on $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$. The counting random measure N_p defined by $N_p((t_1, t_2] \times \Lambda)(\omega) = \sum_{t_1<s\leq t_2} I_\Lambda(p(s))$ for any $\Lambda \in B_\sigma(K)$ is called the Poisson random measure associated with the Poisson point process p. Define the measure $\widetilde{N}$ by $\widetilde{N}(dt, du) = N_p(dt, du) - dt\mu(du)$, where μ is the characteristic measure on K called the compensated Poisson random measure associated with the Poisson point process p. For a Borel set $Z \in B_\sigma(K - \{0\})$, the space $\mathcal{P}^p([0, T] \times Z, X), p \geq 2$ denotes the the space of all predictable mapping $F : [0, T] \times Z \times \Omega \to X$ with

$$\int_0^T \int_Z \mathbb{E}\|F(t, y)\|^p dt\mu(dy) < \infty.$$

Here, $\mathbb{E}$ denotes the integration with respect to a probability measure $\mathbb{P}$, i.e., $\mathbb{E}y = \int_\Omega y d\mathbb{P}$. For a basic study on stochastic differential equation, we refer to the book by Prato and Zabczyk [13], and Applebaum [6].

Let $\{S(t) : t \in \mathbb{R}\} \subset L(X)$ be a strongly continuous cosine family and the corresponding strongly continuous sine family $\{C(t) : t \in \mathbb{R}\} \subset L(X)$ be defined by

$$C(t)y = \int_0^t S(s)yds, \quad t \in \mathbb{R}, \quad y \in X.$$

The generator $A : X \to X$ of $\{S(t) : t \in \mathbb{R}\}$ is given by $Ay = \frac{d^2}{dt^2}S(t)y|_{t=0}$ for all $y \in D(A) = \{y \in X : S(\cdot)y \in C^2(\mathbb{R}, X)\}$.
Consider the second order differential equation

$$\begin{aligned} y''(t) &= Ay(t) + h(t), \quad t \geq 0 && (6)\\ y(0) &= y_0, \quad y'(0) = y_1, && (7) \end{aligned}$$

We shall also define the set $E = \{y \in X : \frac{d}{dt}S(t)y \text{ is continuous}\}$ with the norm $\|y\|_E = \|y\| + \sup_{t\in\mathbb{R}} \|AC(t)\|$. In (6), h is to be an integrable function and $y_0, y_1 \in X$. The existence of the solution of above second order system given by

$$y(t) = S(t)y_0 + C(t)y_1 + \int_0^t C(t-s)h(s)ds, \quad t > 0, \tag{8}$$

has been studied in the Travis and Webb [5]. If $y_0 \in E$, then solution is continuously differentiable on $[0, T],\ T < \infty$ and

$$y'(t) = AC(t)y_0 + S(t)y_1 + \int_0^t S(t-s)h(s)ds, \ t \in [0, T]. \tag{9}$$

For more details, see the articles [5, 21, 22].

Definition 2.1. *The mild solution $y(t)$ of the system (1)-(4) is said to be exponentially stable in the p-th moment for $p \geq 2$, if there exist two positive constants $\gamma > 0$ and $M^* \geq 1$ such that*

$$\mathbb{E}\|y(t)\|^p \leq M^* e^{-\gamma t}, \quad t \geq 0, \quad p \geq 2, \tag{10}$$

for any solution $y(t)$ with initial condition $\phi \in C$.

Lemma 2.2. *[13] For any $p \geq 2$ and for an arbitrary $\mathcal{L}_2^0$-valued predictable process $y(\cdot)$,*

$$\sup_{s\in[0,t]} \mathbb{E}\| \int_0^s y(v)dw(v)\|^p \leq \mathbb{C}_p\Big(\int_0^t \left(\mathbb{E}\|y(v)\|^p_{\mathcal{L}_2^0}\right)^{2/p} dv\Big)^{p/2},$$

where $\mathbb{C}_p = (p(p-1)/2)^{p/2}$ and $\mathcal{L}_2^0$ denotes the space of all Q-Hilbert Schmidt operators from K to X.

Lemma 2.3. *[6, 14] (Kunita's first inequality) For any $p \geq 2$, there exists $\mathbb{C}_p > 0$ such that*

$$\begin{aligned} \sup_{0\leq\tau\leq t} \mathbb{E}\Big[\| \int_0^t \int_Z \mathcal{K}(\tau, x)\widetilde{N}(d\tau, dx)\|\Big]^p \quad &\leq \quad \mathbb{C}_p\Big\{\mathbb{E}\Big[\Big(\int_0^t \int_Z \|\mathcal{K}(s, x)\|^2 \mu dxds\Big)^{p/2}\Big] \\ &\quad + \mathbb{E}\Big[\int_0^t \int_Z \|\mathcal{K}(s, x)\|^p \mu dxds\Big]\Big\}. \end{aligned}$$

Next, the definition of mild solution for the stochastic system (1)-(4) is provided.

Definition 2.4. *A piecewise continuous X-valued stochastic process $y(t),\ t \in \mathbb{R}$ is called a mild solution of a system (1)-(4) if*
(i) $y(t)$ is adapted to $\mathcal{F}_t (t \geq 0)$ and has a cádlág path on $t \geq 0$ almost surely.
(ii) For $t \in [0, +\infty)$ almost surely,

$$y(t) = \begin{cases} \mathcal{S}(t)\phi(0) + C(t)[x_1 + H(0, u_0, \int_{-\infty}^{0} h(\theta, u(\theta))d\theta)] \\ - \int_0^t \mathcal{S}(t-s)H(s, u_s, \int_{-\infty}^{0} h(\theta, u(s+\theta))d\theta))ds + \int_0^t C(t-s)F(s, u_s, \int_{-\infty}^{0} f(\theta, u(s+\theta))d\theta))ds \\ + \int_0^t C(t-s)G(s, u_s, \int_{-\infty}^{0} g(\theta, u(s+\theta))d\theta)dw(s) + \int_0^t \int_3 C(t-s)\mathcal{K}(s, u_s, y)\widetilde{N}(ds, dy) \\ + \sum_{0<t_i<t} \mathcal{S}(t-t_i)I_i(u(t_i^-)) + \sum_{0<t_i<t} C(t-t_i)J_i(u(t_i^-)),\ t \in [0, T], \\ \phi \in C,\ t \in (-\infty, 0]. \end{cases} \tag{11}$$

and $y'(0) = x_1$.

3. Existence Result

This section presents the existence and uniqueness of the mild solution of the impulsive stochastic system (1)-(4). Before proving the results, we make the following assumptions.

($A1$) The function $H, F : [0, \infty) \times C \times X \to X$, $G : [0, \infty) \times C \times X \to \mathcal{L}_2^0(K, X)$ and $h, f, L_g : (-\infty, 0] \times X \to X$ are continuous functions and there exist constants $L_F, L_H, L_G > 0$ and $L_f, L_h, L_g > 0$ such that

$$\begin{aligned} \|F(t, x_1, y_1) - F(t, x_2, y_2)\| &\leq L_F\Big[\int_{-\infty}^{0} k(\vartheta)\|x_1(t+\vartheta) - x_2(t+\vartheta)\|d\vartheta + \|y_1 - y_2\|\Big], \\ \|H(t, x_1, y_1) - H(t, x_2, y_2)\| &\leq L_H\Big[\int_{-\infty}^{0} k(\vartheta)\|x_1(t+\vartheta) - x_2(t+\vartheta)\|d\vartheta + \|y_1 - y_2\|\Big], \\ \|G(t, x_1, y_1) - G(t, x_2, y_2)\| &\leq L_G\Big[\int_{-\infty}^{0} k(\vartheta)\|x_1(t+\vartheta) - x_2(t+\vartheta)\|d\vartheta + \|y_1 - y_2\|\Big], \\ \|f(t, x_1) - f(t, x_2)\| &\leq L_f k(t)\|x_1 - x_2\|, \\ \|h(t, x_1) - h(t, x_2)\| &\leq L_h k(t)\|x_1 - x_2\|, \\ \|g(t, x_1) - g(t, x_2)\| &\leq L_g k(t)\|x_1 - x_2\| \end{aligned}$$

for all $x_1, x_2 \in C, y_1, y_2 \in X$ and $t \in [0, \infty)$ with $\|F(t, 0, 0)\| \leq L_F^1, \|H(t, 0, 0)\| \leq L_H^1, \|G(t, 0, 0)\| \leq L_G^1$, $\|f(t, 0)\| = \|h(t, 0)\| = \|g(t, 0)\| = 0$. Here the function $k : (-\infty, 0] \to [0, \infty)$ is a function satisfying $\int_{-\infty}^{0} k(t)dt = 1$ and $\int_{-\infty}^{0} k(t)e^{-h_1 t}dt < +\infty (h_1 > 0)$.

($A2$) The function $\mathcal{K}(t, \cdot, \cdot)$ is continuous and there exist positive constants $L_\mathcal{K}$ such that

$$\|\mathcal{K}(t, u_1, y) - \mathcal{K}(t, u_2, y)\| \leq L_\mathcal{K} \int_{-\infty}^{0} k(\vartheta)\|k'(u_1(t+\vartheta), y) - k'(u_2(t+\vartheta), y)\|d\vartheta, \tag{12}$$

and for any $y \in Z$ and $u_1, u_2 \in C$, $t \geq 0$

$$\int_Z \|k'(u, y) - k'(v, y)\|^i \mu dy \leq \mathbb{L}_{k'}^i \times \|u - v\|^i,\ k'(0, y) = 0. \tag{13}$$

where $i = 2, 4$.

($A3$) The cosine family of operators $\{\mathcal{S}(t) : t \geq 0\}$ and corresponding sine family $\{C(t) : t \geq 0\}$ satisfy $\|\mathcal{S}(t)\| \leq Me^{-\mu_1 t}$ and $\|C(t)\| \leq Me^{-\mu_2 t}$ $t \geq 0$, where M, μ_1, μ_2 are positive constants.

$(A4)$ The function $I_n, J_n : \mathcal{C} \to X(n = 1, \cdots,)$ are continuous functions and there exist numbers $c_n, d_n > 0(n = 1, 2, \cdots,)$ such that

$$\begin{aligned} \|I_n(u_1) - I_n(u_2)\| &\leq c_n\|u_1 - u_2\|, \\ \|J_n(u_1) - J_n(u_2)\| &\leq d_n\|u_1 - u_2\| \end{aligned} \tag{14}$$

for $u_1, u_2 \in \mathcal{C}$ and $I_n(0) = J_n(0) = 0, \sum_{n=1}^{+\infty} c_n < +\infty, \sum_{n=1}^{+\infty} d_n < +\infty.$

Theorem 3.1. *The system (1) and (4) has a unique fixed point in $\mathcal{B}_T$ if conditions $(A1)$-$(A4)$ are satisfied and*

$$\begin{aligned} &\Big\{M^p L_H^p L_h^p + M^p L_H^p \mu_1^{-p}(1 + L_h)^p T + M^p L_F^p \mu_2^{-p}(1 + L_f)^p T + \mathbb{C}_p M^p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} \\ &\times \mu_2^{-p/2} L_G^p (1 + L_g)^p T + \mathbb{C}_p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} \mu_2^{-p/2} T(L_{\mathcal{K}}^{p/2} + L_{\mathcal{K}})M^p + M^p\Big(\sum_{n=1}^{\infty} c_n\Big)^p \\ &+ M^p\Big(\sum_{n=1}^{\infty} d_n\Big)^p\Big\} < 8^{1-p}. \end{aligned} \tag{15}$$

Proof. First define the space $\mathcal{B}_T$ of all functions $y(t, w) : (-\infty, T] \times \Omega \to X$ such that $y(t, w)$ is measurable in w for each fixed $t \in (-\infty, T]$ and bounded and continuous in t for a.e. fixed $w \in \Omega$ with norm

$$\|y\|_{\mathcal{B}_T} = \Big(\mathbb{E}(\sup_{t \in (-\infty, T]} \|y(t, w)\|^p)\Big)^{1/p}.$$

Clearly, $\mathcal{B}_T$ is a Banach space with the above defined norm. We now define the operator $\Gamma : \mathcal{B}_T \to \mathcal{B}_T$ as

$$\Gamma u(t) = \begin{cases} \mathcal{S}(t)\phi(0) + C(t)[x_1 + H(0, u_0, \int_{-\infty}^0 h(\theta, u(\theta))d\theta)] \\ - \int_0^t \mathcal{S}(t-s)H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta)ds + \int_0^t C(t-s)F(s, u_s, \int_{-\infty}^0 f(\theta, u(s+\theta))d\theta)ds \\ + \int_0^t C(t-s)G(s, u_s, \int_{-\infty}^0 g(\theta, u(s+\theta))d\theta)dw(s) + \int_0^t \int_Z C(t-s)\mathcal{K}(s, u_s, y)\widetilde{N}(ds, dy) \\ + \sum_{0<t_n<t} \mathcal{S}(t-t_n)I_n(u(t_n^-)) + \sum_{0<t_n<t} C(t-t_n)J_n(u(t_n^-)),\ t \in [0, T], \\ \phi \in \mathcal{C},\ t \in (-\infty, 0]. \end{cases} \tag{16}$$

This theorem is proved in several steps.

First, the right continuity of operator Γ on the interval $[0, T]$ is shown. Let $u \in \mathcal{B}_T$ and $|\epsilon|$ be sufficiently small. Then, for $t \in (0, T)$,

$$
\begin{aligned}
&\mathbb{E}\|\Gamma u(t+\epsilon)-\Gamma u(t)\|^p\\
\leq\;& 8^{p-1}\mathbb{E}\|\Big(\mathcal{S}(t+\epsilon)-\mathcal{S}(t)\Big)\phi(0)\|^p+8^{p-1}\mathbb{E}\|\Big(C(t+\epsilon)-C(t)\Big)(x_1+H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta))\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^{t+\epsilon}\mathcal{S}(t+\epsilon-s)H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))ds\\
&\quad-\int_0^t \mathcal{S}(t-s)H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))ds\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^{t+\epsilon}C(t+\epsilon-s)F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))ds\\
&\quad-\int_0^t C(t-s)F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))ds\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^{t+\epsilon}C(t+\epsilon-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta))dw(s)\\
&\quad-\int_0^t C(t-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta))dw(s)\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^{t+\epsilon}\int_Z C(t+\epsilon-s)\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)-\int_0^t\int_Z C(t-s)\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)\|^p\\
&+8^{p-1}\mathbb{E}\|\sum_{0<t_n<t}[\mathcal{S}(t+\epsilon-t_n)-\mathcal{S}(t-t_n)]I_n(u(t_n^-))\|^p\\
&+8^{p-1}\mathbb{E}\|\sum_{0<t_n<t}[C(t+\epsilon-t_n)-C(t-t_n)]J_n(u(t_n^-))\|^p,\\
=\;& 8^{p-1}\sum_{j=1}^{8}\mathbb{J}_j. \qquad (17)
\end{aligned}
$$

From the conditions

$$
\begin{aligned}
\mathbb{J}_1 \;\leq\;& \mathbb{E}\|\mathcal{S}(t+\epsilon)-\mathcal{S}(t)\|^p\times\mathbb{E}\|\phi(0)\|^p, \qquad (18)\\
\mathbb{J}_2 \;\leq\;& 2^{p-1}\mathbb{E}\|C(t+\epsilon)-C(t)\|^p\times[\mathbb{E}\|x_1\|^p+\mathbb{E}\|H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta))\|^p] \qquad (19)\\
\mathbb{J}_3 \;\leq\;& 2^{p-1}\mathbb{E}\|\int_t^{t+\epsilon}\mathcal{S}(t+\epsilon-s)H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))ds\|^p\\
&+2^{p-1}\mathbb{E}\|\int_0^t[\mathcal{S}(t+\epsilon-s)-\mathcal{S}(t-s)]H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))ds\|^p,\\
\leq\;& 2^{p-1}\Big[\int_t^{t+\epsilon}\|\mathcal{S}(t+\epsilon-s)\|^{p/p-1}ds\Big]^{p-1}\int_t^{t+\epsilon}\mathbb{E}\|H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))\|^p ds\\
&+2^{p-1}\Big[\int_0^t\|\mathcal{S}(t+\epsilon-s)-\mathcal{S}(t-s)\|^{\frac{p}{p-1}}ds\Big]^{p-1}\int_0^t\mathbb{E}\|H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))\|^p ds\\
\to\;& 0,\ \text{ as }\ \epsilon\to 0, \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{J}_4 \quad &\leq \quad 2^{p-1}\mathbb{E}\|\int_t^{t+\epsilon} C(t+\epsilon-s)F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))ds\|^p \\
&\quad +\mathbb{E}\|\int_0^t [C(t+\epsilon-s)-C(t-s)]F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))ds\|^p \\
&\leq \quad 2^{p-1}\Big[\int_t^{t+\epsilon}\|C(t+\epsilon-s)\|^{p/p-1}ds\Big]^{p-1}\int_t^{t+\epsilon}\mathbb{E}\|F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))\|^p ds \\
&\quad +2^{p-1}\Big[\int_0^t\|C(t+\epsilon-s)-C(t-s)\|^{\frac{p}{p-1}}ds\Big]^{p-1}\int_0^t\mathbb{E}\|F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))\|^p ds \\
&\to \quad 0, \text{ as } \epsilon\to 0,
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathbb{J}_5 \quad &\leq \quad 2^{p-1}\mathbb{E}\|\int_t^{t+\epsilon} C(t+\epsilon-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta))dw(s)\|^p \\
&\quad +2^{p-1}\mathbb{E}\|\int_0^t [C(t+\epsilon-s)-C(t-s)]G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)dw(s)\|^p \\
&\leq \quad 2^{p-1}\mathbb{C}_p\Big[\int_t^{t+\epsilon}\Big(\mathbb{E}\|C(t+\epsilon-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta))\|^p\Big)^{2/p}ds\Big]^{p/2} \\
&\quad +2^{p-1}\mathbb{C}_p\Big[\int_0^t\Big(\mathbb{E}\|[C(t+\epsilon-s)-C(t-s)]G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta))\|^p\Big)^{2/p}ds\Big]^{p/2} \\
&\to \quad 0, \text{ as } \epsilon\to 0,
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\mathbb{J}_6 \quad &\leq \quad 2^{p-1}\mathbb{E}\|\int_t^{t+\epsilon}\int_Z C(t+\epsilon-s)\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)\|^p \\
&\quad +2^{p-1}\mathbb{E}\|\int_0^t\int_Z [C(t+\epsilon-s)-C(t-s)]\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)\|^p \\
&\leq \quad 2^{p-1}\mathbb{C}_p\mathbb{E}\Big[\int_t^{t+\epsilon}\int_Z\|C(t+\epsilon-s)\mathcal{K}(s,u_s,y)\|^2 ds\mu dy\Big]^{p/2} \\
&\quad +2^{p-1}\mathbb{C}_p\mathbb{E}\Big[\int_t^{t+\epsilon}\int_Z\|C(t+\epsilon-s)\mathcal{K}(s,u_s,y)\|^p ds\mu dy\Big] \\
&\quad +2^{p-1}\mathbb{C}_p\mathbb{E}\Big[\int_0^t\int_Z\|[C(t+\epsilon-s)-C(t-s)]\mathcal{K}(s,u_s,y)\|^2 ds\mu dy\Big]^{p/2} \\
&\quad +2^{p-1}\mathbb{C}_p\mathbb{E}\Big[\int_0^t\int_Z\|[C(t+\epsilon-s)-C(t+\epsilon-s)]\mathcal{K}(s,u_s,y)\|^p ds\mu dy\Big] \\
&\leq \quad 2^{p-1}\mathbb{C}_p\Big[\int_t^{t+\epsilon}\|C(t+\epsilon-s)\|^2\int_Z\mathbb{E}\|\mathcal{K}(s,u_s,y)\|^2\mu(dy)ds\Big]^{p/2} \\
&\quad +2^{p-1}\mathbb{C}_p\int_t^{t+\epsilon}\|C(t+\epsilon-s)\|^p\int_Z\mathbb{E}\|\mathcal{K}(s,u_s,y)\|^p\mu(dy)ds \\
&\quad +2^{p-1}\mathbb{C}_p\Big[\int_0^t\|C(t+\epsilon-s)-C(t-s)\|^2\int_Z\mathbb{E}\|\mathcal{K}(s,u_s,y)\|^2\mu(dy)ds\Big]^{p/2} \\
&\quad +2^{p-1}\mathbb{C}_p\int_0^t\|[C(t+\epsilon-s)-C(t-s)]\|^p\int_Z\mathbb{E}\|\mathcal{K}(s,u_s,y)\|^p\mu(dy)ds \\
&\to \quad 0, \text{ as } \epsilon\to 0,
\end{aligned}
\tag{23}
$$

$$
\mathbb{J}_7 \quad \leq \quad \sum_{0<t_n<t}\|\mathcal{S}(t+\epsilon-t_n)-\mathcal{S}(t-t_n)\|^p\times\mathbb{E}\|I_n(u(t_n^-))\|^p \ \to 0, \text{ as } \epsilon\to 0, \tag{24}
$$

$$
\mathbb{J}_8 \quad \leq \quad \sum_{0<t_n<t}\|C(t+\epsilon-t_n)-C(t-t_n)\|^p\times\mathbb{E}\|J_n(u(t_n^-))\|^p \ \to 0, \text{ as } \epsilon\to 0. \tag{25}
$$

Thus, the above estimations gives

$$\lim_{\epsilon\to 0}\mathbb{E}\|(\Gamma u)(t+\epsilon)-(\Gamma u)(t)\|^p=0. \tag{26}$$

Consequently, we conclude that the function $t\mapsto(\Gamma u)(t)$ is continuous on the interval $(-\infty,T]$.
Next, we show that $\Gamma(\mathcal{B}_T)\subset\mathcal{B}_T$. For $y\in\mathcal{B}_T$, we have
$\mathbb{E}\|\Gamma u(t)\|^p$

$$\begin{aligned}
\leq\; & 8^{p-1}\mathbb{E}\|\mathcal{S}(t)\phi(0)\|^p+8^{p-1}\mathbb{E}\|C(t)[x_1+H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)]\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^t\mathcal{S}(t-s)H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))ds\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^t C(t-s)F(s,u_s,\int_{-\infty}^0 f(\theta,u(s+\theta))d\theta))ds\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^t C(t-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)dw(s)\|^p\\
&+8^{p-1}\mathbb{E}\|\int_0^t\int_Z C(t-s)\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)\|^p\\
&+8^{p-1}\mathbb{E}\|\sum_{0<t_n<t}\mathcal{S}(t-t_n)I_n(u(t_n^-))\|^p+8^{p-1}\mathbb{E}\|\sum_{0<t_n<t}C(t-t_n)J_n(u(t_n^-))\|^p\\
\leq\; & 8^{p-1}\Big\{M^pe^{-p\mu_1t}\|\phi\|^p+2^{p-1}M^pe^{-p\mu_2t}(\|x_1\|^p+\mathbb{E}\|H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p)\\
&+2^{p-1}M^p\mu_1^{1-p}\int_0^t e^{-\mu_1(t-s)}[\mathbb{E}\|H(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))-H(s,0,0)\|^p+\mathbb{E}\|H(s,0,0)\|^p]ds\\
&+2^{p-1}M^p\mu_2^{1-p}\int_0^t e^{-\mu_2(t-s)}[\mathbb{E}\|F(s,u_s,\int_{-\infty}^0 h(\theta,u(s+\theta))d\theta))-F(s,0,0)\|^p+\mathbb{E}\|F(s,0,0)\|^p]ds\\
&+M^p\mathbb{C}_p\Big(\int_0^t\Big[e^{-p\mu_2(t-s)}\mathbb{E}\|G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)\|^p_{L_2^0}\Big]^{2/p}ds\Big)^{p/2}\\
&+\mathbb{C}_p\Big[\mathbb{E}\Big(\int_0^t\int_Z\|C(t-s)\mathcal{K}(s,u_s,y)\|^2ds\mu dy\Big)^{p/2}+\int_0^t\int_Z\|C(t-s)\mathcal{K}(s,u_s,y)\|^pds\mu dy\Big]\\
&+M^p\mathbb{E}\Big(\sum_{c=1}^\infty c_n^{\frac{p-1}{p}}c_n^{1/p}e^{-\mu_1(t-t_n)}\|u(t_n^-)\|\Big)^p+M^p\mathbb{E}\Big(\sum_{n=1}^\infty d_n^{\frac{p-1}{p}}b_n^{1/p}e^{-\mu_2(t-t_n)}\|u(t_n^-)\|\Big)^p\Big\}\\
\leq\; & 8^{p-1}\Big\{M^pe^{-p\mu_1t}\|\phi\|^p+2^{p-1}M^pe^{-p\mu_2t}(\|x_1\|^p+\mathbb{E}\|H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p)\\
&+2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{1-p}\int_0^t e^{-\mu_1(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\\
&+2^{p-1}L_h\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^pds+2^{p-1}M^p(L_H^1)^p\mu_1^{-p}+2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{1-p}\\
&\times\int_0^t e^{-\mu_2(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta+2^{p-1}L_f\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^pds\\
&+2^{p-1}M^p(L_F^1)^p\mu_2^{-p}+\mathbb{C}_pM^p\Big(\int_0^t e^{-\frac{2\mu_2(p-1)}{p-2}(t-s)}ds\Big)^{p/2-1}\\
&\times\int_0^t e^{-\mu_2(t-s)}\mathbb{E}\|G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)\|^p_{L_2^0}ds+\mathbb{C}_pM^p\Big[L_{\mathcal{K}}^p
\end{aligned}$$

$$
\begin{aligned}
&\times\Big(\int_0^t e^{-2\mu_2(t-s)}\int_Z \mathbb{E}\|\mathcal{K}(s,u_s,y)\|^2\mu dyds\Big)^{p/2} + L_{\mathcal{K}}^p\int_0^t e^{-p\mu_2(t-s)}\int_Z \mathbb{E}\|\mathcal{K}(s,u_s,y)\|^p\mu dyds\Big]\\
&+[M^p\Big(\sum_{n=1}^{\infty}c_n\Big)^p + M^p\Big(\sum_{n=1}^{\infty}d_n\Big)^p]\|u(t_n)\|^p\Big\}\\
\leq\ & 8^{p-1}\Big\{M^pe^{-p\mu_1 t}\|\phi\|^p + 2^{p-1}M^pe^{-p\mu_2 t}(\|x_1\|^p + \mathbb{E}\|H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p)\\
&+2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{1-p}(1+2L_h)^p\int_0^t e^{-\mu_1(t-s)}\int_{-\infty}^0 k(\vartheta)\mathbb{E}\|u(s+\vartheta)\|^pd\vartheta ds\\
&+2^{p-1}M^p(L_H^1)^p\mu_1^{-p} + 2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{1-p}(1+2L_f)^p\int_0^t e^{-\mu_2(t-s)}\\
&\times\int_{-\infty}^0 k(\vartheta)\mathbb{E}\|u(s+\vartheta)\|^pd\vartheta ds + 2^{p-1}M^p(L_F^1)^p\mu_2^{-p} + 2^{(p-1)sgn(L_G^1)}\mathbb{C}_pM^pL_G^p\\
&\times\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}(1+L_g)^p\int_0^t e^{-\mu_2(t-s)}\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|^pd\vartheta ds\\
&+2^{p-1}\mathbb{C}_p\mu_2^{-1}M^p(L_G^1)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}} + \mathbb{C}_pM_p\Big[L_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}\int_0^t e^{-\mu_2(t-s)}\\
&\times\int_Z\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|^pd\vartheta\mu dyds + L_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}\\
&\times\int_0^t e^{-\mu_2(t-s)}\int_Z\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|^pd\vartheta\mu dyds\Big] + [M^p\Big(\sum_{n=1}^{\infty}c_n\Big)^p + M^p\Big(\sum_{n=1}^{\infty}d_n\Big)^p]\|u(t_n)\|^p\Big\},\\
\leq\ & 8^{p-1}\Big\{M^pe^{-p\mu_1 t}\|\phi\|^p + 2^{p-1}M^pe^{-p\mu_2 t}(\|x_1\|^p + \mathbb{E}\|H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p)\\
&+2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{-p}(1+2L_h)^p\sup_{t\in(-\infty,T]}\|u(t)\|^p + 2^{p-1}M^p(L_H^1)^p\mu_1^{-p}\\
&+2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{-p}(1+2L_f)^p\sup_{t\in(-\infty,T]}\|u(t)\|^p + 2^{p-1}M^p(L_F^1)^p\mu_2^{-p} + 2^{(p-1)sgn(L_G^1)}\mathbb{C}_pM^pL_G^p\\
&\times\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}(1+L_g)^p\times\frac{1}{\mu_2}\sup_{t\in(-\infty,T]}\|u(t)\|^p + 2^{p-1}\mathbb{C}_p\mu_2^{-1}M^p(L_G^1)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}\\
&+\mathbb{C}_pM_p\Big[L_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}\frac{1}{\mu_2}(\mathbb{L}_{k'}^2)^{p/2}\sup_{t\in(-\infty,T]}\|u(t)\|^p + L_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-\frac{p}{2}}(\mathbb{L}_{k'}^p)^p\\
&\times\frac{1}{\mu_2}\sup_{t\in(-\infty,T]}\|u(t)\|^p] + [M^p\Big(\sum_{n=1}^{\infty}c_n\Big)^p + M^p\Big(\sum_{n=1}^{\infty}d_n\Big)^p]\|u(t_n)\|^p\Big\} < \infty.
\end{aligned}
$$

Since $\mathbb{E}\sup_{t\in(-\infty,T]}\|\Gamma u(t)\|^p \leq \mathbb{E}\sup_{t\in[0,T]}\|\Gamma u(t)\|^p + \mathbb{E}\sup_{\theta\in(-\infty,0]}\|\phi(\theta)\|^2$, therefore, it can be concluded that $\mathbb{E}\sup_{t\in(-\infty,T]}\|\Gamma u(t)\|^p < \infty$.
Next, we show that Γ is a contraction in $\mathcal{B}_T$. Let $u_1, u_2 \in \mathcal{B}_T$ and $t \in [0,T]$. Thus, we have

$\mathbb{E}\|\Gamma u_1(t) - \Gamma u_2(t)\|^p$

$$\begin{aligned}
\leq\ & 7^{p-1}\mathbb{E}\|C(t)[H(0,u_0,\int_{-\infty}^{0} h(\theta,u_1(\theta))d\theta) - H(0,u_0,\int_{-\infty}^{0} h(\theta,u_2(\theta))d\theta)]\|^p \\
& +7^{p-1}\mathbb{E}\|\int_0^t \mathcal{S}(t-s)[H(s,(u_1)_s,\int_{-\infty}^{0} h(\theta,u_1(s+\theta))d\theta)) - H(s,(u_2)_s,\int_{-\infty}^{0} h(\theta,u_2(s+\theta))d\theta))]ds\|^p \\
& +7^{p-1}\mathbb{E}\|\int_0^t C(t-s)[F(s,(u_1)_s,\int_{-\infty}^{0} f(\theta,u_1(s+\theta))d\theta)) - F(s,(u_2)_s,\int_{-\infty}^{0} f(\theta,u_2(s+\theta))d\theta))]ds\|^p \\
& +7^{p-1}\mathbb{E}\|\int_0^t C(t-s)[G(s,(u_1)_s,\int_{-\infty}^{0} g(\theta,u_1(s+\theta))d\theta) - G(s,(u_2)_s,\int_{-\infty}^{0} g(\theta,u_2(s+\theta))d\theta)]dw(s)\|^p \\
& +7^{p-1}\mathbb{E}\|\int_0^t\int_Z C(t-s)[\mathcal{K}(s,(u_1)_s,y) - \mathcal{K}(s,(u_2)_s,y)]\widetilde{N}(ds,dy)\|^p \\
& +7^{p-1}\mathbb{E}\|\sum_{0<t_n<t} \mathcal{S}(t-t_n)[I_n(u_1(t_n^-)) - I_n(u_2(t_n^-))]\|^p \\
& +7^{p-1}\mathbb{E}\|\sum_{0<t_n<t} C(t-t_n)[J_n(u_1(t_n^-)) - J_n(u_2(t_n^-))]\|^p \\
\leq\ & 7^{p-1}\Big\{M^pL_H^pL_h^p + M^pL_H^p\mu_1^{-p}(1+L_h)^pT + M^pL_F^p\mu_2^{-p}(1+L_f)^pT + \mathbb{C}_pM^p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} \\
& \times\mu_2^{-p/2}L_G^p(1+L_g)^pT + \mathbb{C}_p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2}\mu_2^{-p/2}L_{\mathcal{K}}^pT((\mathbb{L}_{k'}^2)^{p/2} + (\mathbb{L}_{k'}^p)^p)M^p + M^p\Big(\sum_{n=1}^{\infty} c_n\Big)^p \\
& +M^p\Big(\sum_{n=1}^{\infty} d_n\Big)^p\Big\} \times \mathbb{E}\sup_{s\in(-\infty,t]}\|u_1(s) - u_2(s)\|^p. \qquad (27)
\end{aligned}$$

Therefore, by condition (15), we have

$$\begin{aligned}
& 7^{p-1}\Big\{M^pL_H^pL_h^p + M^pL_H^p\mu_1^{-p}(1+L_h)^pT + M^pL_F^p\mu_2^{-p}(1+L_f)^pT + \mathbb{C}_pM^p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} \\
& \times\mu_2^{-p/2}L_G^p(1+L_g)^pT + \mathbb{C}_p\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2}\mu_2^{-p/2}L_{\mathcal{K}}^pT((\mathbb{L}_{k'}^2)^{p/2} + (\mathbb{L}_{k'}^p)^p)M^p + M^p\Big(\sum_{n=1}^{\infty} c_n\Big)^p \\
& +M^p\Big(\sum_{n=1}^{\infty} d_n\Big)^p\Big\} < 1. \qquad (28)
\end{aligned}$$

Thus, we conclude that Γ is a contraction operator on $\mathcal{B}_T$. Therefore, by Banach fixed point theorem, (1)-(4) has a unique solution on $[0,T]$. This completes the proof of the theorem. □

4. Exponential Stability

This section studies the p-th exponential stability of the mild solution of impulsive stochastic system (1)-(4). We use the following lemma for establishing the exponentially stability of the mild solution of considered system (1)-(4).

Lemma 4.1. *[9] Let $\Psi : \mathbb{R} \to [0,\infty)$ be a function and assume that there exist some constants $\eta_j > 0 (j = 1,2,3,4,5)$, $\zeta > 0$ and $c_n, d_n (n = 1,2,\cdots)$ such that*

$$\Psi(t) \leq \begin{cases} \eta_1 e^{-\mu_1 t} + \eta_2 e^{-\mu_2 t},\ t \in (-\infty,0], \\ \eta_1 e^{-\mu_1 t} + \eta_2 e^{-\mu_2 t} + \eta_3 \int_0^t e^{-\mu_1(t-s)} \int_{-\infty}^0 k(\vartheta)\Psi(s+\vartheta)d\vartheta ds \\ +\eta_4 \int_0^t e^{-\mu_2(t-s)} \int_{-\infty}^0 k(\vartheta)\Psi(s+\vartheta)d\vartheta ds + \sum_{t_n<t} c_n e^{-\mu_1(t-t_n)}\Psi(t_n^-) \\ + \sum_{t_n<t} d_n e^{-\mu_2(t-t_n)}\Psi(t_n^-) + \zeta \end{cases} \tag{29}$$

holds for $\mu_1, \mu_2 \in (0,\gamma]$, $\gamma > 0$. If

$$\sigma := \frac{\eta_3}{\mu_1} + \frac{\eta_4}{\mu_2} + \sum_{n=1}^{\infty}(c_n + d_n) < 1, \tag{30}$$

then

$$\Psi(t) \leq \delta e^{-\lambda t} + \frac{\eta}{1-\sigma},\ t \in (-\infty,\infty), \tag{31}$$

where $\lambda \in (0, \mu_1 \wedge \mu_2)$ and $\delta \geq \eta_1 + \eta_2$ satisfy

$$\begin{aligned} \sigma_\lambda &:= \left(\frac{\eta_3}{\mu_1 - \lambda} + \frac{\eta_4}{\mu_2 - \lambda}\right)\int_{-\infty}^0 k(\vartheta)e^{-\lambda\vartheta}d\vartheta + \sum_{n=1}^{\infty}(c_n + d_n) < 1, \\ \delta &\geq \frac{\eta_1 + \eta_2}{1 - \sigma_\lambda}; \end{aligned} \tag{32}$$

or $\sigma_\lambda \leq 1$ and

$$\delta \geq \frac{(\mu_1 - \lambda)(\mu_2 - \lambda)[(\eta_1 + \eta_2) - (\frac{\eta_3}{\mu_1} + \frac{\eta_4}{\mu_2})\frac{\eta}{1-\sigma}]}{\int_{-\infty}^0 k(\vartheta)e^{-\lambda\vartheta}d\vartheta(\eta_3(\mu_2 - \lambda) - \eta_4(\mu_1 - \lambda))}, \tag{33}$$

where $\eta_3(\mu_2 - \lambda) \neq \eta_4(\mu_1 - \lambda)$.

Theorem 4.2. *Let us assume that the conditions $(A1)$-$(A4)$ hold and $\mu_1, \mu_2 \in (0,\gamma]$. Then, (1)-(4) is exponentially stable in p-th moment provided*

$$\begin{aligned} &2^{(p-1)sgn(L_H^1)} M^p L_H^p \mu_1^{-p}(1 + L_h)^p + 2^{(p-1)sgn(L_F^1)} M^p L_F^p \mu_2^{-p}(1 + L_f)^p + 2^{(p-1)sgn(L_G^1)} \mathbb{C}_p M^p L_G (1 + L_g)^p \\ &\times \mu_2^{-p/2}\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} + \mathbb{C}_p M^p L_{\mathcal{K}}^p \Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_k^2)^{p/2} + (\mathbb{L}_k^p)^p) + M^p\Big(\sum_{n=1}^{\infty} c_n\Big)^p \\ &+ M^p\Big(\sum_{n=1}^{\infty} d_n\Big)^p < 8^{1-p}. \end{aligned} \tag{34}$$

Proof. From (11), we have

$$\begin{aligned}
&\mathbb{E}\|u(t)\|^p \\
&= \|\mathcal{S}(t)\phi(0) + C(t)[x_1 + H(0, u_0, \int_{-\infty}^0 h(\theta, u(\theta))d\theta)] - \int_0^t \mathcal{S}(t-s)H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta))ds \\
&\quad + \int_0^t C(t-s)F(s, u_s, \int_{-\infty}^0 f(\theta, u(s+\theta))d\theta))ds + \int_0^t C(t-s)G(s, u_s, \int_{-\infty}^0 g(\theta, u(s+\theta))d\theta)dw(s) \\
&\quad + \int_0^t \int_Z C(t-s)\mathcal{K}(s, u_s, y)\widetilde{N}(ds, dy) + \sum_{0<t_n<t} \mathcal{S}(t-t_n)I_n(u(t_n^-)) + \sum_{0<t_n<t} C(t-t_n)J_n(u(t_n^-))\|^p \\
&\leq 8^{p-1}\mathbb{E}\|\mathcal{S}(t)\phi(0)\|^p + 8^{p-1}\mathbb{E}\|C(t)[x_1 + H(0, u_0, \int_{-\infty}^0 h(\theta, u(\theta))d\theta)]\|^p \\
&\quad + 8^{p-1}\mathbb{E}\|\int_0^t \mathcal{S}(t-s)H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta))ds\|^p \\
&\quad + 8^{p-1}\mathbb{E}\|\int_0^t C(t-s)F(s, u_s, \int_{-\infty}^0 f(\theta, u(s+\theta))d\theta))ds\|^p \\
&\quad + 8^{p-1}\mathbb{E}\|\int_0^t C(t-s)G(s, u_s, \int_{-\infty}^0 g(\theta, u(s+\theta))d\theta)dw(s)\|^p \\
&\quad + 8^{p-1}\mathbb{E}\|\int_0^t \int_Z C(t-s)\mathcal{K}(s, u_s, y)\widetilde{N}(ds, dy)\|^p + 8^{p-1}\mathbb{E}\|\sum_{0<t_n<t} \mathcal{S}(t-t_n)I_n(u(t_n^-))\|^p \\
&\quad + 8^{p-1}\mathbb{E}\|\sum_{0<t_n<t} C(t-t_n)J_n(u(t_n^-))\|^p. \qquad (35)
\end{aligned}$$

By the assumption $(A1)$-$(A4)$ and Hölder inequality, we get

$$\begin{aligned}
&\mathbb{E}\|\int_0^t \mathcal{S}(t-s)H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta))ds\|^p \\
&\leq M^p\Big(\int_0^t (e^{-\mu_1(t-s)(p+1)/p})^{\frac{p}{p-1}} ds\Big)^{p-1} \times \int_0^t e^{-\mu_1(t-s)}\mathbb{E}\|H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta))\|^p ds \\
&\leq M^p \mu_1^{1-p} \int_0^t e^{-\mu_1(t-s)}\mathbb{E}\|H(s, u_s, \int_{-\infty}^0 h(\theta, u(s+\theta))d\theta)) - H(s,0,0) + H(s,0,0)\|^p ds \\
&\leq 2^{(p-1)sgn(L_H^1)} M^p L_H^p \mu_1^{1-p}(1+L_h)^p \int_0^t e^{-\mu_1(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds + 2^{p-1}M^p L_H^1 \mu_1^{-p}.
\end{aligned}$$

Similarly, we can estimate

$$\begin{aligned}
&\mathbb{E}\|\int_0^t \mathcal{S}(t-s)F(s, u_s, \int_{-\infty}^0 f(\theta, u(s+\theta))d\theta))ds\|^p \\
&\leq 2^{(p-1)sgn(L_F^1)} M^p L_F^p \mu_2^{1-p}(1+L_f)^p \int_0^t e^{-\mu_2(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds + 2^{p-1}M^p L_F^1 \mu_2^{-p}.
\end{aligned}$$

From the conditions $(A1)$ and $(A4)$, we have

$$\begin{aligned}
\mathbb{E}\|\sum_{0<t_n<t} \mathcal{S}(t-t_n)I_n(u(t_n^-))\|^p &\leq M^p\mathbb{E}\Big(\sum_{n=1}^{\infty} c_n^{\frac{p-1}{p}} c_n^{1/p} e^{-\mu_1(t-t_n)}\|u(t_n^-)\|\Big)^p \\
&\leq M^p\Big(\sum_{0<t_n<t} c_n\Big)^{p-1} \sum_{0<t_n<t} c_n e^{-p\mu_1(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p \\
&\leq M^p\Big(\sum_{0<t_n<t} c_n\Big)^{p-1} \sum_{0<t_n<t} c_n e^{-\mu_1(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p,
\end{aligned}$$

and

$$\mathbb{E}\|\sum_{0<t_n<t} C(t-t_n)J_n(u(t_n^-))\|^p \quad \leq \quad M^p\Big(\sum_{0<t_n<t} d_n\Big)^{p-1}\sum_{0<t_n<t} d_n e^{-\mu_2(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p.$$

By Lemma 2.2,

$$\begin{aligned}
&\mathbb{E}\|\int_0^t C(t-s)G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)dw(s)\|^p\\
&\leq M^p\mathbb{C}_p\Big(\int_0^t (e^{-\mu_2 p(t-s)}\mathbb{E}\|G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)\|^p_{L_2^0})^{2/p}ds\Big)^{p/2}\\
&= M^p\mathbb{C}_p\Big(\int_0^t e^{-2\mu_2(t-s)}(\mathbb{E}\|G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)\|^p_{L_2^0})^{2/p}ds\Big)^{p/2}\\
&\leq \mathbb{C}_p M^p\Big(\int_0^t e^{-\frac{2\mu_2(p-1)}{p-2}(t-s)}ds\Big)^{p/2-1}\int_0^t e^{-\mu_2(t-s)}\mathbb{E}\|G(s,u_s,\int_{-\infty}^0 g(\theta,u(s+\theta))d\theta)\|^p_{L_2^0}ds\\
&\leq 2^{(p-1)sgn(L_G^1)}\mathbb{C}_p M^p L_G(1+L_g)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}\int_0^t e^{-\mu_2(t-s)}\\
&\quad\times\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds+\mathbb{C}_p 2^{p-1}\mu_2^{-1}M^p(L_F^1)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}, \qquad (36)
\end{aligned}$$

where $\mathbb{C}_p = (p(p-1)/2)^{p/2}$.
By Lemma 2.3, we have
$\mathbb{E}\|\int_0^t\int_Z C(t-s)\mathcal{K}(s,u_s,y)\widetilde{N}(ds,dy)\|^p$

$$\begin{aligned}
&\leq \mathbb{C}_p M^p\Big[\mathbb{E}\Big(\int_0^t\int_Z e^{2\mu_2(t-s)}\|\mathcal{K}(s,u_s,y)\|^2 ds\mu dy\Big)^{p/2}+\int_0^t\int_Z e^{-p\mu_2(t-s)}\mathbb{E}\|\mathcal{K}(s,u_s,y)\|^p ds\mu dy\Big]\\
&\leq \mathbb{C}_p M^p\Big[\Big(\int_0^t\int_Z e^{-2\mu_2(t-s)}(\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|d\vartheta)^2 ds\mu dy\Big)^{p/2}\\
&\quad+\int_0^t\int_Z e^{-p\mu_2(t-s)}(\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|d\vartheta)^p ds\mu dy\Big]\\
&\leq \mathbb{C}_p M^p L^p_{\mathcal{K}}\Big[\Big(\int_0^t\int_Z e^{-2\mu_2(t-s)}\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|^2 d\vartheta ds\mu dy\Big)^{p/2}\\
&\quad+\int_0^t\int_Z e^{-p\mu_2(t-s)}\int_{-\infty}^0 k(\vartheta)\|k'(u(s+\vartheta),y)\|^p d\vartheta ds\mu dy\Big]\\
&\leq \mathbb{C}_p M^p L^p_{\mathcal{K}}\Big[(\mathbb{L}^2_{k'})^{p/2}\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}\int_0^t e^{-\mu_2(t-s)}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds\\
&\quad+(\mathbb{L}^p_{k'})^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}\times\int_0^t e^{-\mu_2(t-s)}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds\Big]\\
&\leq \mathbb{C}_p M^p L^p_{\mathcal{K}}\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}^2_{k'})^{p/2}+(\mathbb{L}^p_{k'})^p)\int_0^t e^{-\mu_2(t-s)}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds. \qquad (37)
\end{aligned}$$

Thus, from (35) and Hölder inequality, we get

$$
\begin{aligned}
&\mathbb{E}\|u(t)\|^p \\
\leq\ & 8^{p-1}M^p\mathbb{E}\|\phi\|^p e^{-\mu_1 t} + 8^{p-1}M^p\mathbb{E}\|x_1 + H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p e^{-\mu_2 t} \\
&+8^{p-1}\times 2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{1-p}(1+L_h)^p\int_0^t e^{-\mu_1(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds \\
&+16^{p-1}M^pL_H^1\mu_1^{-p} + 8^{p-1}2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{1-p}(1+L_f)^p\int_0^t e^{-\mu_2(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds \\
&+16^{p-1}M^pL_F^1\mu_2^{-p} + 8^{p-1}2^{(p-1)sgn(L_G^1)}\mathbb{C}_pM^pL_G(1+L_g)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}\int_0^t e^{-\mu_2(t-s)} \\
&\times\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds + 16^{p-1}\mathbb{C}_p\mu_2^{-1}M^p(L_G^1)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2} \\
&+8^{p-1}\mathbb{C}_pM^pL_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_{k'}^2)^{p/2}+(\mathbb{L}_{k'}^p)^p)\int_0^t e^{-\mu_2(t-s)}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|d\vartheta\Big)^p ds \\
&+8^{p-1}M^p\Big(\sum_{0<t_n<t}c_n\Big)^{p-1}\sum_{0<t_n<t}c_ne^{-\mu_1(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p + 8^{p-1}M^p\Big(\sum_{0<t_n<t}d_n\Big)^{p-1} \\
&\times\sum_{0<t_n<t}d_ne^{-\mu_2(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p \\
:=\ & \widehat{\mathbb{K}} + 8^{p-1}M^p\mathbb{E}\|\phi\|^p e^{-\mu_1 t} + 8^{p-1}M^p\mathbb{E}\|x_1 + H(0,u_0,\int_{-\infty}^0 h(\theta,u(\theta))d\theta)\|^p e^{-\mu_2 t} \\
&+\widehat{\mathbb{K}}_3\int_0^t e^{-\mu_1(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|^p d\vartheta\Big)ds + \widehat{\mathbb{K}}_4\int_0^t e^{-\mu_2(t-s)}\mathbb{E}\Big(\int_{-\infty}^0 k(\vartheta)\|u(s+\vartheta)\|^p d\vartheta\Big)ds \\
&+\sum_{0<t_n<t}\widehat{c_n}e^{-\mu_1(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p + \sum_{0<t_n<t}\widehat{d_n}e^{-\mu_2(t-t_n)}\mathbb{E}\|u(t_n^-)\|^p,
\end{aligned}
\tag{38}
$$

where

$$
\begin{aligned}
\widehat{\mathbb{K}} &= 16^{p-1}M^pL_H^1\mu_1^{-p} + 16^{p-1}M^pL_H^1\mu_1^{-p} + 16^{p-1}\mathbb{C}_p\mu_2^{-1}M^p(L_G^1)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}, \\
\widehat{\mathbb{K}}_3 &= 8^{p-1}\times 2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{1-p}(1+L_h)^p, \\
\widehat{\mathbb{K}}_4 &= 8^{p-1}2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{1-p}(1+L_f)^p + 8^{p-1}2^{(p-1)sgn(L_G^1)}\mathbb{C}_pM^pL_G(1+L_g)^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2} \\
&\quad +8^{p-1}\mathbb{C}_pM^pL_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_{k'}^2)^{p/2}+(\mathbb{L}_{k'}^p)^p), \\
\widehat{c_n} &= 8^{p-1}M^p\Big(\sum_{0<t_n<t}c_n\Big)^{p-1}c_n,\ \ \widehat{d_n} = 8^{p-1}M^p\Big(\sum_{0<t_n<t}d_n\Big)^{p-1}d_n.
\end{aligned}
\tag{39}
$$

Therefore, it is obvious that there exist two positive numbers $\mathbb{B}'$ and $\mathbb{B}''$ such that for any $t \in (-\infty, 0]$

$$\mathbb{E}\|u(t)\|^p \leq \mathbb{B}' e^{-\mu_1 t} + \mathbb{B}'' e^{-\mu_2 t}.$$

If inequality (34) holds, i.e. $\frac{\widehat{\mathbb{K}}_3}{\mu_1} + \frac{\widehat{\mathbb{K}}_4}{\mu_2} + \sum_{0<t_n<t}\widehat{c_n} + \sum_{0<t_n<t}\widehat{d_n} < 1$, then condition (30) of Lemma 4.1 holds. Thus, there exist positive constants $\mathbb{B}$ and γ such that $\mathbb{E}\|u(t)\|^p \leq \mathbb{B}e^{-\gamma t},\ t \in [0,\infty)$. This finishes the proof of the theorem. □

When $p = 2$, we have the following result by Theorem 4.2. For this result, we need following assumption and lemma.

(A') The function $\mathcal{K}(t,\cdot,\cdot)$ is continuous and there exist positive constants $L_{\mathcal{K}}, \widetilde{L}_1$ and $\widetilde{L}_2 > 0$ and a continuous function $k' : C \times Z \to X$ such that

$$\|\mathcal{K}(t,x,u) - \mathcal{K}(t,y,u)\| \le L_{\mathcal{K}} \int_{-\infty}^{0} k(\vartheta)\|k'(x(t+\vartheta),u) - k'(y(t+\vartheta),u)\|d\vartheta, \ \ \mathcal{K}(t,0,0) = 0,$$

and for any $z \in Z$ and $x, y \in C$, $t \ge 0$,

$$\int_Z \|k'(x,u) - k'(y,u)\|^i \mu du \le \widetilde{L}_i \|x-y\|^i, \quad i = 2,4, \ \ k'(0,u) = 0.$$

Corollary 4.3. *Suppose assumptions $(A1)-(A2)$,(A'), $(A4)-(A5)$ hold and $\mu_1, \mu_2 \in (0,\gamma]$. Then, (1) is mean square exponentially stable provided*

$$\begin{aligned} &\frac{2^{sgn(L_H^1)}M^2L_H^2(1+L_h)^2}{\mu_1^2} + \frac{2^{sgn(L_F^1)}M^2L_F^2(1+L_f)^2}{\mu_2^2} + 2^{sgn(L_G^1)}M^2L_G^2(1+L_g)^2 \times \mu_2^{-1} \\ &+ \mathbb{C}_2 M^2 L_{\mathcal{K}}^2(\widetilde{L}_2 T + T\sqrt{\widetilde{L}_4})) + M^2\Big(\sum_{n=1}^{\infty} c_n\Big)^2 + M^2\Big(\sum_{n=1}^{\infty} d_n\Big)^2 < \frac{1}{8}. \end{aligned} \tag{40}$$

Remark 4.4. *Here,Corollary 4.3 is different from the lemmas of Chen [17], Hua and Jiang [33], Jiang et al. (2016a), Sakthivel et al. [29], and Chadha and Bora [2], that played an important role in proving the stability of the mild solution of stochastic system (1). The nonnegative constant ζ plays an important role in the lemma 4.2. Lemma 4.1 is the generalization of the paper Lemma 3.2 in [16], (if $\eta_j > 0 (j = 1,2,3,4), \zeta = 0$ in Lemma 4.1) . Meanwhile, in Lemma 4.1 we only require λ to satisfy $\sigma_\lambda \le 1$ or $\sigma < 1$, which can be found more easily than the condition $\sigma_\lambda = 1$ of [16] .*

Remark 4.5. *The inequalities (34), (40) are independent of initial state which is needed in most of existing results. Besides, in this article, the function $k(t)$ satisfies the two important conditions $\int_{-\infty}^{0} k(t)dt = 1$ and $\int_{-\infty}^{0} k(t)e^{-ht}dt < \infty (h > 0)$.*

Remark 4.6. *Equation (1) is more general than those considered in [2, 3, 16, 17, 24, 25, 29, 32, 35]. Sufficient conditions on exponential stability of (1) are derived by establishing a new integral inequality with impulses generalize and improves the results of [2, 29].*

Corollary 4.7. *If the conditions $(A1)-(A4)$ hold with $L_F^1 = L_H^1 = L_G^1 = 0$ and $\mu_1, \mu_2 \in (0,\gamma]$, then (1) is exponentially stable in the p-th moment provided*

$$\begin{aligned} &M^p L_H^p \mu_1^{-p}(1+L_h)^p + M^p L_F^p \mu_2^{-p}(1+L_f)^p + \mathbb{C}_p M^p L_G (1+L_g)^p \times \mu_2^{-p/2}\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} \\ &+\mathbb{C}_p M^p L_{\mathcal{K}}^p \Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_k^2)^{p/2} + (\mathbb{L}_k^p)^p) + M^p\Big(\sum_{j=1}^{\infty} a_j\Big)^p + M^p\Big(\sum_{i=1}^{\infty} b_j\Big)^p < 8^{1-p}. \end{aligned}$$

If $I_j = J_j = 0$, then the system (1)-(4) turns out to be the following system:

$$\begin{aligned} &d[u'(t) + H(t,u_t,\int_{-\infty}^{0} h(\theta,u(t+\theta))d\theta))] = [Au(t) + F(t,u_t,\int_{-\infty}^{0} f(\theta,u(t+\theta))d\theta))]dt \\ &+ G(t,u_t,\int_{-\infty}^{0} g(\theta,u(t+\theta))d\theta)dw(t) + \int_Z \mathcal{K}(t,u_t,y)\widetilde{N}(dt,dy), \ t \ge 0, \ \ t \ne t_i \end{aligned} \tag{41}$$

$$u_0(\cdot) = \varphi \in C, \ \ u'(0) = x_1,. \tag{42}$$

Then, we similarly can get following results by Lemma 4.1.

Corollary 4.8. *Let us assume that* $(A1)$-$(A4)$ *are satisfied and* $\mu_1, \mu_2 \in (0, \gamma]$. *Then, system (1) is p-th exponentially stable provided*

$$2^{(p-1)sgn(L_H^1)}M^pL_H^p\mu_1^{-p}(1+L_h)^p + 2^{(p-1)sgn(L_F^1)}M^pL_F^p\mu_2^{-p}(1+L_f)^p + 2^{(p-1)sgn(L_G^1)}\mathbb{C}_pM^p$$
$$\times L_G^p(1+L_g)^p\mu_2^{-p/2}\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2} + \mathbb{C}_pM^pL_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_k^2)^{p/2} + (\mathbb{L}_k^p)^p) < 6^{1-p}.$$

Corollary 4.9. *Suppose assumptions* $(A1)-(A2)$,(A'), $(A4)$ *hold and* $\mu_1, \mu_2 \in (0, \gamma]$. *Then, (1) is mean square exponentially stable provided*

$$\frac{2^{sgn(L_H^1)}M^2L_H^2(1+L_h)^2}{\mu_1^2} + \frac{2^{sgn(L_F^1)}M^2L_F^2(1+L_f)^2}{\mu_2^2} + 2^{sgn(L_G^1)}M^2L_G^2(1+L_g)^2 \times \mu_2^{-1}$$
$$+ \mathbb{C}_2M^2L_{\mathcal{K}}^2(\widetilde{L}_2T + T\sqrt{\widetilde{L}_4})) < \frac{1}{6}. \tag{43}$$

Corollary 4.10. *If the conditions* $(A1)-(A4)$ *hold with* $L_F^1 = L_H^1 = L_G^1 = 0$ *and* $\mu_1, \mu_2 \in (0, \gamma]$, *then (1) is exponentially stable in the p-th moment provided*

$$M^pL_H^p\mu_1^{-p}(1+L_h)^p + M^pL_F^p\mu_2^{-p}(1+L_f)^p + \mathbb{C}_pM^pL_G(1+L_g)^p \times \mu_2^{-p/2}\Big(\frac{2(p-1)}{p-2}\Big)^{1-p/2}$$
$$+\mathbb{C}_pM^pL_{\mathcal{K}}^p\Big(\frac{2\mu_2(p-1)}{p-2}\Big)^{1-p/2}((\mathbb{L}_k^2)^{p/2} + (\mathbb{L}_k^p)^p) < 6^{1-p}.$$

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