Filomat 32:2 (2018), 531–542
https://doi.org/10.2298/FIL1802531T

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Soliton Solutions for (2+1) and (3+1)-Dimensional Kadomtsev-Petviashvili-Benjamin-Bona-Mahony Model Equations and their Applications

Kalim U. Tariq[a,b], Aly Seadawy[d,c]

[a]*School of Mathematics and Statistics, Huazhong University of Science and Technology, Wuhan 430074, P. R. China*
[b]*Department of Mathematics, Mirpur University of Science and Technology, Mirpur (AJK) 10250, Pakistan*
[c]*Mathematics Department, Faculty of science, Taibah University, Al-Ula, Saudi Arabia*
[d]*Mathematics Department, Faculty of Science, Beni-Suef University, Egypt*

Abstract. The Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) model equations as a water wave model, are governing equations, for fluid flows, describes bidirectional propagating water wave surface. The soliton solutions for (2+1) and (3+1)-Dimensional Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equations have been extracted. The solitary wave ansatz method are adopted to approximate the solutions. The corresponding integrability criteria, also known as constraint conditions, naturally emerge from the analysis of the problem.

1. Introduction

The propagation of nonlinear wave is one of the key phenomenon of nature and a growing interest has been drawn to the study of nonlinear waves in the dynamical system. The nonlinear equations have plenty of applications in sciences and engineering like electrochemistry, electromagnetic theory, fluid dynamics, acoustics, cosmology, astrophysics and plasma physics etc., see for references [1-6].
In the last few eras great improvement have been made in the progress of methods for finding the exact solutions of nonlinear equations but the advancement achieved is inadequate. Taking into account the merits and demerits of analytic methods, it is observed that there is no single outstanding preferable method which can be applied to any kind of nonlinear problems to obtain exact solutions. Consequently, it is apprehended that all of these methods are problem dependent, viz. some approaches work well with certain problems but not the others. Therefore, it is rather substantial to relate some established techniques in the literature to nonlinear partial differential equations; for details see also [7-14].

The solitary wave Ansatz method [15-33] have been adopted to present the solutions of (2+1) and (3+1)-Dimensional Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equations, are respectively

2010 *Mathematics Subject Classification*. 02.30.Jr; 47.10.A-; 52.25.Xz; 52.35.Fp.
Keywords. Solitary wave soliton; shock wave soliton; singular solitons; Exact solutions
Received: 18 January 2017; Accepted: 09 April 2017
Communicated by Mića Stanković
Corresponding Author: Aly Seadway
Email addresses: kalimulhaq@hust.edu.cn (Kalim U. Tariq), aly742001@yahoo.com (Aly Seadawy)

defined by

$$(u_t + \mu_1 u_x - \mu_2 (u^2)_x - \mu_3 u_{xxt})_x + \mu_4 u_{yy} = 0 \tag{1}$$

$$(u_t + \mu_1 u_x - \mu_2 (u^2)_x - \mu_3 u_{xxt})_x + \mu_4 u_{yy} + \mu_5 u_{zz} = 0 \tag{2}$$

Where $\mu_1, \mu_2, \mu_3, \mu_4$ and μ_5 are real parameters. This article is organized as follows. In section 2, the solitary wave solution has been found, while in section 3, the shock wave solutions have been established and the singular forms are discussed in section 4. In last section, the conclusion have been drawn.

2. Solitary wave solitons

2.1. (2+1)-D KP-BBM equation

To calculate the solitary wave solitons for (1.1), suppose

$$u(x, y, t) = \frac{A}{\cosh^\lambda \psi} \quad \text{where} \quad \psi = \alpha x + \beta y - \nu t \tag{3}$$

A is the amplitude; α, β are the inverse widths and ν is the velocity of the solitary wave. λ is to be determined later. By using (2.3)

$$u_{tx} = \frac{A\lambda(\lambda+1)\alpha\nu}{\cosh^{\lambda+2}\psi} - \frac{A\lambda^2\nu}{\cosh^\lambda \psi} \tag{4}$$

$$u_{xx} = \frac{A\lambda^2\alpha^2}{\cosh^\lambda \psi} - \frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi} \tag{5}$$

$$(u^2)_{xx} = \frac{4A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} - \frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi} \tag{6}$$

$$u_{xxtx} = -\frac{A\lambda^4\alpha^3\nu}{\cosh^\lambda \psi} + \frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu}{\cosh^{\lambda+2}\psi} - \frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu}{\cosh^{\lambda+4}\psi} \tag{7}$$

$$u_{yy} = \frac{A\lambda^2\beta^2}{\cosh^\lambda \psi} - \frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi} \tag{8}$$

substituting (2.4)-(2.8) into (1.1)

$$\begin{aligned}
&\frac{A\lambda(\lambda+1)\alpha\nu}{\cosh^{\lambda+2}\psi} - \frac{A\lambda^2\nu}{\cosh^\lambda \psi} + \mu_1\frac{A\lambda^2\alpha^2}{\cosh^\lambda \psi} - \mu_1\frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi} - 4\mu_2\frac{A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi} \\
&+\mu_2\frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi} + \mu_3\frac{A\lambda^4\alpha^3\nu}{\cosh^\lambda \psi} - 2\mu_3\frac{A\lambda(\lambda+1)(\lambda^2-2\mu_3\lambda+2)\alpha^3\nu}{\cosh^{\lambda+2}\psi} \\
&+\mu_3\frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu}{\cosh^{\lambda+4}\psi} + \mu_4\frac{A\lambda^2\beta^2}{\cosh^\lambda \psi} - \mu_4\frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi} = 0
\end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned}
&A\lambda(\lambda+1)\alpha\nu - \mu_1 A\lambda(\lambda+1)\alpha^2 - 4\mu_2 A^2\lambda^2\alpha^2 \\
&-2\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu - \mu_4 A\lambda(\lambda+1)\beta^2 = 0 \\
&2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0
\end{aligned}$$

set $\lambda = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1 + \beta^2\mu_4)}{4\alpha^2\mu_2\mu_3 - \mu_2}, \quad \nu = \frac{\alpha^2\mu_1 + \beta^2\mu_4}{\alpha - 4\alpha^3\mu_3}$$

thus

$$u(x, y, t) = \frac{A}{\cosh^2(\alpha x + \beta y - \nu t)} \tag{9}$$

2.2. (3+1)-D KP-BBM equation

To calculate the solitary wave solitons for (1.2), suppose

$$u(x,y,z,t)=\frac{A}{\cosh^{\lambda}\psi}\quad \text{where}\quad \psi=\alpha x+\beta y+\gamma z-\nu t \tag{10}$$

A is the amplitude; α,β,γ are the inverse widths and ν is the velocity of the solitary wave. λ is to be determined later. By using (2.10)

$$u_{tx} = \frac{A\lambda(\lambda+1)\alpha\nu}{\cosh^{\lambda+2}\psi}-\frac{A\lambda^2\nu}{\cosh^{\lambda}\psi} \tag{11}$$

$$u_{xx} = \frac{A\lambda^2\alpha^2}{\cosh^{\lambda}\psi}-\frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi} \tag{12}$$

$$(u^2)_{xx} = \frac{4A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi}-\frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi} \tag{13}$$

$$u_{xxtx} = -\frac{A\lambda^4\alpha^3\nu}{\cosh^{\lambda}\psi}+\frac{2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu}{\cosh^{\lambda+2}\psi}-\frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu}{\cosh^{\lambda+4}\psi} \tag{14}$$

$$u_{yy} = \frac{A\lambda^2\beta^2}{\cosh^{\lambda}\psi}-\frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi} \tag{15}$$

$$u_{zz} = \frac{A\lambda^2\gamma^2}{\cosh^{\lambda}\psi}-\frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi} \tag{16}$$

substituting (2.4)-(2.9) into (1.2)

$$\begin{aligned}
&\frac{A\lambda(\lambda+1)\alpha\nu}{\cosh^{\lambda+2}\psi}-\frac{A\lambda^2\nu}{\cosh^{\lambda}\psi}+\mu_1\frac{A\lambda^2\alpha^2}{\cosh^{\lambda}\psi}-\mu_1\frac{A\lambda(\lambda+1)\alpha^2}{\cosh^{\lambda+2}\psi}-4\mu_2\frac{A^2\lambda^2\alpha^2}{\cosh^{2\lambda}\psi}\\
&+\mu_2\frac{2A^2\lambda(2\lambda+1)\alpha^2}{\cosh^{2\lambda+2}\psi}+\mu_3\frac{A\lambda^4\alpha^3\nu}{\cosh^{\lambda}\psi}-2\mu_3\frac{A\lambda(\lambda+1)(\lambda^2-2\mu_3\lambda+2)\alpha^3\nu}{\cosh^{\lambda+2}\psi}\\
&+\mu_3\frac{A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu}{\cosh^{\lambda+4}\psi}+\mu_4\frac{A\lambda^2\beta^2}{\cosh^{\lambda}\psi}-\mu_4\frac{A\lambda(\lambda+1)\beta^2}{\cosh^{\lambda+2}\psi}\\
&+\mu_5\frac{A\lambda^2\gamma^2}{\cosh^{\lambda}\psi}-\mu_5\frac{A\lambda(\lambda+1)\gamma^2}{\cosh^{\lambda+2}\psi}=0
\end{aligned}$$

by comparing the powers $2\lambda,\lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned}
&A\lambda(\lambda+1)\alpha\nu-\mu_1A\lambda(\lambda+1)\alpha^2-4\mu_2A^2\lambda^2\alpha^2\\
&-2\mu_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\\
&-\mu_4A\lambda(\lambda+1)\beta^2-\mu_5A\lambda(\lambda+1)\gamma^2=0\\
&2\mu_2A^2\lambda(2\lambda+1)\alpha^2+\mu_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu=0
\end{aligned}$$

set $\lambda=2$

$$A=\frac{6\mu_3(\alpha^2\mu_1+\beta^2\mu_4+\gamma^2\mu_5)}{4\alpha^2\mu_2\mu_3-\mu_2},\quad \nu=\frac{\alpha^2\mu_1+\beta^2\mu_4+\gamma^2\mu_5}{\alpha-4\alpha^3\mu_3}$$

thus

$$u(x,y,z,t)=\frac{A}{\cosh^2(\alpha x+\beta y+\gamma z-\nu t)} \tag{17}$$

3. Shock wave solitons

3.1. (2+1)-D KP-BBM equation

To calculate the shock wave solitons for (1.1), suppose

$$u(x,y,t) = A\tanh^{\lambda}\psi \text{ where } \psi = \alpha x + \beta y - \nu t \text{ and } \lambda > 0 \tag{18}$$

from (3.18)

$$\begin{aligned}
u_{tx} &= -A\lambda(\lambda-1)\alpha\nu\tanh^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\tanh^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\tanh^{\lambda+2}\psi \\
u_{xx} &= A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi - 2A\lambda^2\alpha^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi \qquad (19)\\
(u^2)_{xx} &= 2A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi \qquad (20)\\
u_{xxtx} &= -A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\tanh^{\lambda-4}\psi + 4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\tanh^{\lambda-2}\psi \\
&\quad -2A\lambda^2(3\lambda^2+5)\alpha^3\nu\tanh^{\lambda}\psi + 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\tanh^{\lambda+2}\psi \\
&\quad -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\tanh^{\lambda+4}\psi \qquad (21)\\
u_{yy} &= A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi - 2A\lambda^2\beta^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi \qquad (22)
\end{aligned}$$

substituting (3.19)-(3.23) into (1.1)

$$\begin{aligned}
&-A\lambda(\lambda-1)\alpha\nu\tanh^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\tanh^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\tanh^{\lambda+2}\psi \\
&+\mu_1 A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi - 2\mu_1 A\lambda^2\alpha^2\tanh^{\lambda}\psi + \mu_1 A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi \\
&-\mu_2 2A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi + 8\mu_2 A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi - 2\mu_2 A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi \\
&+\mu_3 A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\tanh^{\lambda-4}\psi - 4\mu_3 A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\tanh^{\lambda-2}\psi \\
&+2\mu_3 A\lambda^2(3\lambda^2+5)\alpha^3\nu\tanh^{\lambda}\psi - 4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\tanh^{\lambda+2}\psi \\
&+\mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\tanh^{\lambda+4}\psi + \mu_4 A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi \\
&-2\mu_4 A\lambda^2\beta^2\tanh^{\lambda}\psi + \mu_4 A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi = 0
\end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned}
&-A\lambda(\lambda+1)\alpha\nu + \mu_1 A\lambda(\lambda+1)\alpha^2 + 8\mu_2 A^2\lambda^2\alpha^2 \\
&-4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4 A\lambda(\lambda+1)\beta^2 = 0 \\
&-2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0
\end{aligned}$$

set $\alpha = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1 + \beta^2\mu_4)}{\mu_2 + 8\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1 + \beta^2\mu_4}{\alpha + 8\alpha^3\mu_3}$$

thus

$$u(x,y,t) = A\tanh^2(\alpha x + \beta y - \nu t) \tag{23}$$

3.2. (3+1)-D KP-BBM equation

To calculate the shock wave solitons for (1.2), suppose

$$u(x,y,z,t) = A\tanh^{\lambda}\psi \text{ where } \psi = \alpha x + \beta y + \gamma z - \nu t \text{ and } \lambda > 0 \tag{24}$$

from (3.25)

$$u_{tx} = -A\lambda(\lambda-1)\alpha\nu \tanh^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\tanh^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\tanh^{\lambda+2}\psi \quad (25)$$

$$u_{xx} = A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi - 2A\lambda^2\alpha^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi \quad (26)$$

$$(u^2)_{xx} = 2A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi \quad (27)$$

$$\begin{aligned} u_{xxtx} = & -A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\tanh^{\lambda-4}\psi + 4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\tanh^{\lambda-2}\psi \\ & -2A\lambda^2(3\lambda^2+5)\alpha^3\nu\tanh^{\lambda}\psi + 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\tanh^{\lambda+2}\psi \\ & -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\tanh^{\lambda+4}\psi \end{aligned} \quad (28)$$

$$u_{yy} = A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi - 2A\lambda^2\beta^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi \quad (29)$$

$$u_{zz} = A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi - 2A\lambda^2\gamma^2\tanh^{\lambda}\psi + A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi \quad (30)$$

substituting (3.26)-(3.31) into (1.2)

$$\begin{aligned} & -A\lambda(\lambda-1)\alpha\nu\tanh^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\tanh^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\tanh^{\lambda+2}\psi \\ & +\mu_1 A\lambda(\lambda-1)\alpha^2\tanh^{\lambda-2}\psi - 2\mu_1 A\lambda^2\alpha^2\tanh^{\lambda}\psi + \mu_1 A\lambda(\lambda+1)\alpha^2\tanh^{\lambda+2}\psi \\ & -\mu_2 2A^2\lambda(2\lambda-1)\alpha^2\tanh^{2\lambda-2}\psi + 8\mu_2 A^2\lambda^2\alpha^2\tanh^{2\lambda}\psi - 2\mu_2 A^2\lambda(2\lambda+1)\alpha^2\tanh^{2\lambda+2}\psi \\ & +\mu_3 A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\tanh^{\lambda-4}\psi - 4\mu_3 A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\tanh^{\lambda-2}\psi \\ & +2\mu_3 A\lambda^2(3\lambda^2+5)\alpha^3\nu\tanh^{\lambda}\psi - 4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\tanh^{\lambda+2}\psi \\ & +\mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\tanh^{\lambda+4}\psi + \mu_4 A\lambda(\lambda-1)\beta^2\tanh^{\lambda-2}\psi \\ & -2\mu_4 A\lambda^2\beta^2\tanh^{\lambda}\psi + \mu_4 A\lambda(\lambda+1)\beta^2\tanh^{\lambda+2}\psi + \mu_5 A\lambda(\lambda-1)\gamma^2\tanh^{\lambda-2}\psi \\ & -2\mu_5 A\lambda^2\gamma^2\tanh^{\lambda}\psi + \mu_5 A\lambda(\lambda+1)\gamma^2\tanh^{\lambda+2}\psi = 0 \end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned} & -A\lambda(\lambda+1)\alpha\nu + \mu_1 A\lambda(\lambda+1)\alpha^2 + 8\mu_2 A^2\lambda^2\alpha^2 \\ & -4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4 A\lambda(\lambda+1)\beta^2 + \mu_5 A\lambda(\lambda+1)\gamma^2 = 0 \\ & -2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0 \end{aligned}$$

set $\alpha = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1 + \beta^2\mu_4 + \gamma^2\mu_5)}{\mu_2 + 8\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1 + \beta^2\mu_4 + \gamma^2\mu_5}{\alpha + 8\alpha^3\mu_3}$$

thus

$$u(x,y,z,t) = A\tanh^2(\alpha x + \beta y + \gamma z - \nu t) \quad (31)$$

4. Singular wave solitons

4.1. Form-I (2+1)-D KP-BBM equation

To calculate the the singular wave Form I solution for (1.1), suppose

$$u(x,y,t) = A\coth^{\lambda}\psi \text{ where } \psi = \alpha x + \beta y - \nu t \text{ and } \lambda > 0 \quad (32)$$

from (4.33)

$$u_{tx} = -A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi \quad (33)$$

$$u_{xx} = A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi - 2A\lambda^2\alpha^2\coth^{\lambda}\psi + A\lambda(\lambda+1)\alpha^2\coth^{\lambda+2}\psi \quad (34)$$

$$(u^2)_{xx} = 2A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\coth^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi \quad (35)$$

$$\begin{aligned} u_{xxtx} = & -A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\coth^{\lambda-4}\psi + 4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\coth^{\lambda-2}\psi \\ & -2A\lambda^2(3\lambda^2+5)\alpha^3\nu\coth^{\lambda}\psi + 4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\coth^{\lambda+2}\psi \\ & -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\coth^{\lambda+4}\psi \end{aligned} \quad (36)$$

$$u_{yy} = A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi - 2A\lambda^2\beta^2\coth^{\lambda}\psi + A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi \quad (37)$$

substituting (4.34)-(4.38) into (1.1)

$$\begin{aligned} & -A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi \\ & +\mu_1 A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi - 2\mu_1 A\lambda^2\alpha^2\coth^{\lambda}\psi + \mu_1 A\lambda(\lambda+1)\alpha^2\coth^{\lambda+2}\psi \\ & -2\mu_2 A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi + 8\mu_2 A^2\lambda^2\alpha^2\coth^{2\lambda}\psi - 2\mu_2 A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi \\ & +\mu_3 A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\coth^{\lambda-4}\psi - 4\mu_3 A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\coth^{\lambda-2}\psi \\ & -4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\coth^{\lambda+2}\psi + 2\mu_3 A\lambda^2(3\lambda^2+5)\alpha^3\nu\coth^{\lambda}\psi \\ & +\mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\coth^{\lambda+4}\psi + \mu_4 A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi \\ & -2\mu_4 A\lambda^2\beta^2\coth^{\lambda}\psi + \mu_4 A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi = 0 \end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

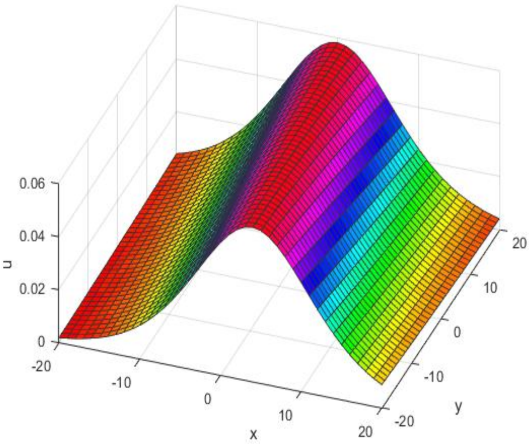
$$\begin{aligned} & -A\lambda(\lambda+1)\alpha\nu + \mu_1 A\lambda(\lambda+1)\alpha^2 + 8\mu_2 A^2\lambda^2\alpha^2 \\ & -4\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4 A\lambda(\lambda+1)\beta^2 = 0 \\ & -2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0 \end{aligned}$$

set $\alpha = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1 + \beta^2\mu_4)}{\mu_2 + 8\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1 + \beta^2\mu_4}{\alpha + 8\alpha^3\mu_3}$$

thus

$$u(x, y, t) = A\coth^2(\alpha x + \beta y - \nu t) \quad (38)$$



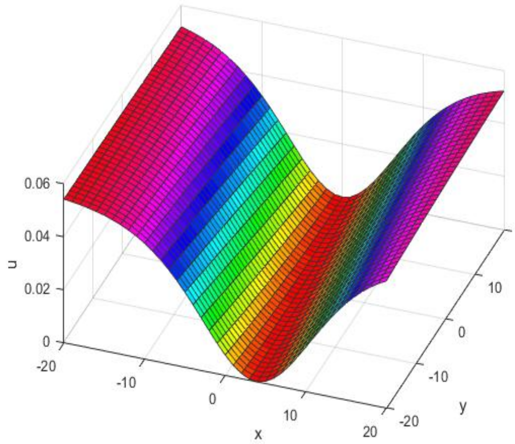
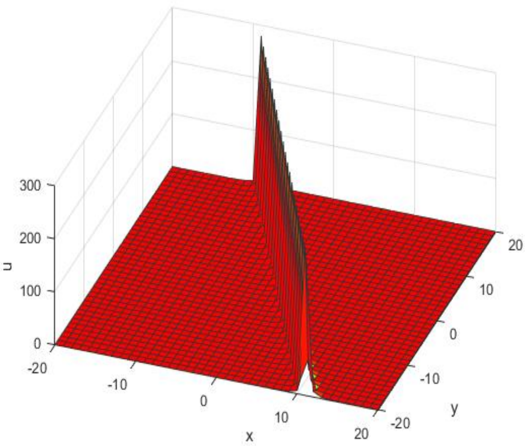


Figure (1) represented solitary wave: μ_1=*1*, μ_2=*1*, μ_3=*-1*, μ_4=*1*, α=*0.1*, β=*0.01*, *t=2* and figure (2) showed shock wave:μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, α=*0.1*, β=*0.01*, *t=2* of the (2+1)-D KP-BBM equation



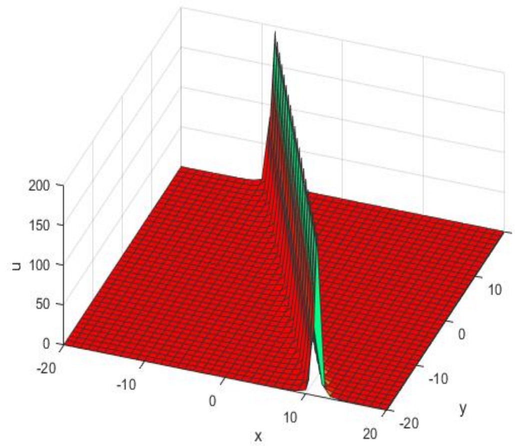


Figure (3) represented Singular Form-I: μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, α=*0.1*, β=*0.05*, *t=1* and figure (4) showed Singular Form-II: μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, α=*0.1*, β=*0.05*, *t=1* of the (2+1)-D KP-BBM equation

4.2. Form-I (3+1)-D KP-BBM equation

To calculate the the singular wave Form I solution for (1.2), suppose

$$u(x, y, z, t) = A \coth^{\lambda} \psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0 \tag{39}$$

from (4.40)

$$\begin{aligned} u_{tx} &= -A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi \\ &\quad -A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi \end{aligned} \tag{40}$$

$$\begin{aligned} u_{xx} &= A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi - 2A\lambda^2\alpha^2\coth^{\lambda}\psi \\ &\quad +A\lambda(\lambda+1)\alpha^2\coth^{\lambda+2}\psi \end{aligned} \tag{41}$$

$$\begin{aligned} (u^2)_{xx} &= 2A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi - 8A^2\lambda^2\alpha^2\coth^{2\lambda}\psi \\ &\quad +2A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi \end{aligned} \tag{42}$$

$$\begin{aligned} u_{xxtx} &= -A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\coth^{\lambda-4}\psi \\ &\quad +4A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\coth^{\lambda-2}\psi - 2A\lambda^2(3\lambda^2+5)\alpha^3\nu\coth^{\lambda}\psi \\ &\quad +4A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\coth^{\lambda+2}\psi \\ &\quad -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\coth^{\lambda+4}\psi \end{aligned} \tag{43}$$

$$\begin{aligned} u_{yy} &= A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi - 2A\lambda^2\beta^2\coth^{\lambda}\psi \\ &\quad +A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi \end{aligned} \tag{44}$$

$$\begin{aligned} u_{zz} &= A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi - 2A\lambda^2\gamma^2\coth^{\lambda}\psi \\ &\quad +A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi \end{aligned} \tag{45}$$

substituting (4.41)-(4.46) into (1.2)

$$\begin{aligned} &-A\lambda(\lambda-1)\alpha\nu\coth^{\lambda-2}\psi + 2A\lambda^2\alpha\nu\coth^{\lambda}\psi - A\lambda(\lambda+1)\alpha\nu\coth^{\lambda+2}\psi \\ &+\mu_1A\lambda(\lambda-1)\alpha^2\coth^{\lambda-2}\psi - 2\mu_1A\lambda^2\alpha^2\coth^{\lambda}\psi + \mu_1A\lambda(\lambda+1)\alpha^2\coth^{\lambda+2}\psi \\ &-2\mu_2A^2\lambda(2\lambda-1)\alpha^2\coth^{2\lambda-2}\psi + 8\mu_2A^2\lambda^2\alpha^2\coth^{2\lambda}\psi - 2\mu_2A^2\lambda(2\lambda+1)\alpha^2\coth^{2\lambda+2}\psi \\ &+\mu_3A\lambda(\lambda-1)(\lambda-2)(\lambda-3)\alpha^3\nu\coth^{\lambda-4}\psi - 4\mu_3A\lambda(\lambda-1)(\lambda^2-2\lambda+2)\alpha^3\nu\coth^{\lambda-2}\psi \\ &-4\mu_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\coth^{\lambda+2}\psi + 2\mu_3A\lambda^2(3\lambda^2+5)\alpha^3\nu\coth^{\lambda}\psi \\ &+\mu_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\coth^{\lambda+4}\psi + \mu_4A\lambda(\lambda-1)\beta^2\coth^{\lambda-2}\psi \\ &-2\mu_4A\lambda^2\beta^2\coth^{\lambda}\psi + \mu_4A\lambda(\lambda+1)\beta^2\coth^{\lambda+2}\psi + \mu_5A\lambda(\lambda-1)\gamma^2\coth^{\lambda-2}\psi \\ &-2\mu_5A\lambda^2\gamma^2\coth^{\lambda}\psi + \mu_5A\lambda(\lambda+1)\gamma^2\coth^{\lambda+2}\psi = 0 \end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned} &-A\lambda(\lambda+1)\alpha\nu + \mu_1A\lambda(\lambda+1)\alpha^2 + 8\mu_2A^2\lambda^2\alpha^2 \\ &-4\mu_3A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4A\lambda(\lambda+1)\beta^2 + \mu_5A\lambda(\lambda+1)\gamma^2 = 0 \\ &-2\mu_2A^2\lambda(2\lambda+1)\alpha^2 + \mu_3A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0 \end{aligned}$$

set $\alpha = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1+\beta^2\mu_4+\gamma^2\mu_5)}{\mu_2+8\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1+\beta^2\mu_4+\gamma^2\mu_5}{\alpha+8\alpha^3\mu_3}$$

thus

$$u(x,y,z,t) = A\coth^2(\alpha x+\beta y+\gamma z-\nu t) \tag{46}$$

4.3. Form-II (2+1)-D KP-BBM equation

To calculate the the singular wave Form II solution for (1.1), suppose

$$u(x,y,t) = A\,\mathrm{csch}^{\lambda}\psi \quad \text{where } \psi = \alpha x+\beta y+\gamma z-\nu t \text{ and } \lambda > 0 \tag{47}$$

from (4.48)

$$\begin{aligned}
u_{tx} &= -A\lambda(\lambda+1)\alpha\nu\,\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\,\mathrm{csch}^{\lambda}\psi && (48)\\
u_{xx} &= A\lambda(\lambda+1)\alpha^2\,\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\alpha^2\,\mathrm{csch}^{\lambda}\psi && (49)\\
(u^2)_{xx} &= 4A^2\lambda^2\alpha^2\,\mathrm{csch}^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\,\mathrm{csch}^{2\lambda+2}\psi && (50)\\
u_{xxtx} &= -A\lambda^4\alpha^3\nu\,\mathrm{csch}^{\lambda}\psi - 2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\,\mathrm{csch}^{\lambda+2}\psi\\
&\quad -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\,\mathrm{csch}^{\lambda+4}\psi && (51)\\
u_{yy} &= A\lambda(\lambda+1)\beta^2\,\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\beta^2\,\mathrm{csch}^{\lambda}\psi && (52)
\end{aligned}$$

substituting (4.49)-(4.53) into (1.1)

$$\begin{aligned}
&-A\lambda(\lambda+1)\alpha\nu\,\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\,\mathrm{csch}^{\lambda}\psi\\
&+\mu_1 A\lambda(\lambda+1)\alpha^2\,\mathrm{csch}^{\lambda+2}\psi + \mu_1 A\lambda^2\alpha^2\,\mathrm{csch}^{\lambda}\psi\\
&-4\mu_2 A^2\lambda^2\alpha^2\,\mathrm{csch}^{2\lambda}\psi - 2\mu_2 A^2\lambda(2\lambda+1)\alpha^2\,\mathrm{csch}^{2\lambda+2}\psi\\
&+\mu_3 A\lambda^4\alpha^3\nu\,\mathrm{csch}^{\lambda}\psi + 2\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\,\mathrm{csch}^{\lambda+2}\psi\\
&+\mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\,\mathrm{csch}^{\lambda+4}\psi\\
&+\mu_4 A\lambda(\lambda+1)\beta^2\,\mathrm{csch}^{\lambda+2}\psi + \mu_4 A\lambda^2\beta^2\,\mathrm{csch}^{\lambda}\psi = 0
\end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned}
&-A\lambda(\lambda+1)\alpha\nu + \mu_1 A\lambda(\lambda+1)\alpha^2 - 4\mu_2 A^2\lambda^2\alpha^2\\
&+2\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4 A\lambda(\lambda+1)\beta^2 + \mu_5 A\lambda(\lambda+1)\gamma^2 = 0\\
&-2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0
\end{aligned}$$

set $\lambda = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1+\beta^2\mu_4)}{\mu_2 - 4\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1+\beta^2\mu_4}{\alpha - 4\alpha^3\mu_3}$$

thus

$$u(x,y,t) = A\,\mathrm{csch}^2(\alpha\mathrm{x} + \beta\mathrm{y}\text{-}\nu\mathrm{t}) \qquad (53)$$

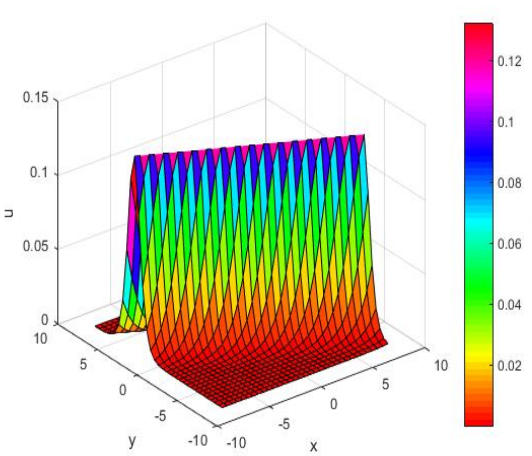
4.4. Form-II (3+1)-D KP-BBM equation

To calculate the the singular wave Form II solution for (1.2), suppose

$$u(x,y,z,t) = A\,\mathrm{csch}^{\lambda}\psi \quad \text{where} \quad \psi = \alpha x + \beta y + \gamma z - \nu t \quad \text{and} \quad \lambda > 0 \qquad (54)$$

from (4.55)

$$\begin{aligned}
u_{tx} &= -A\lambda(\lambda+1)\alpha\nu\,\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\,\mathrm{csch}^{\lambda}\psi && (55)\\
u_{xx} &= A\lambda(\lambda+1)\alpha^2\,\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\alpha^2\,\mathrm{csch}^{\lambda}\psi && (56)\\
(u^2)_{xx} &= 4A^2\lambda^2\alpha^2\,\mathrm{csch}^{2\lambda}\psi + 2A^2\lambda(2\lambda+1)\alpha^2\,\mathrm{csch}^{2\lambda+2}\psi && (57)\\
u_{xxtx} &= -A\lambda^4\alpha^3\nu\,\mathrm{csch}^{\lambda}\psi - 2A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\,\mathrm{csch}^{\lambda+2}\psi\\
&\quad -A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\,\mathrm{csch}^{\lambda+4}\psi && (58)\\
u_{yy} &= A\lambda(\lambda+1)\beta^2\,\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\beta^2\,\mathrm{csch}^{\lambda}\psi && (59)\\
u_{zz} &= A\lambda(\lambda+1)\gamma^2\,\mathrm{csch}^{\lambda+2}\psi + A\lambda^2\gamma^2\,\mathrm{csch}^{\lambda}\psi && (60)
\end{aligned}$$



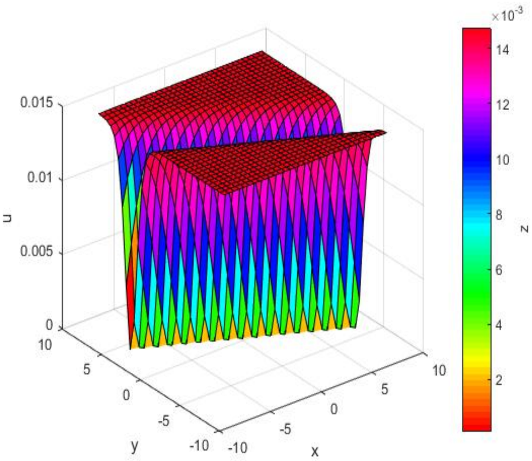
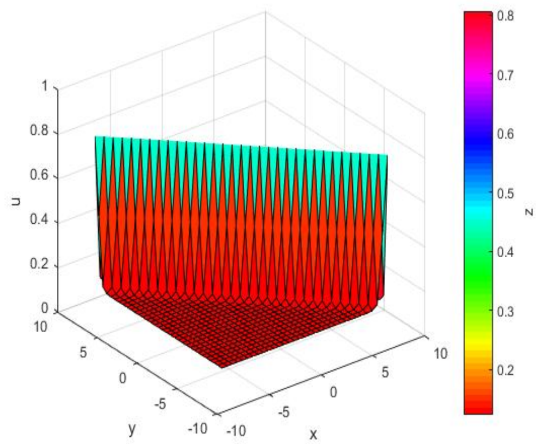


Figure (5) represented Solitary wave: μ_1=*1*, μ_2=*1*, μ_3=*-1*, μ_4=*1*, μ_5=*1*, α=*0.05*, β=*0.1*, γ=*0.1*, *t=2* and figure (6) showed Shock wave:μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, μ_5=*-1*, α=*0.05*, β=*0.1*, γ=*0.1*, *t=2* of the (3+1)-D KP-BBM equation



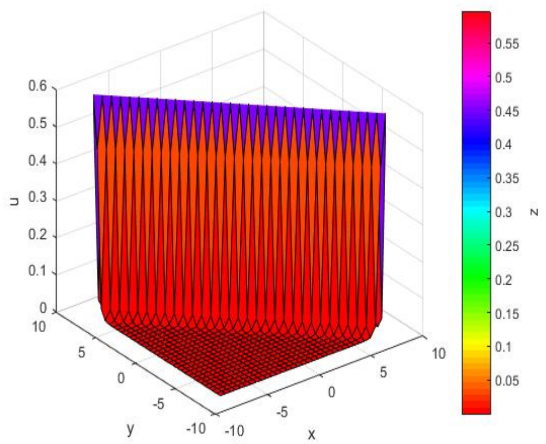


Figure (7) represented Singular Form-I: μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, μ_5=*1*, α=*0.1*, β=*0.1*, γ=*0.05*,*t=2* and figure (8) showed Singular Form-II: μ_1=*1*, μ_2=*1*, μ_3=*1*, μ_4=*1*, μ_5=*1*, α=*0.1*, β=*0.1*, γ=*0.05*, *t=2* of the (3+1)-D KP-BBM equation substituting (4.56)-(4.61) into (1.2)

$$\begin{aligned}
&-A\lambda(\lambda+1)\alpha\nu\,\mathrm{csch}^{\lambda+2}\psi - A\lambda^2\alpha\nu\,\mathrm{csch}^{\lambda}\psi \\
&+\mu_1 A\lambda(\lambda+1)\alpha^2\,\mathrm{csch}^{\lambda+2}\psi + \mu_1 A\lambda^2\alpha^2\,\mathrm{csch}^{\lambda}\psi \\
&-4\mu_2 A^2\lambda^2\alpha^2\,\mathrm{csch}^{2\lambda}\psi - 2\mu_2 A^2\lambda(2\lambda+1)\alpha^2\,\mathrm{csch}^{2\lambda+2}\psi \\
&+\mu_3 A\lambda^4\alpha^3\nu\,\mathrm{csch}^{\lambda}\psi + 2\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu\,\mathrm{csch}^{\lambda+2}\psi \\
&+\mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu\,\mathrm{csch}^{\lambda+4}\psi \\
&+\mu_4 A\lambda(\lambda+1)\beta^2\,\mathrm{csch}^{\lambda+2}\psi + \mu_4 A\lambda^2\beta^2\,\mathrm{csch}^{\lambda}\psi \\
&+\mu_5 A\lambda(\lambda+1)\gamma^2\,\mathrm{csch}^{\lambda+2}\psi + \mu_5 A\lambda^2\gamma^2\,\mathrm{csch}^{\lambda}\psi = 0
\end{aligned}$$

by comparing the powers $2\lambda, \lambda+2$ and $\lambda+4, 2\lambda+2$

$$\begin{aligned}
&-A\lambda(\lambda+1)\alpha\nu + \mu_1 A\lambda(\lambda+1)\alpha^2 - 4\mu_2 A^2\lambda^2\alpha^2 \\
&+2\mu_3 A\lambda(\lambda+1)(\lambda^2+2\lambda+2)\alpha^3\nu + \mu_4 A\lambda(\lambda+1)\beta^2 + \mu_5 A\lambda(\lambda+1)\gamma^2 = 0 \\
&-2\mu_2 A^2\lambda(2\lambda+1)\alpha^2 + \mu_3 A\lambda(\lambda+1)(\lambda+2)(\lambda+3)\alpha^3\nu = 0
\end{aligned}$$

set $\lambda = 2$

$$A = \frac{6\mu_3(\alpha^2\mu_1 + \beta^2\mu_4 + \gamma^2\mu_5)}{\mu_2 - 4\alpha^2\mu_2\mu_3}, \quad \nu = \frac{\alpha^2\mu_1 + \beta^2\mu_4 + \gamma^2\mu_5}{\alpha - 4\alpha^3\mu_3}$$

thus

$$u(x,y,z,t) = A\,\mathrm{csch}^2(\alpha \mathrm{x} + \beta \mathrm{y} + \gamma \mathrm{z} - \nu \mathrm{t}) \tag{61}$$

5. Physical description

In Fig.1, we take μ_1=1, μ_2=1, μ_3=1, μ_4=1, α=0.1, β=0.01, t=2 for solitary wave and μ_1=1, μ_2=1, μ_3=-1, μ_4=1, α=0.1, β=0.01, t=2 for shock waves; while μ_1=1, μ_2=1, μ_3=1, μ_4=1, α=0.1, β=0.05, t=1 for singular wave Form-I and μ_1=1, μ_2=1, μ_3=1, μ_4=1, α=0.1, β=0.05, t=1 for singular wave Form-II, to study the behaviour of (2+1) dimensional KP equation . In Fig.2, we choose μ_1=1, μ_2=1, μ_3=1, μ_4=1,α=0.1, β=0.05, t=2 for solitary wave and μ_1=1, μ_2=1, μ_3=-1, μ_4=1, α=0.1, β=0.05, t=2 for shock waves, while μ_1=1, μ_2=1, μ_3=1, μ_4=1, α=0.1, β=0.05, t=1 for singular wave Form-I and μ_1=1, μ_2=1, μ_3=1, μ_4=1, α=0.1, β=0.05, t=1 for singular wave Form-II of (2+1) dimesional Boussinesq equation.

6. Conclusion

In this article, the solitary wave ansatz method are successfully employed to (2+1) and (3+1)-Dimensional Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equations. The solitary wave ansatz method is used which is rather heuristic and processes significant features that make it practical for the determination of single soliton solutions for a wide class of nonlinear evolution equations. The constraint conditions for the existence of solutions are also listed.

References

[1] S. Yomosa, Soliton excitations in deoxyribonucleic acid (DNA) double helices, Physical Review A 27 (1983) 2120-2125.
[2] M. Younis, S. Ali, Solitary wave and shock wave solitons to the transmission line model for nano-ionic currents along microtubules, Applied Mathematics and Computation, 246 (2014) 460-463.
[3] M. Younis, S. Ali, S.A. Mahmood, Solitons for compound KdVBurgers equation with variable coefficients and power law nonlinearity, Nonlinear Dynamics 81(3) (2014) 1191-1196.
[4] H. Bulut, Y. Pandir, S. T. Demiray. Exact Solutions of time-fractional KdV equations by using generalized Kudryashov method, Inter. J. Model. Optim. 4 (2014) 315-320.
[5] A. Sardar, S.M Husnine, S.T.R. Rizvi, M. Younis, K. Ali, Multiple travelling wave solutions for electrical transmission line model, Nonlinear Dynamics 82(3) (2015) 1317-1324.

[6] M Younis, STR Rizvi, Optical Soliton Like-Pulses in Ring-Cavity Fiber Lasers of Carbon Nanotubes, Journal of Nanoelectronics and Optoelectronics, 11(3), (2016) 276-279.
[7] N. Cheemaa, S.A. Mehmood, S.T.R. Rizvi, M. Younis, Single and combined optical solitons with third order dispersion in Kerr media, Optik - International Journal for Light and Electron Optics 127(20) (2016) 8203-8208.
[8] C.H. Su, C.S. Gardner, Drivation of the Korteweg-de Vries and Burgers Equation, J. Math. Phy, 10 (1969) 536-539.
[9] H. Gard, P. N. Hu, Unified shock profile in plasma, Phys. Fluids, 10 (1967) 2596-2602.
[10] H. Grad, P. N. Hu, Collisional theory of shock and nonlinear waves in plasma,J. phys.Fluids, 15 (1972) 845-864.
[11] R. S. johnson, A nonlinear equation incorporating damping and dispersion, J. Phys. Fluids Mech, 42 (1970) 60-94.
[12] L. Zhibin, W. Mingliang, Travelling wave solutions to the two-dimensional Kdv-Burgers equation, J. Phys. A: Math. Gen. 26 (1993) 6027-6031.
[13] A. R. Seadawy, A. Sayed, Travelling wave solution of two-dimensional nonlinear KdV-Burgers equation, Applied Mathematical Sciences, 7 (2013), 3367-3377.
[14] Z. Zhu, Exact solutions for a two-dimensional KdV-Burgers-type equation, Chinese Journal of Physics, 34 (1996), 1101-1105.
[15] A.M.Wazwaz, Variants of the two-dimensional Boussinesq equation with compactons, solitons and periodic solutions, Comput. Math. Appl., 49(2005), No. 2-3, 295-301
[16] D. G. Natsis, Solitary wave solutions of the one-dimensional Boussinesq equations Numerical Algorithms (2007), DOI 10.1007/s11075-007-9102-6
[17] Bin Zheng, Traveling Wave Solutions For The (2+1) Dimensional Boussinesq Equation And The Two-Dimensional Burgers Equation By (G'/G)-expansion method Wseas Transactions on Computers (2010), ISSN: 1109-2750
[18] Figen, Haci M.Baskonus, Hasan Bulut, On the Complex and Hyperbolic Structures for the (2 + 1)-Dimensional Boussinesq Water Equation Entropy (2015),17, 82678277; doi:10.3390/e17127878
[19] A. R. Seadawy, The Solutions of the Boussinesq and Generalized Fifth-Order KdV Equations by Using the Direct Algebraic Method, Applied Mathematical Sciences, Vol. 6, 2012, no. 82, 4081 - 4090
[20] Gideon P. Daspan and Michael M. Tom, Comparison of KP and BBM-KP Models, Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Sciences Volume 2007, Article ID 37274, doi:10.1155/2007/37274
[21] Xiao-Ling Gai, Yi-Tian Gao, Xin Yu, Zhi-Yuan Sun, Soliton Interactions for the Generalized (3+1)-Dimensional Boussinesq Equation, International Journal of Modern Physics B Vol. 26, No. 7 (2012) 1250062, World Scientific Publishing Company DOI: 10.1142/S0217979212500622
[22] A.R. Seadawy, Stability analysis solutions for nonlinear three-dimensional modified Korteweg-de Vries-Zakharov-Kuznetsov equation in a magnetized electron-positron plasma, Physica A: Statistical Mechanics and its Applications 455 (2016) 4451.
[23] A.R. Seadawy, Three-dimensional nonlinear modified Zakharov-Kuznetsov equation of ion-acoustic waves in a magnetized plasma, Computers and Mathematics with Applications 71 (2016) 201212.
[24] M.A. Helal and A.R. Seadawy, Variational method for the derivative nonlinear Schrodinger equation with computational applications, Physica Scripta, 80 (2009) Article ID 035004.
[25] M.A. Helal, A.R. Seadawy, Exact soliton solutions of a D-dimensional nonlinear Schrodinger equation with damping and diffusive terms, Zeitschrift fur Angewandte Mathematik und Physik, 62 (2011) 839-847.
[26] A.R. Seadawy, New exact solutions for the KdV equation with higher order nonlinearity by using the variational method, Comput. Math. Appl., 62 (2011) 3741-3755.
[27] M.A. Helal, A.R. Seadawy, Benjamin-Feir instability in nonlinear dispersive waves, comput. Math. Appl., 64 (2012) 3557-3568.
[28] Khater, A. H., Callebaut D. K., Malfliet, W. and Seadawy A. R., Nonlinear Dispersive Rayleigh-Taylor Instabilities in Magnetohydro-dynamic Flows, Physica Scripta, 64 (2001) 533-547.
[29] Khater, A. H., Callebaut D. K. and Seadawy A. R., "Nonlinear Dispersive Kelvin-Helmholtz Instabilities in Magnetohydrodynamic Flows" Physica Scripta, 67 (2003) 340-349.
[30] Khater, A. H., Callebaut D. K. and Seadawy A. R., General soliton solutions of an n-dimensional Complex Ginzburg-Landau equation, Physica Scripta, Vol. 62 (2000) 353-357.
[31] Khater, A. H., Helal M. A. and Seadawy A. R., General soliton solutions of n-dimensional nonlinear Schrodinger equation" IL Nuovo Cimento 115B, (2000) 1303-1312.
[32] Khater, A. H., Callebaut D. K., Helal, M. A. and Seadawy A. R., Variational Method for the Nonlinear Dynamics of an Elliptic Magnetic Stagnation Line, The European Physical Journal D, 39, (2006) 237-245.
[33] Khater, A. H., Callebaut D. K., Helal, M. A. and Seadawy A. R., General Soliton Solutions for Nonlinear Dispersive Waves in Convective Type Instabilities, Physica Scripta, 74, (2006) 384.