Filomat 32:3 (2018), 1025–1033
https://doi.org/10.2298/FIL1803025A

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Asymptotic Method for Solution of Identification Problem of the Nonlinear Dynamic Systems

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Abstract. A dynamic system, when the motion of the object is described by the system of nonlinear ordinary differential equations, is considered. The right part of the system involves the phase coordinates as a unknown constant vector-parameter and a small number. The statistical data are taken from practice: the initial and final values of the object coordinates. Using the method of quasilinearization the given equation is reduced to the system of linear differential equations, where the coefficients of the coordinate and unknown parameter, also of the perturbations depend on a small parameter linearly. Then, by using the least-squares method the unknown constant vector-parameter is searched in the form of power series on a small parameter and for the coefficients of zero and the first orders the analytical formulas are given. The fundamental matrices both in a zero and in the first approach are constructed approximately, by means of the ordinary Euler method. On an example the determination of the coefficient of hydraulic resistance (CHR) in the lift in the oil extraction by gas lift method is illustrated, as the obtained results in the first approaching coincides with well-known results on order of 10^{-2}.

1. Introduction

The problem of identification [3, 9, 13] of the dynamic systems [7, 8, 11, 16] has a lot of practical applications, one of them is the applying in the oil fields, for example, in the oil extraction [8, 13, 20]. Really, in the serve of oil by means of pipelines, in the extraction of oil by the gas lift method or by the rod-pumping setting and other methods - the determination of CHR [3, 6, 9], in layers - the determination of parameters of formation of gas-liquid mixture [10] and other, requires development of numerical methods for the solving of corresponding problems of identification [18] of the dynamic systems. In [9] the gradient method on the basis of Gramm-Schmidt orthogonalization for determination of CHR is given. In [4, 9] the asymptotic method is given for determination of CHR in the first approaching relatively to small parameter, where a small parameter is accepted by inverse to the value of depth of the well. It is shown, that if the use of ordinary Gramm-Schmidt method requires enough large machine time, then an asymptotic method allows to calculate the solution of CHR in the first approaching analytically and the numerical results coincide

2010 *Mathematics Subject Classification*. Primary 49J15; Secondary 49J35
Keywords. Coefficient of hydraulic resistance, Least-squares method, quasilinearization, identification of the dynamic systems, asymptotic method
Received: 26 December 2016; Revised: 14 April 2017; Accepted: 20 April 2017
Communicated by Ljubiša D.R. Kočinac
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with the sought solution to the 10^{-2} order. Coming from these results, the authors of [3, 5] generalized the results obtained in [4] for multidimensional case. Further, using the methods of quazilinearization [14] and least-squares [17] the computational algorithm is suggested for solving of general identification problem of the dynamic systems for determination of the constant vector-parameter, which can be used for finding the CHR in the lift and the parameters for formation of gas-liquid mixture in the layers of the wells in the oil production.

The results of calculations show that in simplest case, when the sought (unknown) constant vector-parameter - the subject to determination is scalar, then the method of Gramm-Schmidt requires 2 hours in the simple example (with exactness 10^{-8} order). Therefore, makes the sense to develop an asymptotic method for solving of identification problem of the dynamic systems, when the small parameter is included to the right part of corresponding differential equations, where on the example of the oil fields it is the inverse value of depth of the wells.

In this work it is assumed, that some series of initial and final values (statistical data) of phase coordinate of nonlinear ordinary differential equation[1)] (to the right part are included the small parameter and unknown constant vector) are given. It is required to find the small paremeter and unknown constant vector in such way that their solutions in the end point coincided (with certain exactness) with statistical data. Using the methods of least-squares and quazilinearization is given an iterative scheme for the construction of asymptotic solutions in the first approach relatively to the small parameter.

The results are illustrated on the example of the oil extraction by gaslift method for determination of CHR, where the small parameter ε is accepted inverse to the value of depth of the well. Also for the coefficients of asymptotic expressions on ε the analytical formulas are given, where the linear differential equation is changed by the discrete approximation using ordinary Euler method. The given numerical results for the values of CHR, which are differs on order of 10^{-2} from the [1, 3, 9] can be used as a good initial approach for iterative schemes [3].

2. Problem Statement

Let a nonlinear ordinary differential equation

$$\dot{y}(x) = f(y(x), \alpha, \varepsilon) \tag{1}$$

and some sets of initial and final values of the n-dimensional phase vector $y(x)$ are given

$$y_k(0) = y_{0k}, \quad y_k(l) = y_{lk}, \quad k = 0, ..., N-1. \tag{2}$$

Here α is a constant unknown vector, ε is a small parameter, f is the n-dimensional function is differentiable on y, α, ε. It is required to find such vector-parameter $\alpha = \alpha^*$, that the end value $y^i(l, \alpha^*, \varepsilon)$(the solution of equation (1) with an initial value y_{0k}) exactly enough coincided with y_{lk} $(k = 0, 1, ..., N-1)$. Such problem further we will call the identification of the dynamic systems (1).

Let the initial approach $y^i(x), \alpha^i$ be given. Using the method of quazilinearization [3, 17] we present the equation (1) in the linear form relatively $y^i(x), \alpha^i$ and ε, in the following form

$$\begin{aligned} \dot{y}^i(x) &= \left(A_0\left(y^{i-1}(x), \alpha^{i-1}\right) + \varepsilon A_1\left(y^{i-1}(x), \alpha^{i-1}\right)\right) y^i(x) + \\ &+ \left(B_0\left(y^{i-1}(x), \alpha^{i-1}\right) + \varepsilon B_1\left(y^{i-1}(x), \alpha^{i-1}\right)\right) \alpha^i + \\ &+ \left(C_0\left(y^{i-1}(x), \alpha^{i-1}\right) + \varepsilon C_1\left(y^{i-1}(x), \alpha^{i-1}\right)\right), \end{aligned} \tag{3}$$

where $A_i\left(y^{i-1}(x), \alpha^{i-1}\right), B_i\left(y^{i-1}(x), \alpha^{i-1}\right), \ C_i\left(y^{i-1}(x), \alpha^{i-1}\right)$, are the results of decomposition of Taylor in the

[1)]In the spatial case [2] the motion of the equation describing by the hyperbolic equations the problem is reduced the solution of the corresponding problem, where the discrete model is considered.

first approach and determined in the following form

$$\begin{aligned}
&A_0\left(y^{i-1}(x),\alpha^{i-1}\right)=f'_y\left(y_0^{i-1},\alpha_0^{i-1},0\right),\quad A_1\left(y^{i-1}(x),\alpha^{i-1}\right)=f''_{y\varepsilon}\left(y_0^{i-1},\alpha_0^{i-1},0\right),\\
&B_0\left(y^{i-1}(x),\alpha^{i-1}\right)=f'_\alpha\left(y_0^{i-1},\alpha_0^{i-1},0\right),\quad B_1\left(y^{i-1}(x),\alpha^{i-1}\right)=f''_{\alpha\varepsilon}\left(y_0^{i-1},\alpha_0^{i-1},0\right),\\
&C_0\left(y^{i-1}(x),\alpha^{i-1}\right)=f\left(y_0^{i-1},\alpha_0^{i-1},0\right)-f'_y\left(y_0^{i-1},\alpha_0^{i-1},0\right)y_0^{i-1}-f_\alpha\left(y_0^{i-1},\alpha_0^{i-1},0\right)\alpha_0^{i-1},\\
&C_1\left(y^{i-1}(x),\alpha^{i-1}\right)=f'_\varepsilon\left(y_0^{i-1},\alpha_0^{i-1},0\right)-f''_{y\varepsilon}\left(y_0^{i-1},\alpha_0^{i-1},0\right)y_0^{i-1}-f''_{\alpha\varepsilon}\left(y_0^{i-1},\alpha_0^{i-1},0\right)\alpha_0^{i-1}.
\end{aligned}\tag{4}$$

Now we present the solution of linear equation [19] in the following form

$$y^i(t)=\left(\Phi_0^{0,i-1}(t)+\varepsilon\Phi_0^{1,i-1}\right)y^i(0)+\left(\Phi_1^{0,i-1}(t)+\varepsilon\Phi_1^{1,i-1}\right)\alpha^i+\left(\Phi_2^{0,i-1}(t)+\varepsilon\Phi_2^{1,i-1}\right),\tag{5}$$

where $\Phi_0^{0,i-1}(t,0)$,$\Phi_0^{1,(i-1)}(t,0)$ are determined from the next linear differential equations

$$\begin{aligned}
&\Phi_0^{0,i-1}(t,0)=\mathrm{A}_0\left(y^{i-1}(x),\alpha^{i-1}\right)\Phi_0^{0,i-1}(t,0),\quad \Phi_0^{0,i-1}(0,0)=\mathrm{E},\\
&\Phi_0^{1,i-1}(t,0)=\mathrm{A}_0\left(y^{i-1}(x),\alpha^{i-1}\right)\Phi_0^{1,i-1}(t,0)+\mathrm{A}_1\left(y^{i-1}(x),\alpha^{i-1}\right)\Phi_0^{0,i-1}(t,0),\\
&\Phi_0^{1,i-1}(0,0)=0
\end{aligned}\tag{6}$$

and

$$\Phi_0^{i-1}(t,0)=\Phi_0^{0,i-1}(t,0)+\varepsilon\Phi_0^{1,i-1}(t,0)$$

is the fundamental matrix of the homogeneous equation (5) in the first approach relatively small parameter ε, and $\Phi_1^{n,i-1}(t,0)$, $\Phi_2^{n,i-1}(t,0)$, $(n=0,1)$ from (5) are determined in the following form [5]

$$\begin{aligned}
&\Phi_1^{0,i-1}(t,0)=\int_0^t\Phi_0^{0,i-1}(\tau,0)\,B_0\left(y^{i-1},\ \alpha^{i-1}\right)d\tau,\\
&\Phi_1^{1,i-1}(t,0)=\int_0^t\left(\Phi_0^{0,i-1}(\tau,0)\,B_1\left(y^{i-1},\ \alpha^{i-1}\right)+\Phi_0^{1,i-1}(\tau,0)\,B_0\left(y^{i-1},\ \alpha^{i-1}\right)d\tau,\right.\\
&\Phi_2^{0,i-1}(t,0)=\int_0^t\Phi_0^{0,i-1}(\tau,0)\,C_0\left(y^{i-1},\ \alpha^{i-1}\right)d\tau,\\
&\Phi_2^{1,i-1}(t,0)=\int_0^t\left(\Phi_0^{0,i-1}(\tau,0)\,C_1\left(y^{i-1},\ \alpha^{i-1}\right)+\Phi_0^{1,i-1}(\tau,0)\,C_0\left(y^{i-1},\ \alpha^{i-1}\right)d\tau.\right.
\end{aligned}\tag{7}$$

Further for the solutions (5) in the point $x=l$ and for the final values from (2) we construct the functional

$$I^i=\sum_{k=0}^{N-1}\left(y_k^i(l)-y_{lk}^i\right)'A\left(y_k^i(l)-y_{lk}^i\right),\tag{8}$$

where A is the well-known weight matrix, $y_k^i(l)$ is the solution of the equation (5) at statistical data $y_k(0)=y_{0k}$ from (2). Thus, if we can choose such α^i, that the functional (8) get the minimum value, then we in fact provide the closeness of the solution $y(x,\alpha,\varepsilon)$ in the point $x=l$ with $y_k^i(l)=y_{lk}^i$ from (2).

3. Calculation of the Gradient of the Functional (8)

To obtain the formula for the gradient from the functional (8) first we put$y_k^i(l)$ from (5) into (8):

$$
\begin{aligned}
I^i = &\sum_{k=0}^{N-1}\left\{\left[y_0'^{\,i}\left(\Phi_0^{0,i-1}+\varepsilon\Phi_0^{1,i-1}\right)'+\alpha'^{i}\left(\Phi_1^{0,i-1}+\varepsilon\Phi_1^{1,i-1}\right)'+\left(\Phi_2^{0,i-1}+\varepsilon\Phi_2^{1,i-1}\right)'-y_{lk}'^{\,i}\right]A\times\right.\\
&\left.\times\left[\left(\Phi_0^{0,i-1}+\varepsilon\Phi_0^{1,i-1}\right)y_0^i+\left(\Phi_1^{0,i-1}+\varepsilon\Phi_1^{1,i-1}\right)\alpha^i+\left(\Phi_2^{0,i-1}+\varepsilon\Phi_2^{1,i-1}\right)-y_{lk}^i\right]\right\}=\\
= &\sum_{k=0}^{N-1}\left\{\left[y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-\right.\right.\\
&-y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}y_{lk}^i+\alpha'^{i}\Phi_1'^{\,0,i-1}A\Phi_0^{0,i-1}y_0^i+\alpha'^{i}\Phi_1'^{\,0,i-1}A\Phi_1^{0,i-1}\alpha^i+\alpha'^{i}\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}\\
&-\alpha'^{i}\Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-\\
&\left.-\Phi_2'^{\,0,i-1}\mathrm{A}y_{lk}^i-y_{lk}'^{\,i}\mathrm{A}\Phi_0^{0,i-1}y_0^i-y_{lk}'^{\,i}\mathrm{A}\Phi_1^{0,i-1}y_0^i-y_{lk}'^{\,i}\mathrm{A}\;\Phi_2^{0,i-1}+y_{lk}'^{\,i}\mathrm{A}y_{lk}^i\right]+\\
&+\varepsilon\left[y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{1,i-1}y_0^i+y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}\alpha^i+y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+\right.\\
&+y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_0^{1,i-1}\alpha^i+y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}y_{lk}^i+\\
&+\alpha'^{i}\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_0^{1,i-1}y_0^i+\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}\alpha^i+\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+\alpha'^{i}\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+\\
&+\alpha'^{i}\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+\alpha'^{i}\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\alpha'^{i}\Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_0^{1,i-1}y_0^i+\\
&+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}\alpha^i+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+\Phi_2'^{\,1,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+\Phi_2'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+\\
&\left.\left.+\Phi_2'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\Phi_2'^{\,1,i-1}\mathrm{A}y_{lk}^i-y_{lk}'^{\,i}\mathrm{A}\Phi_0^{1,i-1}y_0^i-y_{lk}'^{\,i}A\Phi_1^{1,i-1}\alpha^i-y_{lk}'^{\,i}\mathrm{A}\Phi_2^{1,i-1}\right]\right\}=
\end{aligned}
$$

$$
\begin{aligned}
= &\sum_{k=0}^{N-1}\left\{y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^i+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-\right.\\
&-2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}y_{lk}^i+\alpha'^{i-1}\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i-2\alpha'^{i-1}\Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i+\\
&+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-2\Phi_2'^{\,0,i-1}\mathrm{A}y_{lk}^i-y_{lk}'^{\,i}\mathrm{A}y_{lk}^i+\\
&+\varepsilon\left[2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{1,i-1}y_0^i+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}\alpha^i+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+\right.\\
&+2y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^i+2y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-2y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}y_{lk}^i+2\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha^{i2}+\\
&+2\alpha'^{i}\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-2'\alpha^{i}\Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i+2\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+\\
&\left.\left.+2\;\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}\alpha^i-2\Phi_2'^{\,1,i-1}\mathrm{A}y_{lk}^i\right]\right\}=\\
= &\sum_{k=0}^{N-1}\left\{\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{0,i-1}y_0^{i\,2}+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}y_{lk}^i+2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-2y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}y_0^iy_{lk}^i+\right.\\
&+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}-2\Phi_2'^{\,0,i-1}\mathrm{A}y_{lk}^i-y_{lk}'^{\,i}\mathrm{A}y_{lk}^i+\\
&\\
&+2\varepsilon\left(y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_0^{1,i-1}y_0^i+y_0'^{\,i}\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}+y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\right.\\
&\left.-\;y_0'^{\,i}\Phi_0'^{\,1,i-1}\mathrm{A}y_{lk}^i+\Phi_2'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}-\Phi_2'^{\,1,i-1}\mathrm{A}y_{lk}^i\right)+\\
&+\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i-\Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i+2\;\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}+\right.\\
&+\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}y_0^i+\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i+\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\right.\\
&\left.\left.-\Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i+\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}\right)\right)\alpha^i+\\
&\left.+\left(\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}+\;\varepsilon\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)\alpha^{i2}\right\}.
\end{aligned}
\tag{9}
$$

Now we take the derivate on an unknown constant vector α^i from (9) where $I_\alpha'^{\,i}$ is determined in the following form:

$$
\begin{aligned}
I_\alpha'^{\,i}= &\sum_{k=0}^{N-1}\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i-\Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i+2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}+\right.\\
&+\varepsilon\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}y_0^i+\Phi_0'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i+\right.\\
&\left.+\;\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i+\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1}\right)+\\
&\left.+\;2\left(\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}+\;\varepsilon\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)\alpha^i\right).
\end{aligned}
\tag{10}
$$

We equate to zero the first derivate $I_{\alpha}^{i\,\prime}$ and search α^iin the form

$$y^0(t) = 1. \tag{11}$$

For determination α_0^i and α_1^i we have the next algebraic equations:

$$\sum_{k=0}^{N-1}\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i - \Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i + 2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1} + 2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha_0^i\right) = 0,$$

$$\begin{gathered}\sum_{k=0}^{N-1}\left\{\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}y_0^i + \Phi_0'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i + \Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1} - \Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i + \right.\\ \left.+\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1} + 2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha_0^i + 2\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\alpha_1^i\right\} = 0.\end{gathered} \tag{12}$$

After solving the equations (12) relatively to and α_1^i, correspondingly, we have[2)]:

$$\alpha_0^i = -\frac{1}{2}\sum_{k=0}^{N-1}\left\{\left(\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)^{-1}\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i - \Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i + 2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}\right)\right\}, \tag{13}$$

$$\begin{gathered}\alpha_1^i = -\tfrac{1}{2}\sum_{k=0}^{N-1}\left\{\left(\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)^{-1}\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{1,i-1}y_0^i + \Phi_0'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i + \Phi_1'^{\,1,i-1}\mathrm{A}\Phi_2^{0,i-1}-\right.\right.\\ -\Phi_1'^{\,1,i-1}\mathrm{A}y_{lk}^i + \Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{1,i-1} - \Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\sum_{k=0}^{N-1}\left(\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)^{-1}\times\\ \left.\left.\times\left(\Phi_0'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}y_0^i - \Phi_1'^{\,0,i-1}\mathrm{A}y_{lk}^i + 2\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_2^{0,i-1}\right)\right)\right\}.\end{gathered} \tag{14}$$

Thus calculating α in the first approach on ξ we obtain: $\alpha^i \approx \alpha_0^i + \varepsilon\alpha_1^i$.

Summarizing the above it is possible to present the computational algorithm [12] for solving of the problem of identification (1), (2), (8)

Algorithm 1.

1. The nonlinear function δ_2 from (1), the initial $y_k(0)$ and final $y_k(l)$, the given weight matrices A_i, the initial approaches $y^{i-1}(x), \alpha^{i-1}$ and small number α^i are forming.
2. $f_y(y_0, \alpha_0, 0), f_\alpha(y_0, \alpha_0, 0), f_y'(y_0, \alpha_0, 0), f_\alpha'(y_0, \alpha_0, 0), f_{yz}''(y_0, \alpha_0, 0), f_{\alpha z}''(y_0, \alpha_0, 0)$ are calculating.
3. $A_0\left(y^{i-1}(x), \alpha^{i-1}\right)$, $A_1\left(y^{i-1}(x), \alpha^{i-1}\right)$, $B_0\left(y^{i-1}(x), \alpha^{i-1}\right)$, $B_1\left(y^{i-1}(x), \alpha^{i-1}\right)$, $C_0\left(y^{i-1}(x), \alpha^{i-1}\right)$, α_0^i are forming from (3).
4. The fundamental matrices $\Phi_0^{0,i-1}(t,0)$, y_l are calculating [19] at the initial $\Phi_0^{0,i-1}(0,0) = \mathrm{E}$, α_1^{i-1} from (6).
5. By using of the fundamental matrices Δ, $\alpha^i \approx \alpha_0^i + \varepsilon\alpha_1^i$, y_{l-1}, $\Phi_2^{1,i-1}(t,0)$ are calculating the integrals.
6. α_0^i and α_1^i are calculating from (11).
7. $\Phi_1^{1,i-1}(t,0)$ we accept $y^0(x)$ as the initial iteration. If it is satisfied, we go to the step 2. Otherwise the calculation process is stopped.

[2)]Here it is assumed that $\left(\Phi_1'^{\,0,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)^{-1}$ and $\left(\Phi_1'^{\,1,i-1}\mathrm{A}\Phi_1^{0,i-1}\right)^{-1}$ are exist. If these conditions are not exist, we can use the regularization method [15].

4. Ordinary Algorithm of Euler for Solving (1) and the Approximate Formulas for $\Phi_0^{0,i-1}(t,0)$, $\Phi_2^{0,i-1}(t,0)$, $\Phi_1^{0,i-1}(t,0)$, $\Phi_1^{1,i-1}(t,0)$, $\Phi_2^{0,i-1}(t,0)$, $\Phi_2^{1,i-1}(t,0)$

We note that at calculating α^i over the algorithm 1 one of the difficulties is the procedure of finding of $\Phi_0^{0,i-1}(t,0)$, $\Phi_0^{1,i-1}(t,0)$and$\Phi_1^{0,i-1}(t,0)$, $\Phi_1^{1,i-1}(t,0)$, $\Phi_2^{0,i-1}(t,0)$, $\Phi_2^{1,i-1}(t,0)$. However by help of approximate methods it is possible to restore them.

Now discretizing on the step Δ the equation (3) on the first Euler method [8, 20], we obtain:

$$\begin{aligned}&y_{i+1} = \left(\mathrm{E} + \Delta\left(\mathrm{A}_0\left(y^{i-1},\alpha^{i-1}\right) + \varepsilon \mathrm{A}_1\left(y^{i-1},\alpha^{i-1}\right)\right)\right) y_k^i +\\ &+\Delta\left(B_0\left(y^{i-1},\alpha^{i-1}\right) + \varepsilon B_1\left(y^{i-1},\alpha^{i-1}\right)\right)\alpha_k^i + \Delta\left(C_0\left(y^{i-1},\alpha^{i-1}\right) + \varepsilon C_1\left(y^{i-1},\alpha^{i-1}\right)\right).\end{aligned} \tag{15}$$

It is now possible to express y_l through an initial condition y_0. First we show this expressions for y_1, y_2

$$\begin{aligned}&y_1 = ((\mathrm{E} + \Delta\mathrm{A}_0) + \Delta\mathrm{A}\varepsilon)\, y_0 + (\mathrm{B}_0 + \varepsilon\mathrm{B}_1)\,\alpha^i + (C_0 + \varepsilon C_1)\,,\\ &y_2 = \left((\mathrm{E} + \Delta\mathrm{A}_0)^2 + 2\varepsilon\Delta\,(\mathrm{E} + \Delta\mathrm{A}_0)\,\mathrm{A}_1\right) y_0 +\\ &+ [(\mathrm{E} + \Delta\mathrm{A}_0)\,\mathrm{B}_0 + \mathrm{B}_0 + \varepsilon\,((\mathrm{E} + \Delta\mathrm{A}_0)\,\mathrm{B}_1 + \mathrm{B}_1 + \Delta\mathrm{A}_1\mathrm{B}_0)]\,\alpha^i +\\ &+ [(\mathrm{E} + \Delta\mathrm{A}_0)]\,C_0 + C_0 + \varepsilon\,\left((\mathrm{E} + \Delta\mathrm{A}_0) + 2\varepsilon\Delta\,(\mathrm{E} + \Delta\mathrm{A}_0)\,C_1 + C_1 + \Delta\mathrm{A}_1 C_0\right)].\end{aligned}$$

Let for y_{l-1} are true the relations

$$\begin{aligned}&y_{l-1} = \left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-1} + N\Delta\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,\varepsilon\right) y_0 +\\ &+ \left[\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,\mathrm{B}_0 + \varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,\mathrm{B}_1\right.\right.\right. +\\ &+\mathrm{N}\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,\Delta\mathrm{A}_1\mathrm{B}_0\Big]\,\alpha^i + \varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,C +\right.\\ &+\varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,C_1 + N\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-2}\,\Delta\mathrm{A}_1 C_0\right).\end{aligned}$$

By mathematical induction we will prove easily, that

$$\begin{aligned}&y_l = \left((\mathrm{E} + \Delta\mathrm{A}_0)^{N} + N\Delta\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\varepsilon\right) y_0 +\\ &+ \left[\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\mathrm{B}_0 + \varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\mathrm{B}_1\right.\right.\right. +\\ &+\mathrm{N}\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\Delta\mathrm{A}_1\mathrm{B}_0\Big]\,\alpha^i + \varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,C +\right.\\ &+\varepsilon\left((\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,C_1 + N\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\Delta\mathrm{A}_1 C_0\right).\end{aligned} \tag{16}$$

Now from (16) we define the approximate formulas for $\Phi_i^j(0,l)$.

Note that the finding of α_0^i , α_1^i from (14) makes difficulty from calculations of fundamental matrices $\Phi_0^n(t,0)$, $\Phi_1^n(t,0)$, $\Phi_2^n(t,0)(n=0,1)$ from the system of linear differential equations (5). Therefore in the next point by means of ordinary method of Euler we discretize the equation (5) and restore approximately the fundamental matrices $\Phi_i^n(t,0)$, $(n=0,1)(i=0,1,2)$ in the first approach. As is shown from (13) - (14) for renewal of α_0^i , α_1^i it is necessary to take into account the higher $\Phi_i^j(0,l)$. The relation (16) allows to find them. Therefore comparing (16) with (5) we have:

$$\begin{aligned}&\Phi_0^{0,i-1} = (\mathrm{E} + \Delta\mathrm{A}_0)^N\,,\\ &\Phi_0^{1,i-1} = N\Delta\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,,\\ &\Phi_1^{0,i-1} = (\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\mathrm{B}_0,\\ &\Phi_1^{1,i-1} = (\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\mathrm{B}_1 + \mathrm{N}\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\Delta\mathrm{A}_1\mathrm{B}_0,\\ &\Phi_2^{0,i-1} = (\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,C_0,\\ &\Phi_2^{1,i-1} = (\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,C_1 + \mathrm{N}\,(\mathrm{E} + \Delta\mathrm{A}_0)^{N-1}\,\Delta\mathrm{A}_1 C_0.\end{aligned} \tag{17}$$

Thus it is simpler to calculate α_0^i, α_1^i from (13), (14) by means of approximate formulas.

We note that with the help of (17) we can easily find for α_0^{i-1}, α_1^{i-1} from (13) and (14) the next expressions in an obvious form

$$\begin{aligned}\alpha_0^i = &-\tfrac{1}{2}\sum_{k=0}^{N-1}\left\{\left(B_0'\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}B_0\right)^{-1}\times\right.\\ &\times\left(y_0'^{\,i}\left(E+\Delta A_0'\right)^{N}A\left(E+\Delta A_0'\right)^{N-1}B_0-B_0'\left(E+\Delta A_0'\right)^{N-1}Ay_{lk}+\right.\\ &\left.\left.+\ y_0'^{\,i}\left(E+\Delta A_0'\right)^{N}A\left((E+\Delta A_0)^{N-1}B_1+N\left(E+\Delta A_0'\right)^{N-1}\Delta A_1B_0\right)\right)\right\},\end{aligned} \tag{18}$$

$$\begin{aligned}\alpha_1^i = &-\tfrac{1}{2}\sum_{k=0}^{N-1}\left\{\left(B_0'\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}B_0\right)^{-1}\times\right.\\ &\times\left(y_0'^{\,i}B_0'\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}B_0++B_1'\left(E+\Delta A_0'\right)^{N-1}+\right.\\ &+B_0'A_1'\Delta N\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}C_0-B_1'\left(E+\Delta A_0'\right)^{N-1}+\\ &+B_0'A_1'\Delta N\left(E+\Delta A_0'\right)^{N-1}Ay_{lk}+B_0'\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}C_1+\\ &N\left(E+\Delta A_0'\right)^{N-1}\Delta A_1C_0+\left(E+\Delta A_0'\right)^{N-1}B_1+\\ &\left.+N\left(E+\Delta A_0'\right)^{N-1}A_1B_0A\left(E+\Delta A_0'\right)^{N-1}B_0\right)\times\\ &\times\left(B_0'\left(E+\Delta A_0'\right)^{N-1}A\left(E+\Delta A_0'\right)^{N-1}B_0\right)^{-1}\times\\ &\left(y_0'^{\,i}\left(E+\Delta A_0'\right)^{N}A\left(E+\Delta A_0'\right)^{N-1}B_0-B_0'\left(E+\Delta A_0'\right)^{N-1}Ay_{lk}+\right.\\ &\left.\left.+y_0'^{\,i}\left(E+\Delta A_0'\right)^{N}A\left((E+\Delta A_0)^{N-1}B_1+N\left(E+\Delta A_0'\right)^{N-1}\Delta A_1B_0\right)\right)\right\}.\end{aligned} \tag{19}$$

We note that for the obtaining of more exact values it is necessary to make the discretization on the method of Runge-Kutta and other. Thus, summarizing the above results we can offer the next computational algorithm for finding of coincidence $\alpha^i \approx \alpha_0^i + \varepsilon\alpha_1^i$.

Algorithm 2.

1. The function $f(y,x,\alpha)$ and the statistical data y_0^i, y_l^i from (2) and (3) ($i = 0,1,...,N$) are given.
2. The derivatives f_y, f_α are calculating
3. $A_0, A_1, B_0, B_1, C_0, C_1$ are calculating from (4).
4. $\Phi_0^{0,i-1}, \Phi_0^{1,i-1}, \Phi_1^{0,i-1}, \Phi_1^{1,i-1}, \Phi_2^{0,i-1}, \Phi_2^{1,i-1}$ are calculating from (7) (or (17)).
5. α_0^i and α_1^i are calculating by the formulas (13) and (14), correspondingly (or (18), (19)).
6. $\alpha^i \approx \alpha_0^i + \varepsilon\alpha_1^i$ are calculating
7. The condition $I_i > I_{i+1}$ is checked up. If the condition is not satisfied, we go to the step 2. Otherwise the calculation process is stopped .

We note that formulas (18) and (19) allow approximately to find α.

We consider the following example.

5. Example

Let us consider the gas lift process for the oil production where the motion equation is described by the following nonlinear ordinary differential equation [3, 6, 8, 10]

$$\dot{Q} = \frac{2a(\lambda_c)\rho F Q^2}{\varepsilon^2 c^2 \rho^2 F^2 - Q^2}, \quad Q(0) = u, \tag{20}$$

where $c >> \omega_c$, except $Q = \rho\omega_c F$ all values are constant, F is the cross-sectional area of pump-compressor pipes, that is constant relatively to axes.

Here it is assumed that the transition from the end of ring pipe through the layer to beginning of the lift ($x = l$) is executed on the following difference equation:

$$\begin{aligned} &Q(l+0) = \gamma Q(l-0) + \gamma_1(Q(l-0))\bar{Q}, \\ &\gamma_1(Q(l-0)) = -\delta_3(Q(l-0) - \delta_2)^2 + \delta_1, \end{aligned} \tag{21}$$

where $\gamma,\ \delta_1,\ \delta_2,\ \delta_3$ are constant real numbers, which are subject to determination.

For simplicity we suppose that parameters $\gamma,\ \delta_1,\ \delta_2,\ \delta_3$ are known and it is required to find the CHR λ_c, included to (20) through $\alpha(\lambda_c)$.

Further some nominal trajectory $Q^0(x)$ and parameter α^0 are searching supposing that k^{th} iteration is already done. Linearizing the equation (20) near these data we have

$$\dot{Q}^k(x) = A\left(Q^{k-1}, \alpha^{k-1}\right) Q^k(x) + B\left(Q^{k-1}, \alpha^{k-1}\right)\alpha^k + C\left(Q^{k-1}, \alpha^{k-1}\right), \tag{22}$$

where

$$A_0\left(y^{i-1}(x), \alpha^{i-1}\right) = 0, \qquad A_1\left(y^{i-1}(x), \alpha^{i-1}\right) = 4a^{i-1}c^2\rho^3F^3Q^{i-1},$$

$$B_0\left(y^{i-1}(x), \alpha^{i-1}\right) = -2\rho F, \qquad B_1\left(y^{i-1}(x), \alpha^{i-1}\right) = 2\rho^3F^3c^2$$

$$C_0\left(y^{i-1}(x), \alpha^{i-1}\right) = 2\rho F a^{i-1}, \quad C_1\left(y^{i-1}(x), \alpha^{i-1}\right) = 2a^{i-1}c^2\rho^3F^3\left(2\left(Q^{i-1}\right)^2 + 1\right).$$

Note that by help of relations (17), (18) the matrices Φ^{k-1}, $\Phi_1^{k-1}(x,0)$, $\Phi_2^{k-1}(x,0)$ are calculating in the following form

$$\Phi_1^{k-1}(x,0) = \left(\sum_{j=N+2}^{2N-1}\left(\prod_{1=2N-1}^{j}\left(E + A\left(Q^{k-1}(x_{i_1}), \alpha^{k-1}\right)\right)h\right)B\left(Q^{k-1}(x_{j-1}), \alpha^{k-1}\right)h\right) +$$

$$+B\left(Q^{k-1}(x_{2N-1}), \alpha^{k-1}\right)h,$$

$$\Phi_2^{k-1}(x,0) = \left(\sum_{j=N+2}^{2N-1}\left(\prod_{1=2N-1}^{j}\left(E + A\left(Q^{k-1}(x_{i_1}), \alpha^{k-1}\right)\right)h\right)C\left(Q^{k-1}(x_{j-1}), \alpha^{k-1}\right)h\right) +$$

$$+C\left(Q^{k-1}(x_{2N-1}), \alpha^{k-1}\right)h,$$

where h s an enough small number, which is the step of integration.

Let the statistical data, that are the results of measuring of debit $\tilde{Q}_{2n}^i$ on leaving with the given initial volume of gas are given, i.e.$\tilde{Q}_0^i$, $\tilde{Q}_{2n}^i$, $i = \overline{1,5}$ are known.

Table 1.

y_i^{l+0}	5.5698	5.5732	5.5761	5.5810	5.5848	5.5852	5.5824
y_i^{2l}	4.4242	4.4248	4.4254	4.4262	4.4266	4.4263	4.4251

Then the functional from (8) has the following form:

$$I = \frac{1}{2}\sum_{i=1}^{5}\left|Q_0^i - Q_l^i\right|^2,$$

where Q_l^i is the solution of the equation (3) for the initial conditions Q_0^i. Let the parameters of the equation (20) look like as:

$$\begin{aligned} &at \quad 0 \le x < l-0: \ \ l = 1485\ m,\ \ s = 331 m/s\ ,\ \ \rho = \tfrac{0{,}717kg}{m^3},\ \ d = \sqrt{114^2 - 73^2}10^{-3}m,\ \ \lambda = 0.01, \\ &at \quad l+0 < x \le 2l: \ \ s = 850 m/s,\ \ \rho = \tfrac{700kg}{\text{m}^3},\ \ d = 0.073\,m,\ \ \lambda = 0.23. \end{aligned} \tag{23}$$

Now we pass to implementation of the above algorithm.

The initial value of CHR λ_c^0 we accept equal to 1. Accepting $y^0(t) = 1$ and repeating the procedure 1-5 from the algorithm 2 we determine the values of λ_c^1, $y^1(t)$. After 44 iterations the following result was obtained:

$$\lambda_c \approx 0.29834,$$

that coincides with λ from (23) within 10^{-2}.

Note that such approach can be satisfactory to finding of initial iterations of ordinary gradient method, for finding of CHR [3], linearizing [1] and other.

References

[1] F.A. Aliev, The Methods of Solving of Applied Problems of Optimization of Dynamic Systems, Baku, Elm, 1989, 327 p.
[2] F.A. Aliev, N.A. Aliev, N.A. Safarova, et al. Sweep method for solving the Roesser type equation describing the motion in the pipeline, Appl. Math. Comput. 295 (2017) 16–23.
[3] F.A. Aliev, N.A. Ismailov, Inverse problem to determine the hydraulic resistance coefficient in the gas lift process, Appl. Comput. Math. 12 (2013) 306–313.
[4] F.A. Aliev, N.A. Ismailov, On a problem of identification in linear stationary case, Reports of NAS of Azerbaijan 46 (2010) 6-14.
[5] F.A. Aliev, N.A. Ismailov, Optimization problems with a periodic boundary-value condition and boundary control for gas-lifting wells, Neliniini Kolyvannya (Nonlinear Oscillations) 17 (2014) 558–574 (Translated in: J. Math. Sci. 208 (2015) 467–476.
[6] F.A. Aliev, N.A. Ismailov, H. Haciyev, M.F. Guliev, A method of determine the coefficient of hydraulic resistance in different areas of pump-compressor pipes, TWMS J. Pure Appl. Math. 7 (2016) 211–217.
[7] F.A. Aliev, N.A. Ismailov, E.V. Mamedova, N.S. Mukhtarova, Computational algorithm for solving problem of optimal boundary-control with nonseparated boundary conditions, J. Comput. Syst. Sci. Int. 55 (2016) 700–711.
[8] F.A. Aliev, N.A. Ismailov, N.S. Mukhtarova, Algorithm to determine the optimal solution of a boundary control problem, Automat. Remote Control 76 (2015) 627–633.
[9] F.A. Aliev, N.A. Ismailov, A.A. Namazov, Asymptotic method for finding the coefficient of hydraulic resistance in lifting of fluid on tubing, J. Inverse Ill-Posed Probl. 23 (2015) 511–518.
[10] F.A. Aliev, N.A. Ismailov, A.A. Namazov, M.F. Rajabov, Algorithm for calculating the parameters of formation of gas-liquid mixture in the shoe of gas-lift well, Appl. Comput. Math. 15 (2016) 370–376.
[11] F.A. Aliev, M.M. Mutallimov, N.A. Ismailov, M.F. Radzhabov, Algorithms for constructing optimal controllers for gaslift operation, Automat. Remote Control 73 (2012) 1279–1289.
[12] T.A. Anake, S.A. Bishop, O.O. Agboola, On a hybrid numerical algorithm for the solutions of higher order ordinary differential equations, TWMS J. Pure Appl. Math. 6 (2015) 186–193.
[13] N. Bakhtizin, .R. Latypov Estimation of order of linear objects on experimental information, Automat. Remote Control 53 (1992) 416–419.
[14] P.E. Bellman, P.E. Kalaba, Quazilinearization and Nonlinear Boundary Problems, Moscow, Mir, 1968.
[15] Y. Huang, Z.H. Liu, R. Wang, Quasilinearization for higher order impulsive fractional differential equations, Appl. Comput. Math. 15 (2016) 159–171
[16] K. Jbilou, A survey of Krylov-based methods for model reduction in large-scale MIMO dynamical systems, Appl. Comput. Math. 15 (2016) 117–148.
[17] L.U. Jian, The Least Square Method and Its Application, Science and Technology of West China, 2007.
[18] A.M. Mabrok, A.M. Haggag, I.R. Petersen, System identification algorithm for negative imaginary systems, Appl. Comput. Math. 14 (2015) 336–348.
[19] N.I. Mahmudov, M.A. McKibben, On approximately controllable systems (survey), Appl. Comput. Math. 15 (2016) 247–264.
[20] A.Kh. Mirzadjanzadeh, I.M. Akhmedov, A.M. Khasaev, V.I. Gusev, Technology and Technique of Oil Production, M., Nedra, 1986 (in Russian).