



The Use of Soft Matrices on Soft Multisets and their Applications in Optimal Decision Process

Arzu Erdem Coşkun^a, Çiğdem Gündüz Aras^a, Ayşe Sönmez^b

^a Kocaeli University, Faculty of Arts and Sciences, Department of Mathematics, Umuttepe Campus, Kocaeli, Turkey

^b Gebze Institute of Technology, Department of Mathematics, Cayirova Campus, 41400, Gebze, Kocaeli, Turkey

Abstract. In this paper, we introduce a concept of a soft matrix on a soft multiset, and investigate how to use soft matrices to solve decision making problems. An algorithm for a multiple choose selection problem is also provided. Finally, we demonstrate an illustrative example to show the decision making steps.

Introduction

One has to make a choose between alternative actions in almost every days and make decisions. Most decisions involve multiple objectives. For example, a developer and manufacturer of computer equipment, the X Industry would like to manufacture a large quantity of a product if the consumers demand for the product adequately high. Unfortunately, sometimes the technology development is difficult and there are some crucial elements such as target time for getting market, competitors, etc. The Development Group of the X industry argues that an untested product is introduced and they propose moving up the production start, and putting the device into production before final tests, meanwhile launching a high-priced advertising campaign offering the units as available now. This shows a decision event activity. When we mention decision making one needs to figure out what to do in the face of difficult circumstances. Molodtsov [18] has introduced 'Soft Set Theory' in which we can use parametrization tools for dealing with this kind of uncertainty and difficulties in the process of decision making. Maji, Biswas and Roy [15] defined equality of two soft sets, a subset and a super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. In the study, soft binary operations like AND, OR and the operations of union, intersection were also characterized. Sezgin and Atagün [21] proved that certain De Morgan's law hold in soft set theory with respect to different operations on soft sets. Ali, Feng, Liu, Min and Shabir [1] introduced some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets. The relationship among soft sets, soft rough sets and topologies was established by Li and Xie in [14]. Based on the novel granulation structures called soft approximation spaces, soft rough approximations and soft rough sets were introduced by Feng, Liu, Fotea and Jun in [8]. Jiang, Tang, Chen, Liu and Tang presented an extended fuzzy soft set theory by using the concepts of fuzzy description logics to act as the parameters of fuzzy soft sets. Gunduz and

2010 *Mathematics Subject Classification.* Primary 40A05; Secondary 40A35, 26A15

Keywords. Sequences, series, summability, continuity, soft matrix, soft multiset

Received: 07 December 2016; Revised: 21 September 2017; Accepted: 23 September 2017

Communicated by Ljubiša D.R. Kočinac

Email addresses: erdem.arzu@gmail.com (Arzu Erdem Coşkun), carasgunduz@gmail.com (Çiğdem Gündüz Aras), asonmez@gyte.edu.tr (Ayşe Sönmez)

Bayramaov in [9] introduced some important properties of fuzzy soft topological spaces. An alternative approach to attribute reduction in multi-valued information system under soft set theory was presented by Herawan, Ghazali and Deris in [13]. Gunduz, Sonmez and Cakalli in [10] introduced soft open and soft closed mappings, soft homeomorphism concept. An application of soft sets to a decision making problem with the help of the theory of soft sets was studied in [5, 12, 16, 17, 22, 24].

When we consider a multiple objective decision making problem with m criteria and n alternatives, it is easy to show the decision making methodology as a decision table. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer. Soft matrices which were a matrix representation of the soft sets were introduced in [4, 6, 7, 19, 20, 23]. However, some decision making problems include several decision-makers such as a group decision making problem. As a generalization of Molodtsov's soft set, the definition of a soft multiset was introduced by Alkhazaleh and Salleh in [2]. In the research, they gave basic operations such as complement, union and intersection with examples and then in [3] they introduced the definition of fuzzy soft multiset as a combination of soft multiset and fuzzy set and study its properties and operations.

In this paper we improve a soft multiset concept in decision-making problems and apply soft matrices concept to group decision making problems.

1. Preliminaries

First of all, we recall some basic concepts and notions, which are necessary foundations of group decision making methods.

Definition 1.1. ([18]) Let U be an initial universe, $P(U)$ be the power set of U , E be a set of all parameters and $A \subset E$. A soft set (f_A, E) on the universe U is defined by the set of ordered pairs

$$(f_A, E) := \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$$

where $f_A : E \rightarrow P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$.

Example 1.2. Let us consider a soft set (f_E, E) which describes the "color of the shirts" that Mrs. X is considering to buy. Suppose that there are five shirts in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $E = \{e_1, e_2, e_3, e_4\}$ is a set of decision parameters. For each e_i , $i = 1, 2, 3, 4$, denotes the parameters "white", "purple", "red" and "blue", respectively. Let $A = \{e_1, e_3\} \subset E$ and

$$\begin{aligned} f_A(e_1) &= U, \\ f_A(e_3) &= \{s_2, s_4\}. \end{aligned}$$

Then we can view the soft set (f_A, E) as consisting of the following collection of approximations:

$$(f_A, E) = \{(e_1, U), (e_3, \{s_2, s_4\})\}.$$

Definition 1.3. ([2]) Let $\{U_i : i \in I\}$ be a collection of universes such that $\bigcap_{i \in I} U_i = \emptyset$, $\{E_i = E_{U_i} : i \in I\}$ be a collection of sets of parameters, $E = \prod_{i \in I} E_i$, $U = \prod_{i \in I} P(U_i)$, $A \subset E$. A pair (F_A, E) is called a soft multiset over U , where F_A is a mapping given by $F_A : A \rightarrow U$.

This paper will focus on the situation that universe sets U_i and parameter sets E_i are both finite sets for each $i \in I$.

Example 1.4. Suppose that there are three universes U_1, U_2, U_3 . Let us consider a soft multiset (F_A, E) which describes the "attractiveness of houses", "attractiveness of cars" and "attractiveness of hotels" that Mrs.

X is considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively.

$$\begin{aligned}
 U_1 &= \{h_1, h_2, h_3, h_4, h_5, h_6\} \\
 U_2 &= \{c_1, c_2, c_3, c_4, c_5\} \\
 U_3 &= \{v_1, v_2, v_3, v_4\} \\
 E_1 &= E_{U_1} = \{e_{11} = \text{"expensive"}, e_{12} = \text{"cheap"}, e_{13} = \text{"4 bedroom flat"}, e_{14} = \text{"3 bedroom and terraced house"}, \\
 e_{15} &= \text{"located in the heart of the city"}\} \\
 E_2 &= E_{U_2} = \{e_{21} = \text{"expensive"}, e_{22} = \text{"cheap"}, e_{23} = \text{"friendly technology"}, \\
 e_{24} &= \text{"better performance"}, e_{25} = \text{"luxury"}, e_{26} = \text{"Made in Germany"}\} \\
 E_3 &= E_{U_3} = \{e_{31} = \text{"expensive"}, e_{32} = \text{"cheap"}, e_{33} = \text{"in İstanbul"}, e_{34} = \text{"located in the historic centre"}, \\
 e_{35} &= \text{"neoclassic hotel"}\} \\
 E &= \prod_{i \in I} E_i, U = \prod_{i \in I} P(U_i) \\
 A &= \{a_1 = (e_{11}, e_{21}, e_{31}), a_2 = (e_{11}, e_{22}, e_{34}), a_3 = (e_{12}, e_{23}, e_{35}), a_4 = (e_{15}, e_{24}, e_{32}), \\
 a_5 &= (e_{14}, e_{23}, e_{33}), a_6 = (e_{12}, e_{25}, e_{32}), a_7 = (e_{13}, e_{21}, e_{31}), a_8 = (e_{11}, e_{26}, e_{32})\} \subset E
 \end{aligned}$$

Suppose that

$$\begin{aligned}
 F_A(a_1) &= (\{h_3, h_4, h_5, h_6\}, \{c_1, c_2, c_3\}, \{v_2, v_3\}), \\
 F_A(a_2) &= (\{h_3, h_4, h_5, h_6\}, \{c_4, c_5\}, \{v_1, v_2\}), \\
 F_A(a_3) &= (\{h_1, h_2\}, \emptyset, \{v_2, v_3\}), \\
 F_A(a_4) &= (U_1, \{c_3, c_4\}, \{v_1, v_4\}), \\
 F_A(a_5) &= (\{h_3, h_4, h_5\}, \emptyset, \{v_2, v_4\}), \\
 F_A(a_6) &= (\{h_1, h_2\}, U_2, \{v_1, v_4\}), \\
 F_A(a_7) &= (\emptyset, \{c_1, c_2, c_3\}, \{v_2, v_3\}), \\
 F_A(a_8) &= (\{h_3, h_4, h_5, h_6\}, \{c_4, c_5\}, \{v_1, v_4\}),
 \end{aligned}$$

Then we can view the soft multiset (F_A, E) as consisting of the following collection of approximations

$$\begin{aligned}
 (F_A, E) &= \{((e_{11}, e_{21}, e_{31}), (\{h_3, h_4, h_5, h_6\}, \{c_1, c_2, c_3\}, \{v_2, v_3\})), \\
 &((e_{11}, e_{22}, e_{34}), (\{h_3, h_4, h_5, h_6\}, \{c_4, c_5\}, \{v_1, v_2\})), \\
 &((e_{12}, e_{23}, e_{35}), (\{h_1, h_2\}, \emptyset, \{v_2, v_3\})), \\
 &((e_{15}, e_{24}, e_{32}), (U_1, \{c_3, c_4\}, \{v_1, v_4\})), \\
 &((e_{14}, e_{23}, e_{33}), (\{h_3, h_4, h_5\}, \emptyset, \{v_2, v_4\})), \\
 &((e_{12}, e_{25}, e_{32}), (\{h_1, h_2\}, U_2, \{v_1, v_4\})), \\
 &((e_{13}, e_{21}, e_{31}), (\emptyset, \{c_1, c_2, c_3\}, \{v_2, v_3\})), \\
 &((e_{11}, e_{26}, e_{32}), (\{h_3, h_4, h_5, h_6\}, \{c_4, c_5\}, \{v_1, v_4\}))\}
 \end{aligned}$$

Definition 1.5. ([2]) For any soft multiset (F_A, E) , a pair (e_{ij}, F_{ij}) is called a U_i - soft multiset part for $\forall e_{ij} \in a_k$, and $F_{ij} \subset F_A(A)$ is an approximate value set, where $a_k \in A, k = 1, 2, 3, \dots, r, i = 1, 2, \dots, m_i, j = 1, 2, \dots, n_j$.

Example 1.6. Consider the soft multiset given in Exam 1.4. Then,

$$\begin{aligned}
 (e_{1j}, F_{1j}) &= \{(e_{11}, \{h_3, h_4, h_5, h_6\}), (e_{12}, \{h_1, h_2\}), (e_{13}, \emptyset), \\
 &(e_{14}, \{h_3, h_4, h_5\}), (e_{15}, U_1)\}
 \end{aligned}$$

is a U_1 - soft multiset part of (F_A, E) .

Definition 1.7. (i) Let a pair (e_{ij}, F_{ij}) be a U_i - soft multiset part of soft multiset (F_A, E) , $A_i \subset E_i$. Then a subset of $U_i \times E_i$ is called a relation form of U_i - soft multiset part of (F_A, E) which is uniquely defined by

$$R_{A_i} = \{(u_{ij}, e_{ij}) : e_{ij} \in A_i, u_{ij} \in F_{ij}(e_{ij})\}$$

The characteristic function of $\chi_{R_{A_i}}$ is written by

$$\chi_{R_{A_i}} : U_i \times E_i \rightarrow \{0, 1\}, \chi_{R_{A_i}}(u_{ij}, e_{ij}) := \begin{cases} 1, & (u_{ij}, e_{ij}) \in R_{A_i} \\ 0, & (u_{ij}, e_{ij}) \notin R_{A_i} \end{cases}$$

(ii) If $U_i = \{u_{i1}, u_{i2}, \dots, u_{im_i}\}$, $E_i = \{e_{i1}, e_{i2}, \dots, e_{in_i}\}$, then we call a matrix $[a_{lk}^i] = \chi_{R_{A_i}}(u_{ik}, e_{il})$, $1 \leq k \leq m_i, 1 \leq l \leq n_i$ as an $m_i \times n_i$ soft matrix of U_i - soft multiset part of (F_A, E) .

Example 1.8. Let us consider Exam 1.6. Then

$$R_{A_1} = \{(h_3, e_{11}), (h_4, e_{11}), (h_5, e_{11}), (h_6, e_{11}), (h_1, e_{12}), (h_2, e_{12}), (h_3, e_{14}), (h_4, e_{14}), (h_5, e_{14}), (h_1, e_{15}), (h_2, e_{15}), (h_3, e_{15}), (h_4, e_{15}), (h_5, e_{15}), (h_6, e_{15})\}$$

Then R_{A_1} is presented by a table as in the following form:

R_{A_1}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
h_1	$\chi_{R_{A_1}}(h_1, e_{11})$	$\chi_{R_{A_1}}(h_1, e_{12})$	$\chi_{R_{A_1}}(h_1, e_{13})$	$\chi_{R_{A_1}}(h_1, e_{14})$	$\chi_{R_{A_1}}(h_1, e_{15})$
h_2	$\chi_{R_{A_1}}(h_2, e_{11})$	$\chi_{R_{A_1}}(h_2, e_{12})$	$\chi_{R_{A_1}}(h_2, e_{13})$	$\chi_{R_{A_1}}(h_2, e_{14})$	$\chi_{R_{A_1}}(h_2, e_{15})$
h_3	$\chi_{R_{A_1}}(h_3, e_{11})$	$\chi_{R_{A_1}}(h_3, e_{12})$	$\chi_{R_{A_1}}(h_3, e_{13})$	$\chi_{R_{A_1}}(h_3, e_{14})$	$\chi_{R_{A_1}}(h_3, e_{15})$
h_4	$\chi_{R_{A_1}}(h_4, e_{11})$	$\chi_{R_{A_1}}(h_4, e_{12})$	$\chi_{R_{A_1}}(h_4, e_{13})$	$\chi_{R_{A_1}}(h_4, e_{14})$	$\chi_{R_{A_1}}(h_4, e_{15})$
h_5	$\chi_{R_{A_1}}(h_5, e_{11})$	$\chi_{R_{A_1}}(h_5, e_{12})$	$\chi_{R_{A_1}}(h_5, e_{13})$	$\chi_{R_{A_1}}(h_5, e_{14})$	$\chi_{R_{A_1}}(h_5, e_{15})$
h_6	$\chi_{R_{A_1}}(h_6, e_{11})$	$\chi_{R_{A_1}}(h_6, e_{12})$	$\chi_{R_{A_1}}(h_6, e_{13})$	$\chi_{R_{A_1}}(h_6, e_{14})$	$\chi_{R_{A_1}}(h_6, e_{15})$

Hence soft matrix $[a_{lk}^1]$ of U_1 soft multiset part of (F_A, E) is written by

$$[a_{lk}^1]_{6 \times 5} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 5}.$$

Definition 1.9. ([7]) $[a_{lk}]_{m \times n}$ is called a zero soft matrix, denoted by $[0]$, if $a_{lk} = 0$ for all $1 \leq l \leq m$ and $1 \leq k \leq n$.

2. Soft Matrices on Soft Multisets

In this section, inspired by the above definitions to soft matrices and soft multisets, first we will begin defining soft matrices on soft multisets and its product and then we will give examples for these concepts.

Definition 2.1. Let $U_i = \{u_{i1}, u_{i2}, \dots, u_{im_i}\}$ be universes, $E_i = \{e_{i1}, e_{i2}, \dots, e_{in_i}\}$ be parameters for each $i \in I$, R_{A_i} be a relation form of U_i - soft multiset part of (F_A, E) , $[a_{lk}^i]_{m_i \times n_i}$, $1 \leq k \leq m_i, 1 \leq l \leq n_i$ be soft matrix of U_i - soft multiset part of (F_A, E) . Then

$$[A_{lk}]_{m \times n} = \begin{bmatrix} [a_{lk}^1]_{m_1 \times n_1} & [0]_{m_1 \times n_2} & \dots & [0]_{m_1 \times n_N} \\ [0]_{m_2 \times n_1} & [a_{lk}^2]_{m_2 \times n_2} & \dots & [0]_{m_2 \times n_N} \\ \dots & \dots & \ddots & \dots \\ [0]_{m_N \times n_1} & [0]_{m_N \times n_2} & \dots & [a_{lk}^N]_{m_N \times n_N} \end{bmatrix}_{m \times n}$$

is called a soft matrix of (F_A, E) , where $m = m_1 + m_2 + \dots + m_N$, $n = n_1 + n_2 + \dots + n_N$.

Example 2.2. Let us consider Example 1.6. The soft matrix $[a_{lk}^1]$ of U_1 - soft multiset part of (F_A, E) is given in Example 1.8. Similarly, we can obtain the soft matrices $[a_{lk}^2]$ of U_2 - and $[a_{lk}^3]$ of U_3 - soft multiset parts of (F_A, E) as shown below:

$$[a_{lk}^2]_{5 \times 6} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}_{5 \times 6}, \quad [a_{lk}^3]_{4 \times 5} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}_{4 \times 5}.$$

Then soft matrix $[A_{lk}]_{m \times n}$ of (F_A, E) can be written

$$[A_{lk}]_{15 \times 16} = \left(\begin{array}{c|c|c} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 5} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6 \times 6} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6 \times 5} \\ \hline \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 5} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}_{5 \times 6} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 5} \\ \hline \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 5} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 6} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}_{4 \times 5} \end{array} \right)_{15 \times 16}$$

We make the following product definitions for soft matrices on soft multisets, which are adapted from Definitions 7-10 in [7].

Definition 2.3. Let $[a_{lk}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_A, E) , $[b_{lj}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_B, E) . Then **And** product of $[a_{lk}^i]_{m_i \times n_i}$ and $[b_{lj}^i]_{m_i \times n_i}$ is defined by

$$[a_{lk}^i]_{m_i \times n_i} \wedge [b_{lj}^i]_{m_i \times n_i} = [c_{lp}^i]_{m_i \times n_i^2}$$

where $c_{lp}^i = \min\{a_{lk}^i, b_{lj}^i\}$ such that $p = n_i(k-1) + j$.

Definition 2.4. Let $[a_{lk}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_A, E) , $[b_{lj}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_B, E) . Then **Or** product of $[a_{lk}^i]_{m_i \times n_i}$ and $[b_{lj}^i]_{m_i \times n_i}$ is defined by

$$[a_{lk}^i]_{m_i \times n_i} \vee [b_{lj}^i]_{m_i \times n_i} = [c_{lp}^i]_{m_i \times n_i^2}$$

where $c_{lp}^i = \max\{a_{lk}^i, b_{lj}^i\}$ such that $p = n_i(k-1) + j$.

Definition 2.5. Let $[a_{lk}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_A, E) , $[b_{lj}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_B, E) . Then **And-Not** product of $[a_{lk}^i]_{m_i \times n_i}$ and $[b_{lj}^i]_{m_i \times n_i}$ is defined by

$$[a_{lk}^i]_{m_i \times n_i} \bar{\wedge} [b_{lj}^i]_{m_i \times n_i} = [c_{lp}^i]_{m_i \times n_i^2}$$

where $c_{lp}^i = \min\{a_{lk}^i, 1 - b_{lj}^i\}$ such that $p = n_i(k - 1) + j$.

Definition 2.6. Let $[a_{lk}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_A, E) , $[b_{lj}^i]_{m_i \times n_i}$ be soft matrix of U_i - soft multiset part of (F_B, E) . Then **Or-Not** product of $[a_{lk}^i]_{m_i \times n_i}$ and $[b_{lj}^i]_{m_i \times n_i}$ is defined by

$$[a_{lk}^i]_{m_i \times n_i} \vee [b_{lj}^i]_{m_i \times n_i} = [c_{lp}^i]_{m_i \times n_i^2}$$

where $c_{lp}^i = \max\{a_{lk}^i, 1 - b_{lj}^i\}$ such that $p = n_i(k - 1) + j$.

Example 2.7. As in Example 1.4, Mr. X is joining with Mrs. X for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. The set of choice parameters for him is

$$\begin{aligned} B &= \{b_1 = (e_{11}, e_{25}, e_{31}), b_2 = (e_{13}, e_{22}, e_{33}), b_3 = (e_{12}, e_{26}, e_{32}), b_4 = (e_{14}, e_{24}, e_{34}), \\ b_5 &= (e_{15}, e_{23}, e_{35}), b_6 = (e_{11}, e_{21}, e_{31})\} \subset E. \end{aligned}$$

Then the soft multiset (F_B, E) is given by

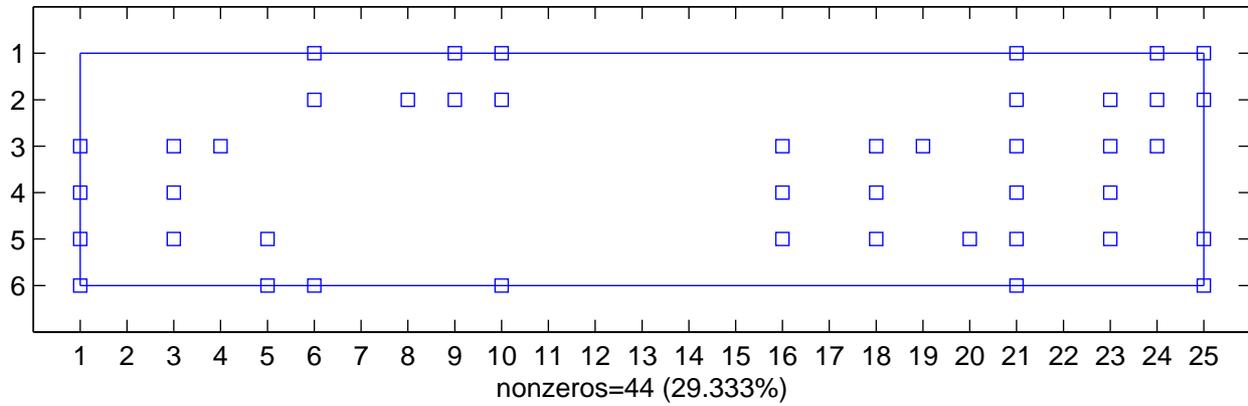
$$\begin{aligned} (F_B, E) &= \{((e_{11}, e_{25}, e_{31}), (U_1, \{c_4, c_5\}, \{v_1, v_2, v_3\})), \\ &((e_{13}, e_{22}, e_{33}), (\{h_2, h_3, h_4, h_5\}, \{c_1, c_2\}, \{v_2, v_4\})), \\ &((e_{12}, e_{26}, e_{32}), (\emptyset, \{c_4, c_5\}, \{v_4\})), \\ &((e_{14}, e_{24}, e_{34}), (\{h_1, h_2, h_3\}, \{c_4, c_5\}, \{v_2, v_3\})), \\ &((e_{15}, e_{23}, e_{35}), (\{h_1, h_2, h_5, h_6\}, \{c_1, c_2, c_3, c_4\}, \{v_2\})), \\ &((e_{11}, e_{21}, e_{31}), (U_1, \{c_3, c_4, c_5\}, \{v_1, v_2, v_3\}))\}, \end{aligned}$$

and the soft matrix $[b_{lk}^1]$ of U_1 - soft multiset part of (F_B, E) is obtained as shown below:

$$[b_{lk}^1]_{6 \times 5} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 5}.$$

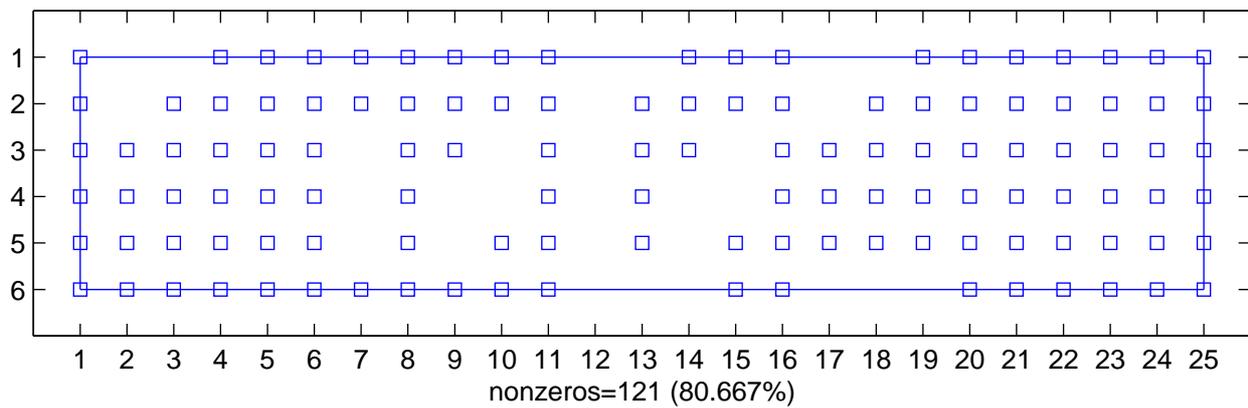
Then we have **And** product of $[a_{ij}^1]_{m \times n}$ of U_1 - soft multiset part of (F_A, E) and $[b_{ik}^1]_{m \times n}$ of U_1 - soft multiset part of (F_B, E) in the following:

And Product of $[a_{ij}^1]$ and $[b_{ij}^1]$ as U_1 – multiset part of (F_A, E) and (F_B, E)



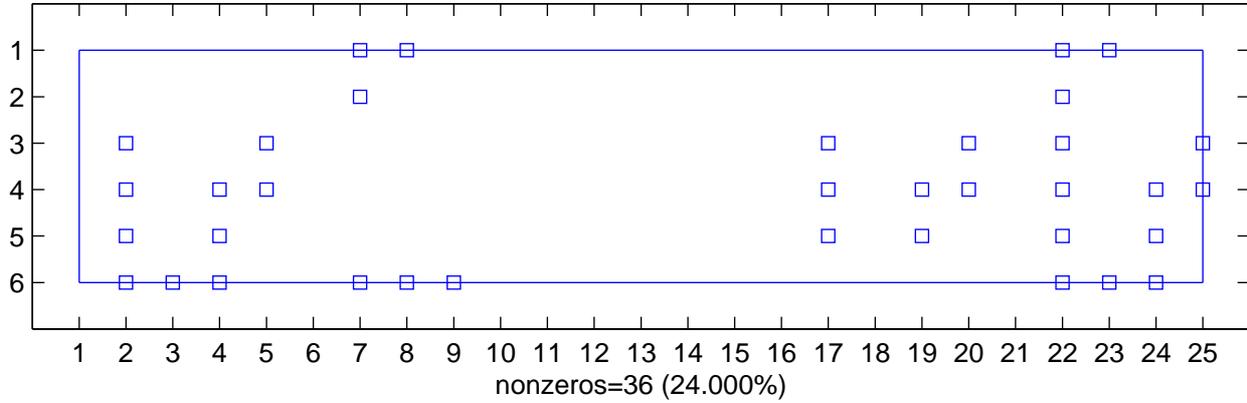
Or product of $[a_{ij}^1]_{m \times n}$ of U_1 soft multiset part of (F_A, E) and $[b_{ik}^1]_{m \times n}$ of U_1 soft multiset part of (F_B, E) in the following:

Or Product of $[a_{ij}^1]$ and $[b_{ij}^1]$ as U_1 – multiset part of (F_A, E) and (F_B, E)



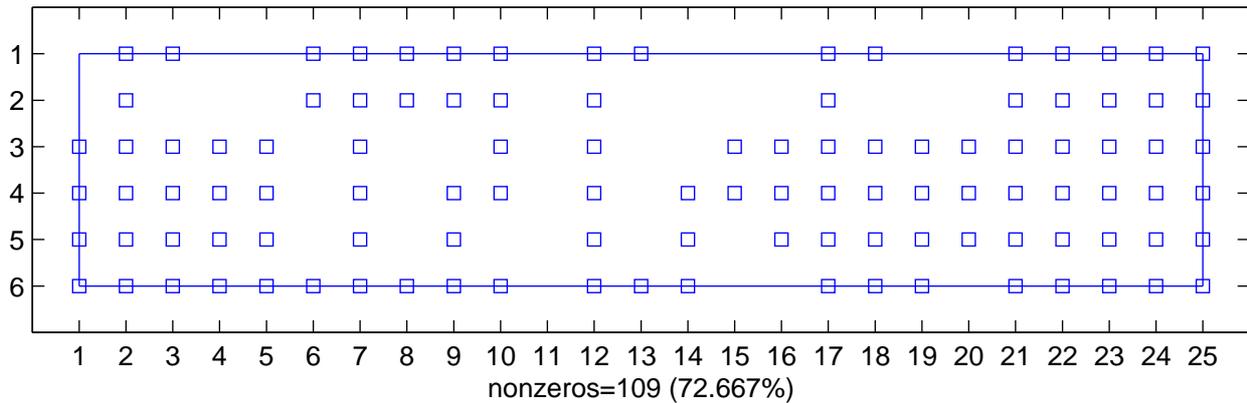
And-Not product of $[a_{ij}^1]_{m \times n}$ of U_1 soft multiset part of (F_A, E) and $[b_{ik}^1]_{m \times n}$ of U_1 soft multiset part of (F_B, E) in the following:

And–Not Product of $[a_{ij}^1]$ and $[b_{ij}^1]$ as U_1 – multiset part of (F_A, E) and (F_B, E)



Or–Not product of $[a_{ij}^1]_{m \times n}$ of U_1 soft multiset part of (F_A, E) and $[b_{ik}^1]_{m \times n}$ of U_1 soft multiset part of (F_B, E) in the following:

Or–Not Product of $[a_{ij}^1]$ and $[b_{ij}^1]$ as U_1 – multiset part of (F_A, E) and (F_B, E)



Here, squares show the value 1 for the product of $[a_{ij}^1]_{m \times n}$ of U_1 – soft multiset part of (F_A, E) and $[b_{ik}^1]_{m \times n}$ of U_1 – soft multiset part of (F_B, E) .

Definition 2.8. Let $[A_{lk}]_{m \times n}, [B_{lk}]_{m \times n}$ be the soft matrices of soft multisets (F_A, E) and (F_B, E) , respectively. Then **And** product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ is defined by

$$[A_{lk}]_{m \times n} \wedge [B_{lk}]_{m \times n} = \begin{bmatrix} [a_{lk}^1]_{m_1 \times n_1} \wedge [b_{lk}^1]_{m_1 \times n_1} & [0]_{m_1 \times n_2} & \cdots & [0]_{m_1 \times n_N} \\ [0]_{m_2 \times n_1} & [a_{lk}^2]_{m_2 \times n_2} \wedge [b_{lk}^2]_{m_2 \times n_2} & \cdots & [0]_{m_2 \times n_N} \\ \cdots & \cdots & \ddots & \cdots \\ [0]_{m_N \times n_1} & [0]_{m_N \times n_2} & \cdots & [a_{lk}^N]_{m_N \times n_N} \wedge [b_{lk}^N]_{m_N \times n_N} \end{bmatrix}_{m \times n}$$

Or product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ is defined by

$$[A_{lk}]_{m \times n} \vee [B_{lk}]_{m \times n} = \left[\begin{array}{c|c|c|c} [a_{lk}^1]_{m_1 \times n_1} \vee [b_{lk}^1]_{m_1 \times n_1} & [0]_{m_1 \times n_2} & \cdots & [0]_{m_1 \times n_N} \\ \hline [0]_{m_2 \times n_1} & [a_{lk}^2]_{m_2 \times n_2} \vee [b_{lk}^2]_{m_2 \times n_2} & \cdots & [0]_{m_2 \times n_N} \\ \hline \cdots & \cdots & \ddots & \cdots \\ \hline [0]_{m_N \times n_1} & [0]_{m_N \times n_2} & \cdots & [a_{lk}^N]_{m_N \times n_N} \vee [b_{lk}^N]_{m_N \times n_N} \end{array} \right]_{m \times ns}$$

And-Not product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ is defined by

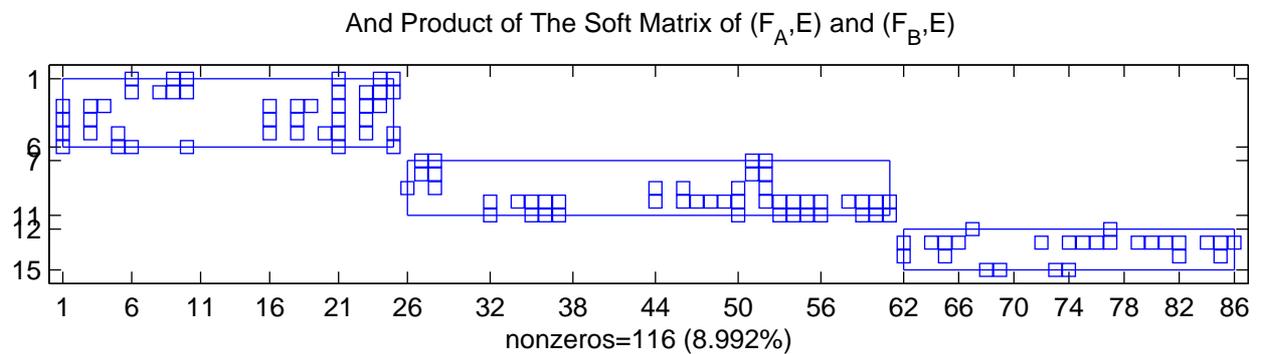
$$[A_{lk}]_{m \times n} \bar{\wedge} [B_{lk}]_{m \times n} = \left[\begin{array}{c|c|c|c} [a_{lk}^1]_{m_1 \times n_1} \bar{\wedge} [b_{lk}^1]_{m_1 \times n_1} & [0]_{m_1 \times n_2} & \cdots & [0]_{m_1 \times n_N} \\ \hline [0]_{m_2 \times n_1} & [a_{lk}^2]_{m_2 \times n_2} \bar{\wedge} [b_{lk}^2]_{m_2 \times n_2} & \cdots & [0]_{m_2 \times n_N} \\ \hline \cdots & \cdots & \ddots & \cdots \\ \hline [0]_{m_N \times n_1} & [0]_{m_N \times n_2} & \cdots & [a_{lk}^N]_{m_N \times n_N} \bar{\wedge} [b_{lk}^N]_{m_N \times n_N} \end{array} \right]_{m \times ns}$$

Or-Not product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ is defined by

$$[A_{lk}]_{m \times n} \vee\bar{\wedge} [B_{lk}]_{m \times n} = \left[\begin{array}{c|c|c|c} [a_{lk}^1]_{m_1 \times n_1} \vee\bar{\wedge} [b_{lk}^1]_{m_1 \times n_1} & [0]_{m_1 \times n_2} & \cdots & [0]_{m_1 \times n_N} \\ \hline [0]_{m_2 \times n_1} & [a_{lk}^2]_{m_2 \times n_2} \vee\bar{\wedge} [b_{lk}^2]_{m_2 \times n_2} & \cdots & [0]_{m_2 \times n_N} \\ \hline \cdots & \cdots & \ddots & \cdots \\ \hline [0]_{m_N \times n_1} & [0]_{m_N \times n_2} & \cdots & [a_{lk}^N]_{m_N \times n_N} \vee\bar{\wedge} [b_{lk}^N]_{m_N \times n_N} \end{array} \right]_{m \times ns}$$

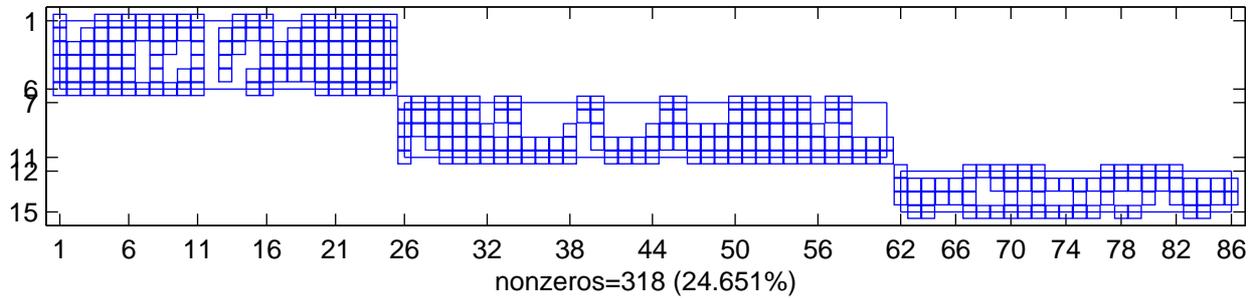
where $m = m_1 + m_2 + \cdots + m_N$, $ns = n_1^2 + n_2^2 + \cdots + n_N^2$.

Example 2.9. Let us consider Example 2.2 and 2.7. **And** product of $[A_{lk}]_{15 \times 16}$ and $[B_{lk}]_{15 \times 16}$ is 15×86 soft matrix whose visualize sparsity pattern is given in the following:



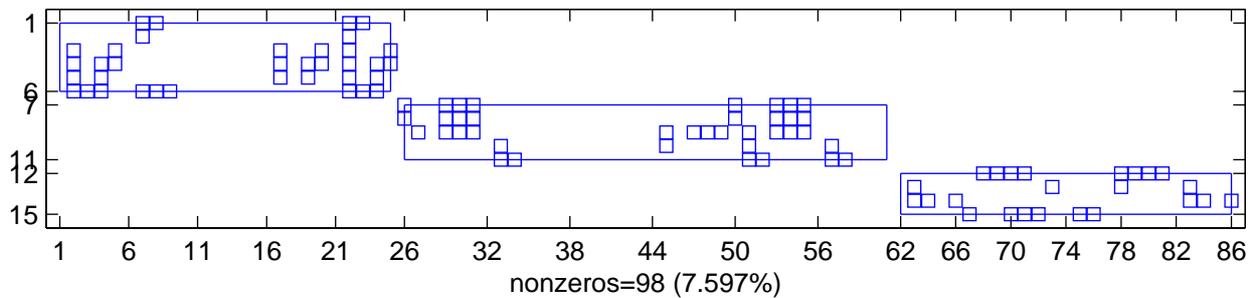
Or product of $[A_{lk}]_{15 \times 16}$ and $[B_{lk}]_{15 \times 16}$ is 15×86 soft matrix whose visualize sparsity pattern is given in the following:

Or Product of The Soft Matrix of (F_A, E) and (F_B, E)



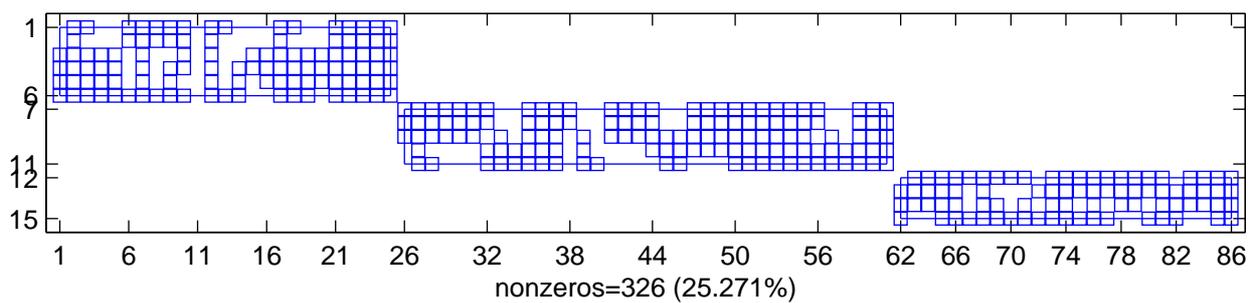
And-Not product of $[A_{lk}]_{15 \times 16}$ and $[B_{lk}]_{15 \times 16}$ is 15×86 soft matrix whose visualize sparsity pattern is given in the following:

And–Not Product of The Soft Matrix of (F_A, E) and (F_B, E)



Or-Not product of $[A_{lk}]_{15 \times 16}$ and $[B_{lk}]_{15 \times 16}$ is 15×86 soft matrix whose visualize sparsity pattern is given in the following:

Or–Not Product of The Soft Matrix of (F_A, E) and (F_B, E)



Here, squares show the value 1 for the product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ as the soft matrices of soft multisets (F_A, E) and (F_B, E) , respectively.

3. Algorithm for Selection of the House, Car and Hotel

The following algorithm is followed by Mr. X and Mrs. X to select the house, car and hotel they wish to purchase and hold a wedding celebration.

Step 1 input the set A of choice parameters of Mrs. X and the set B of choice parameters of Mr. X which are both subsets of E ,

Step 2 find the soft multisets $(F_A, E), (F_B, E)$,

Step 3 find the soft matrix $[a_{lk}^i]$ of U_i - soft multiset part of (F_A, E) and the soft matrix $[b_{lk}^i]$ of U_i - soft multiset part of (F_B, E) ,

Step 4 construct the soft matrix $[A_{lk}]_{m \times n}$ of soft set (F_A, E) and the soft matrix $[B_{lk}]_{m \times n}$ of soft set (F_B, E) by using Definition 2.1,

Step 5 construct **And** product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ by using Definition 2.8, denote by $[C_{lj}]_{m \times ns}$

Step 6 define $I_k^{(i)} = \{j | \exists i : c_{lj} \neq 0, n_1^2 + n_2^2 + \dots + n_{i-1}^2 + (k-1) * n_i < j \leq n_1^2 + n_2^2 + \dots + n_{i-1}^2 + k * n_i, m_1 + m_2 + \dots + m_{i-1} + 1 \leq i \leq m_1 + m_2 + \dots + m_i\}, k \in \{1, 2, \dots, n_i\}, i = 1, 2, \dots, N$ and construct decision function as follows, [7]:

$$w_{l,k}^{(i)} = \begin{cases} \min_{j \in I_k^{(i)}} \{c_{lj}\}, & \text{if } I_k^{(i)} \neq \emptyset \\ 0, & \text{if } I_k^{(i)} = \emptyset \end{cases}$$

$$v_l^{(i)} = \max_k w_{l,k}^{(i)}$$

Step 7 Find the optimum set of U_i in the following way:

$$opt(U_i) = \{u_{ij} \in U_i : 1 \leq j \leq n_i, v_l^{(i)} = 1\}$$

4. Application

Now we use the algorithm to solve our original problem.

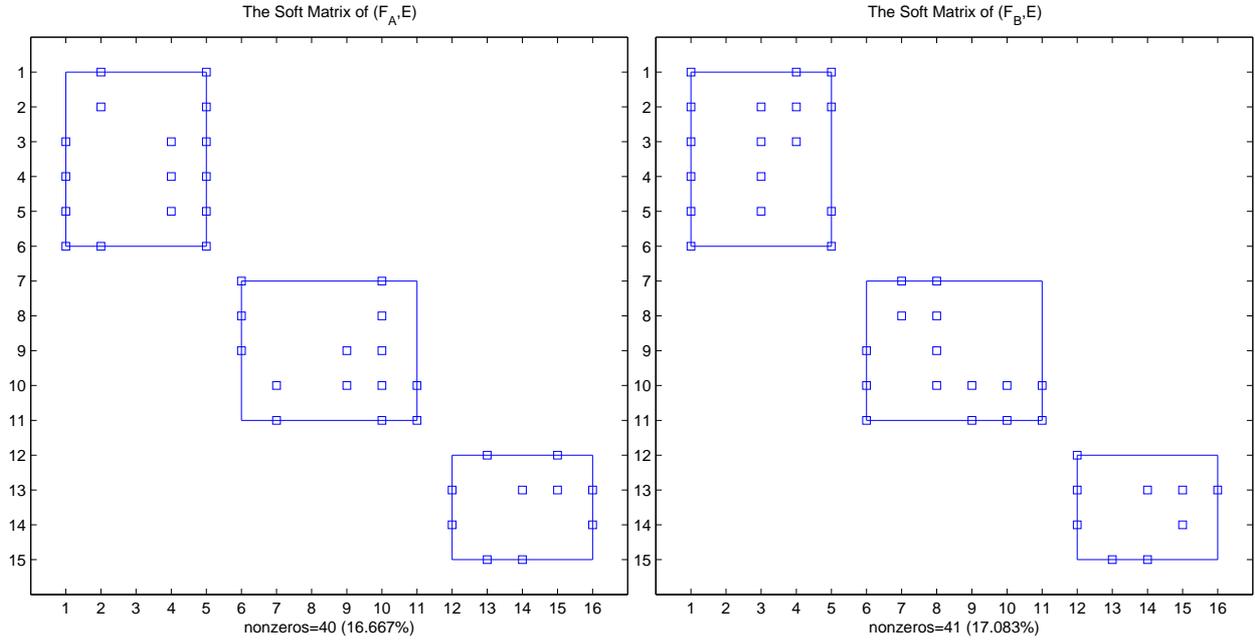
Example 4.1. Suppose that a married couple, Mr. X and Mrs. X , are considering for accommodation purchase, transportation purchase, and venue to hold a wedding celebration respectively. The set of choice parameters for them are given in Example 1.4 and Example 2.7. We now select the house(s), car(s) and hotel(s) on the sets of partners' parameters by using the above algorithm as follows:

Step 1 First, Mrs. X and Mr. X have to choose the sets of their parameters, given in Example 1.4 and Example 2.7, respectively;

Step 2 Then we can write the soft multisets $(F_A, E), (F_B, E)$, given in Example 1.4 and Example 2.7, respectively;

Step 3 Next we find the soft matrix $[a_{lk}^i]$ of U_i soft multiset part of (F_A, E) and the soft matrix $[b_{lk}^i]$ of U_i soft multiset part of (F_B, E) , given in Example 2.2 and Example 2.7, respectively;

Step 4 By using Definition 2.1, we construct the soft matrix $[A_{lk}]_{m \times n}$ of soft set (F_A, E) and the soft matrix $[B_{lk}]_{m \times n}$ of soft set (F_B, E) ;



(a) Visualize Sparsity Pattern of the Soft Matrix of (F_A, E) (b) Visualize Sparsity Pattern of the Soft Matrix of (F_B, E)

Figure 9

Step 5 We construct **And** product of $[A_{lk}]_{m \times n}$ and $[B_{lk}]_{m \times n}$ by using Definition 2.8, denote by $[C_{ij}]_{m \times ns}$, given by Example 2.9;

Step 6 We find the sets $I_k^{(i)}$ as

$$\begin{aligned}
 I_1^{(1)} &= \{1; 3; 4; 5\}, I_2^{(1)} = \{6; 8; 9; 10\}, I_3^{(1)} = \emptyset, I_4^{(1)} = \{16; 18; 19; 20\}, I_5^{(1)} = \{21; 23; 24; 25\} \\
 I_1^{(2)} &= \{26; 27; 28\}, I_2^{(2)} = \{32; 34; 35; 36; 37\}, I_3^{(2)} = \emptyset, I_4^{(2)} = \{44; 46; 47; 48; 49\}, \\
 I_5^{(2)} &= \{50; 51; 52; 53; 54; 55\}, I_6^{(2)} = \{56; 58; 59; 60; 61\} \\
 I_1^{(3)} &= \{62; 64; 65; 66\}, I_2^{(3)} = \{67; 68; 69\}, I_3^{(3)} = \{72; 73; 74; 75; 76\}, \\
 I_4^{(3)} &= \{77; 79; 80; 81\}, I_5^{(3)} = \{82; 84; 85; 86\}
 \end{aligned}$$

and construct decision function

$$\begin{aligned}
 [w_{l,k}^{(1)}] &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6 \times 5}, [w_{l,k}^{(2)}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5 \times 6}, [w_{l,k}^{(3)}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{4 \times 5} \\
 [v_l^{(1)}] &= \begin{bmatrix} 0 \\ \mathbf{1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{matrix}, [v_l^{(2)}] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \mathbf{1} \\ 0 \end{bmatrix} \rightarrow \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{matrix}, [v_l^{(3)}] = \begin{bmatrix} 0 \\ \mathbf{1} \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix}
 \end{aligned}$$

Step 7 We find the optimum set of $U_1 = \{h_2\}$, $U_2 = \{c_4\}$, $U_3 = \{v_2\}$

References

- [1] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.* 57 (2009) 1547–1553.
- [2] S. Alkhalzaleh, A.R. Salleh, Soft multisets theory, *Appl. Math. Sci.* 5 (2011) 3561–3573.
- [3] S. Alkhalzaleh, A.R. Salleh, Fuzzy soft multiset theory, *Abstr. Appl. Anal.* 2012/350603 (2012), 20 pages.
- [4] T.M. Basu, N.K. Mahapatra, S.K. Mondal, Different types of matrices in fuzzy soft set theory and their application in decision making problems, *IRACST–Engin. Sci. Technology* 2 (2012) 389–398.
- [5] N. Çağman, S. Enginoğlu, Soft set theory and uni-int decision making, *European J. Oper. Res.* 207 (2010) 848–855.
- [6] N. Çağman, S. Enginoğlu, Fuzzy soft matrix theory and its application in decision making, *Iranian J. Fuzzy Syst.* 9 (2012) 109–119.
- [7] N. Çağman, S. Enginoğlu, Soft matrix theory and its decision making, *Comput. Math. Appl.* 59 (2010) 3308–3314.
- [8] F. Feng, X. Liu, V.L. Fotea, Y.B. Jun, Soft sets and soft rough sets, *Inform. Sci.* 181 (2011) 1125–1137.
- [9] C.A. Gunduz, S. Bayramov, Some results on fuzzy soft topological spaces, *Math. Problems Engin.* 2013 (2013), 10 pages.
- [10] C.A. Gunduz, A. Sonmez, H. Cakalli, On soft mappings, arXiv:1305.4545(2013), 12 pages.
- [11] A.E. Coskun, C.A. Gunduz, H. Cakalli, A. Sonmez, *AIP Conf. Proc. 3rd Internat. Conf. Anal. Appl. Math. (ICAAM)* 1759 (020099) (2016); doi: 10.1063/1.4959713.
- [12] B.H. Han, S.L. Geng, Pruning method for optimal solutions of int m-int n decision making scheme, *European J. Oper. Res.* 231 (2013) 779–783.
- [13] T. Herawan, R. Ghazali, M.M. Deris, Soft set theoretic approach for dimensionality reduction, *Intern. J. Database Theory Appl.* 3:2 (2010) 47–60.
- [14] Z. Li, T. Xie, The relationship among soft sets, soft rough sets and topologies, *Soft Comput.* 10.1007/s00500-013-1108-5 (2013).
- [15] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, *Comput. Math. Appl.* 45 (2003) 555–562.
- [16] P.K. Maji, A.R. Roy, An application of soft sets in a decision making problem, *Comput. Math. Appl.* 44 (2002) 1077–1083.
- [17] R. Mamat, T. Herawan, M.M. Deris, MAR: Maximum Attribute Relative of soft set for clustering attribute selection, *Knowledge-Based Syst.* 52 (2013) 11–20.
- [18] D. Molodtsov, Soft set theory: first result, *Comput. Math. Appl.* 37 (1999) 19–31.
- [19] J.I. Mondal, T.K. Roy, Theory of fuzzy soft matrix and its multi criteria in decision making based on three basic t -norm operators, *Intern. J. Innov. Res. Sci., Engin. Techn.* 2 (2013) 5715–5723.
- [20] S. Mondal, M. Pal, Soft matrices, *African J. Math. Comput. Sci. Res.* 4 (2011) 379–388.
- [21] A. Sezgin, A.O. Atagün, On operations of soft sets, *Comput. Math. Appl.* 61 (2011) 1457–1467.
- [22] A.K. Singh, P. Parida, Soft computing in financial decision making, *Global J. Manag. Business Stud.* 3 (2013) 103–110.
- [23] S. Vijayabalaji, A. Ramesh, A new decision making theory in soft matrices, *Intern. J. Pure Appl. Math.* 86 (2013) 927–939.
- [24] Z. Zhang, C. Wang, D. Tian, K. Li, A novel approach to interval-valued intuitionistic fuzzy soft set based decision making, *Appl. Math. Modell.*, in press.