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Periodic Solution of a Stochastic Non-Autonomous Lotka-Volterra Cooperative System with Impulsive Perturbations

Chuan Lv[a], Daqing Jiang[a,b,*], Ruihua Wu[a]

[a] *College of Science, China University of Petroleum (East China) Qingdao 266555, P.R. China*
[b] *Nonlinear Analysis and Applied Mathematics (NAAM)-Resaerch Group, king Abdulaziz University, Jeddah, Saudi Arabia*

Abstract. This paper is concerned with a stochastic non-autonomous Lotka-Volterra cooperative model with impulsive effects. The main purpose of this paper is to explore the existence of periodic solution of the system provided that the coefficients of the system are continuous periodic functions. By constructing appropriate Lyapunov functions and using the theory of Khasminskii, sufficient conditions under which the existence of the periodic solution of the system are obtained. Our results illustrate that the existence of the periodic solution has close relations with the white noise and the impulsive perturbations.

1. Introduction

In recent decades, stochastic Lotka-Volterra cooperative population systems driven by Brownian motion have received great attentions and have been studied a lot because of their importance in theory and practice [1–7]. The classical two-species non-autonomous stochastic Lotka-Volterra cooperative system is given by

$$\begin{cases} \mathrm{d}x_1(t) &= x_1(t)\Big[r_1(t) - a_{11}(t)x_1(t) + a_{12}(t)x_2(t)\Big]\mathrm{d}t + \sigma_1(t)x_1(t)\mathrm{d}B_1(t) \\ \mathrm{d}x_2(t) &= x_2(t)\Big[r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\Big]\mathrm{d}t + \sigma_2(t)x_2(t)\mathrm{d}B_2(t) \end{cases} \tag{1}$$

where $r_i(t)$ denotes the intrinsic growth rate of the ith population at time t, $a_{ij}(t)$ represents the action of species j upon the growth rate of species i at time t, particularly, $a_{ii}(t)$ stands for the intraspecific competition coefficient of the ith population at time t, $B_i(t)(i = 1, 2)$ are independent standard Brownian motions, $\sigma_i(t)$ is the coefficients of the effects of environmental stochastic perturbations on the population.

However, in the real world, the environment presents seasonal changes due to the factors such as seasonal effects of weather, mating habits, harvesting and so on. So it is reasonable and necessary to assume the periodicity of parameters in the systems. There are some results about periodic solutions with periodic coefficients system, see [8–13] and the references cited therein. As we have seen already, the research on the periodic solution of the stochastic system is not much, see [14].

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Corresponding author: Daqing Jiang
Email addresses: lvchuan501@163.com. (Chuan Lv), daqingjiang2010@hotmail.com. (Daqing Jiang), wu_ruihua@hotmail.com. (Ruihua Wu)

On the other hand, in the real world, due to some natural and man-made factors, the growth of species usually suffers some discrete changes of relatively short time interval at some fixed times, such as drought, harvesting, hunting etc. These phenomena can not be considered continually, so in this case, stochastic system (1) can not be suitable. In order to describe these phenomena more accurately, the impulsive effects should be taken into account [15, 16]. A variety of population dynamical systems with impulsive effects have been proposed and studied extensively, see [17–20].

Inspired by above discussions, in system (1), the impulse effects and seasonal changes are taken into account, we get the following system:

$$\begin{cases} \mathrm{d}x_1(t) &=& x_1(t)\Big[r_1(t) - a_{11}(t)x_1(t) + a_{12}(t)x_2(t)\Big]\mathrm{d}t + \sigma_1(t)x_1(t)\mathrm{d}B_1(t), \;\; t \neq t_k, k \in N, \\ \mathrm{d}x_2(t) &=& x_2(t)\Big[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\Big]\mathrm{d}t + \sigma_2(t)x_2(t)\mathrm{d}B_2(t), \;\; t \neq t_k, k \in N, \\ x_1(t_k^+) &-& x_1(t_k) = b_{1k}x_1(t_k), \;\; k \in N, \\ x_2(t_k^+) &-& x_2(t_k) = b_{2k}x_2(t_k), \;\; k \in N, \end{cases} \tag{2}$$

where $r_i(t), a_{ij}(t), \sigma_i(t)$ are all periodic functions on $\mathbb{R}_+ = [0, +\infty)$ with period T, N denotes the set of positive integers, $0 < t_1 < t_2 < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$. There exist $T > 0$ and $p \in N$ such that $t_{k+p} = t_k + T$, $b_{1k} = b_{1(k+p)}$, $b_{2k} = b_{2(k+p)}$, $[0, T) \cap \{t_k, k \in N\} = \{t_1, t_2, \cdots, t_p\}$. For biological meanings, we impose the following restriction on system (2):

$$1 + b_{ik} > 0, \;\; i = 1, 2, \;\; k \in N.$$

The impulse can be explained as follows: if $b_{ik} > 0$, the impulsive effects may denote the planting of the species, while $b_{ik} < 0$ may mean the harvesting.

The main aim of this paper is to investigate the existence of periodic solution of the system and to illustrate how the impulse affects the periodic solutions of the system. The rest of this paper is organized as follows. In Section 2, we show the existence of the positive solutions of system (2). In Section 3, we present sufficient conditions for the existence of the periodic solution.

2. Global positive solutions

Throughout this paper, we adopt the following notations. If $f(t)$ is a continuous periodic function with period T, define $f^u = \sup_{t\in[0,T]} f(t)$, $f^l = \inf_{t\in[0,T]} f(t)$, $\prod_{i=1,2} x(i) = x(1)x(2)$. Moreover, we assume that a product equals unity if the number of factors is zero.

In order to study the properties of the solutions to population system (2), we should guarantee the existence of the positive solution. We first give an equivalent theorem which can transfer system (2) with impulse into a system without impulse.

Theorem 2.1. *Consider the following system without impulse:*

$$\begin{cases} \mathrm{d}y_1(t) &=& y_1(t)\Big[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1 + b_{1k}) - a_{11}(t)A_1(t)y_1(t) + a_{12}(t)A_2(t)y_2(t)\Big]\mathrm{d}t + \sigma_1(t)y_1(t)\mathrm{d}B_1(t) \\ \mathrm{d}y_2(t) &=& y_2(t)\Big[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1 + b_{2k}) + a_{21}(t)A_1(t)y_1(t) - a_{22}(t)A_2(t)y_2(t)\Big]\mathrm{d}t + \sigma_2(t)y_2(t)\mathrm{d}B_2(t) \end{cases}. \tag{3}$$

If $(y_1(t), y_2(t))$ is a solution to system (3), then $(x_1(t), x_2(t)) = (A_1(t)y_1(t), A_2(t)y_2(t))$ is a solution to system (2) with the same initial value $(y_1(0), y_2(0)) = (x_1(0), x_2(0))$, where $A_i(t) = [\prod_{j=1}^{p}(1 + b_{ij})]^{-\frac{t}{T}} \prod_{t_k<t}(1 + b_{ik})$, $i = 1, 2$.

Proof. We can easily see that $x_i(t)$ is continuous on $(0, t_1)$ and each interval $(t_k, t_{k+1}) \subset [0, \infty)$, $k \in N$. For

$t \neq t_k$,

$$\begin{aligned} \mathrm{d}x_1(t) &= y_1(t)\mathrm{d}A_1(t) + A_1(t)\mathrm{d}y_1(t) \\ &= -\frac{1}{T}\sum_{j=1}^{p}\ln(1+b_{1j})A_1(t)y_1(t) + \sigma_1(t)A_1(t)y_1(t)\mathrm{d}B_1(t) \\ &\quad +A_1(t)y_1(t)\Big[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - a_{11}(t)A_1(t)y_1(t) + a_{12}(t)A_2(t)y_2(t)\Big]\mathrm{d}t \\ &= x_1(t)\Big[r_1(t) - a_{11}(t)x_1(t) + a_{12}(t)x_2(t)\Big]\mathrm{d}t + \sigma_1(t)x_1(t)\mathrm{d}B_1(t). \end{aligned}$$

Similarly, we have

$$\mathrm{d}x_2(t) = x_2(t)\Big[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t)\Big]\mathrm{d}t + \sigma_2(t)x_2(t)\mathrm{d}B_2(t).$$

And for every $k \in N$ and $t_k \in \mathbb{R}_+$,

$$\begin{aligned} x_i(t_k^+) &= \lim_{t\to t_k^+} x_i(t) = \lim_{t\to t_k^+} A_i(t)y_i(t) = \prod_{j=1}^{p}(1+b_{ij})]^{-\frac{t_k}{T}}(1+b_{ik})\prod_{t_j<t_k}(1+b_{ij})y_i(t_k) \\ &= (1+b_{ik})A_i(t_k)y_i(t_k) = (1+b_{ik})x_i(t_k). \end{aligned}$$

Moreover,

$$x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t) = \lim_{t\to t_k^-} A_i(t)y_i(t) = A_i(t_k)y_i(t_k) = x_i(t_k).$$

This completes the proof. □

Remark 2.2. *We can easily prove that the functions $A_i(t)$, $(i = 1, 2)$ are periodic functions with period T. In fact*

$$\frac{A_i(t+T)}{A_i(t)} = \frac{[\prod_{j=1}^{p}(1+b_{ij})]^{-\frac{t+T}{T}}\prod_{t_k<t+T}(1+b_{ik})}{[\prod_{j=1}^{p}(1+b_{ij})]^{-\frac{t}{T}}\prod_{t_k<t}(1+b_{ik})} = [\prod_{j=1}^{p}(1+b_{ij})]^{-1}\prod_{t\le t_k<t+T}(1+b_{ik}) = 1.$$

From the above equivalent theorem 2.1, we can conclude that we only need to consider the asymptotic properties of system (3), then system (2) has the similar properties.

In the following, we use the stochastic comparison theorem to obtain the existence of the positive solution for model (3).

Lemma 2.3. *Let $(y_1(t), y_2(t))$ be a positive solution of system (3), then $y_1(t) \geq \phi_1(t)$, $y_2(t) \geq \phi_2(t)$, where $\phi_i(t)$, $(i = 1, 2)$ is a solution of the following system:*

$$\begin{cases} \mathrm{d}\phi_i(t) &= \phi_i(t)\Big[r_i(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) - a_{ii}(t)\phi_i(t)\Big]\mathrm{d}t + \sigma_i(t)\phi_i(t)\mathrm{d}B_i(t) \\ \phi_i(0) &= y_i(0) \end{cases}, \quad i = 1, 2.$$

By the stochastic comparison theorem, we can easily reach the above result.

Lemma 2.4. *Suppose the parameters of system (3) satisfying $a_{11}^l a_{22}^l \geq a_{12}^u a_{21}^u$, then*

$$y_1^{c_1}(t)y_2^{c_2}(t) \leq y_1^{c_1}(0)y_2^{c_2}(0)e^{\int_0^t c(s)\mathrm{d}s + c_1\sigma_1(s)\mathrm{d}B_1(s) + c_2\sigma_2(s)\mathrm{d}B_2(s)},$$

where $c(t) := c_1[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{1}{2}\sigma_1^2(t)] + c_2[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{1}{2}\sigma_2^2(t)]$, *and* c_1, c_2 *satisfy*

$$\frac{a_{21}^u}{a_{11}^l} \leq \frac{c_1}{c_2} \leq \frac{a_{22}^l}{a_{12}^u}.$$

Proof. Applying Itô formula to system (3), we see that

$$\mathrm{d}\ln y_1(t) = \Big[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{1}{2}\sigma_1^2(t) - a_{11}(t)A_1(t)y_1(t) + a_{12}(t)A_2(t)y_2(t)\Big]\mathrm{d}t + \sigma_1(t)\mathrm{d}B_1(t),$$

$$\mathrm{d}\ln y_2(t) = \Big[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{1}{2}\sigma_2^2(t) + a_{21}(t)A_1(t)y_1(t) - a_{22}(t)A_2(t)y_2(t)\Big]\mathrm{d}t + \sigma_2(t)\mathrm{d}B_2(t).$$

Further, we have

$$\begin{aligned}
&\mathrm{d}[c_1\ln y_1(t) + c_2\ln y_2(t)]\\
= &\left\{c_1[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{1}{2}\sigma_1^2(t)] + c_2[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{1}{2}\sigma_2^2(t)]\right\}\mathrm{d}t\\
&-[c_1a_{11}(t) - c_2a_{21}(t)]A_1(t)y_1(t)\mathrm{d}t - [c_2a_{22}(t) - c_1a_{12}(t)]A_2(t)y_2(t)\mathrm{d}t + c_1\sigma_1(t)\mathrm{d}B_1(t) + c_2\sigma_2(t)\mathrm{d}B_2(t)\\
\leq &\left\{c_1[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{1}{2}\sigma_1^2(t)] + c_2[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{1}{2}\sigma_2^2(t)]\right\}\mathrm{d}t\\
&-[c_1a_{11}^l - c_2a_{21}^u]A_1(t)y_1(t)\mathrm{d}t - [c_2a_{22}^l - c_1a_{12}^u]A_2(t)y_2(t)\mathrm{d}t + c_1\sigma_1(t)\mathrm{d}B_1(t) + c_2\sigma_2(t)\mathrm{d}B_2(t)\\
\leq &\ c(t) + c_1\sigma_1(t)\mathrm{d}B_1(t) + c_2\sigma_2(t)\mathrm{d}B_2(t).
\end{aligned}$$

Then

$$y_1^{c_1}(t)y_2^{c_2}(t) \leq y_1^{c_1}(0)y_2^{c_2}(0)e^{\int_0^t c(s)\mathrm{d}s + c_1\sigma_1(s)\mathrm{d}B_1(s) + c_2\sigma_2(s)\mathrm{d}B_2(s)}$$

is obtained. □

By Lemma 2.4, we can arrive at the following lemma.

Lemma 2.5. *Let $a_{11}^l a_{22}^l \geq a_{12}^u a_{21}^u$ be hold, then*

$$y_1^{c_1}(t) \leq \phi_2^{-c_2}(t)y_1^{c_1}(0)y_2^{c_2}(0)e^{\int_0^t c(s)\mathrm{d}s + c_1\sigma_1(s)\mathrm{d}B_1(s) + c_2\sigma_2(s)\mathrm{d}B_2(s)},$$

$$y_2^{c_2}(t) \leq \phi_1^{-c_1}(t)y_1^{c_1}(0)y_2^{c_2}(0)e^{\int_0^t c(s)\mathrm{d}s + c_1\sigma_1(s)\mathrm{d}B_1(s) + c_2\sigma_2(s)\mathrm{d}B_2(s)}.$$

By the stochastic comparison theorem, Lemmas 2.3 and 2.5, we can reach the existence of the global positive solutions of system (3).

Theorem 2.6. *If the parameters of system (3) satisfy $a_{11}^l a_{22}^l \geq a_{12}^u a_{21}^u$, then system (3) has a unique global positive solution with positive initial value $(y_1(0), y_2(0))$.*

Remark 2.7. *Our condition for the existence of positive solutions of system (3) is $a_{11}^l a_{22}^l \geq a_{12}^u a_{21}^u$. However, the condition in references [21, 22] is $a_{11}^l a_{22}^l > a_{12}^u a_{21}^u$, which indicates that our condition is better.*

Theorem 2.8. *If the conditions $a_{11}^l a_{22}^l \geq a_{12}^u a_{21}^u$ and $c := \frac{1}{T}\int_0^T c(s)\mathrm{d}s < 0$ are satisfied, then system (3) is nonpersistent, in the sense that $\lim_{t\to\infty} y_1^{c_1}(t)y_2^{c_2}(t) = 0$, a.s.*

This result can be obtained by Lemma 2.5.

3. Existence of periodic solution

The study of the periodic solution is an important content of the dynamical system. In this part, we will seek conditions under which the considered periodic system would have a periodic solution. We first give the definition of the periodic solution under the meaning of distribution.

Definition 3.1 ([23]). *A stochastic process $\xi(t) = \xi(t,\omega)(-\infty < t < +\infty)$ is said to be periodic with period T if for every finite sequence of numbers $t_1, t_2, \cdots, t_n$, the joint distribution of random variables $\xi(t_1+h), \xi(t_2+h), \cdots, \xi(t_n+h)$ is independent of h, where $h = kT(k = \pm 1, \pm 2, \ldots)$.*

Consider the following equation

$$X(t) = X(t_0) + \int_{t_0}^{t} b(s, X(s))\mathrm{d}s + \sum_{r=1}^{k} \int_{t_0}^{t} \sigma_r(s, X(s))\mathrm{d}B_r(s), \tag{4}$$

we assume that the coefficients $b(s,x)$, $\sigma_1(s,x)$, $\sigma_2(s,x), \ldots, \sigma_k(s,x)$ satisfy the following conditions:

$$|b(s,x) - b(s,y)| + \sum_{r=1}^{k} |\sigma_r(s,x) - \sigma_r(s,y)| \le B|x-y|, \quad |b(s,x)| + \sum_{r=1}^{k} |\sigma_r(s,x)| \le B(1+|x|), \tag{5}$$

where B is a constant.

Lemma 3.2 ([23]). *Suppose that the coefficients of system (4) are T-periodic in t and satisfy condition (5) in every cylinder $I \times U$, further suppose that there exists a function $V(t,x) \in C^2$ of T-periodic in t satisfying the following conditions*

$$\inf_{|x|>R} V(t,x) \to \infty \quad as \quad R \to \infty, \tag{6}$$

$$LV(t,x) \le -1 \tag{7}$$

outside some compact set. Then there exists a solution of system (4) which is a T-periodic Markov process.

Remark 3.3. *From the proof the Lemma 3.2, we can see that conditions (5) only guarantee the existence and uniqueness of the solution of (4).*

Now we give main theorem of this section.

Theorem 3.4. *Let conditions $a_{11}^l a_{22}^l > a_{12}^u a_{21}^u$ and $\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t + \sum_{k=1}^{p} \ln(1+b_{ik}) > 0$, $(i = 1,2)$ be satisfied, then there exists a T-periodic solution of system (3).*

Proof. The existence and uniqueness of the solution of system (3) has been guaranteed by Theorem 2.6. According to Lemma 3.2, we only need to prove conditions (6) and (7) are satisfied. Define a C^2-function

$$V(y_1, y_2, t) = \bar{C}_1 \ln y_1 + e^{\omega_1(t)} y_1^{-\theta} + \bar{C}_2 \ln y_2 + e^{\omega_2(t)} y_2^{-\theta},$$

where the constants $\bar{C}_1$ and $\bar{C}_2$ satisfy $(a_{21}^u/a_{11}^l) < (\bar{C}_1/\bar{C}_2) < (a_{22}^l/a_{12}^u)$, $\omega_i(t) \in C^1(R^+, R)$ is a T-periodic function which will be determined in the following proof, and θ is a sufficiently small positive constant such that $\frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t + \frac{1}{T}\sum_{k=1}^{p} \ln(1+b_{ik}) - \frac{\theta}{2}(\sigma_i^u)^2 > 0$. We can easily deduce that $V(y_1, y_2, t) \to \infty$, when

$y_1 \to 0^+$, $y_2 \to 0^+$, $y_1 \to +\infty$, or $y_2 \to +\infty$, then condition (6) is satisfied. By Itô formula, we deduce that

$$\begin{aligned} LV &= -\theta e^{\omega_1(t)} y_1^{-\theta}[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{\theta+1}{2}\sigma_1^2(t) - \frac{1}{\theta}\omega_1'(t)] + \theta e^{\omega_1(t)} a_{11}(t)A_1(t)y_1^{1-\theta} \\ &\quad -\theta e^{\omega_1(t)} a_{12}(t)A_2(t)y_1^{-\theta}y_2 - \bar{C}_1 a_{11}(t)A_1(t)y_1 + \bar{C}_1 a_{12}(t)A_2(t)y_2 - \frac{\bar{C}_1}{2}\sigma_1^2(t) + \bar{C}_1[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k})] \\ &\quad -\theta e^{\omega_2(t)} y_2^{-\theta}[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{\theta+1}{2}\sigma_2^2(t) - \frac{1}{\theta}\omega_2'(t)] + \theta e^{\omega_2(t)} a_{22}(t)A_2(t)y_2^{1-\theta} \\ &\quad -\theta e^{\omega_2(t)} a_{21}(t)A_1(t)y_2^{-\theta}y_1 - \bar{C}_2 a_{22}(t)A_2(t)y_2 + \bar{C}_2 a_{21}(t)A_1(t)y_1 - \frac{\bar{C}_2}{2}\sigma_2^2(t) + \bar{C}_2[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k})] \\ &\le \bar{C}_1[r_1^u + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{1}{2}(\sigma_1^l)^2] + \bar{C}_2[r_2^u + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{1}{2}(\sigma_2^l)^2] \\ &\quad -\theta e^{\omega_1(t)} y_1^{-\theta}[r_1(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{1k}) - \frac{\theta+1}{2}\sigma_1^2(t) - \frac{1}{\theta}\omega_1'(t)] + \theta e^{\omega_1(t)} a_{11}(t)A_1(t)y_1^{1-\theta} \\ &\quad -\theta e^{\omega_2(t)} y_2^{-\theta}[r_2(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{2k}) - \frac{\theta+1}{2}\sigma_2^2(t) - \frac{1}{\theta}\omega_2'(t)] + \theta e^{\omega_2(t)} a_{22}(t)A_2(t)y_2^{1-\theta} \\ &\quad -(\bar{C}_1 a_{11}^l - \bar{C}_2 a_{21}^u)A_1^l y_1 - (\bar{C}_2 a_{22}^l - \bar{C}_1 a_{12}^u)A_2^l y_2 \end{aligned}$$

Let

$$\omega_i'(t) = \theta\left[r_i(t) - \frac{1}{2}\sigma_i^2(t) - \frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t\right].$$

Then $\omega_i(t)$ is a T-periodic function. In fact,

$$\begin{aligned} \omega_i(t+T) - \omega_i(t) &= \int_t^{t+T}\omega_i'(s)\mathrm{d}s = \theta\int_t^{t+T}[r_i(s) - \frac{1}{2}\sigma_i^2(s) - \frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t]\mathrm{d}s \\ &= \theta\int_t^{t+T}[r_i(s) - \frac{1}{2}\sigma_i^2(s)]\mathrm{d}s - \theta\int_0^T [r_i(t) - \frac{1}{2}\sigma_i^2(t)]\mathrm{d}t = 0. \end{aligned}$$

By the conditions of theorem, we have

$$\begin{aligned} & r_i(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) - \frac{\theta+1}{2}\sigma_i^2(t) - \frac{1}{\theta}\omega_i'(t) \\ = \; & r_i(t) + \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) - \frac{\theta+1}{2}\sigma_i^2(t) - r_i(t) + \frac{1}{2}\sigma_i^2(t) + \frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t \\ = \; & \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) + \frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t - \frac{\theta}{2}\sigma_i^2(t) \\ > \; & \frac{1}{T}\sum_{k=1}^{p}\ln(1+b_{ik}) + \frac{1}{T}\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t - \frac{\theta}{2}(\sigma_i^u)^2 > 0. \end{aligned}$$

Now, we can conclude that $LV \le -\infty$, when $y_1 \to 0^+$, $y_2 \to 0^+$, $y_1 \to +\infty$, or $y_2 \to +\infty$. Therefore, we can choose a sufficiently small $\varepsilon > 0$ such that $LV < -1$, when $(y_1, y_2) \in U_\varepsilon^c$, where

$$U_\varepsilon = \left\{(y_1, y_2) \in R_2^+, \varepsilon \le y_1 \le \frac{1}{\varepsilon}, \varepsilon \le y_2 \le \frac{1}{\varepsilon}\right\}.$$

□

Remark 3.5. *From the condition* $\int_0^T (r_i(t) - \frac{1}{2}\sigma_i^2(t))\mathrm{d}t + \sum_{k=1}^{p} \ln(1 + b_{ik}) > 0$, *we can see that the white noise and negative impulse are disadvantageous for the existence of the periodic solutions. However, the positive impulse are advantageous for the existence of the periodic solutions.*

4. Conclusions and further discussions

This paper is concerned with a stochastic non-autonomous Lotka-Volterra cooperative system with impulsive effects. This kind of model is more applicable. Our key contributions are as follows.

(a) We give the condition of the existence of the positive solutions for system 2 which is better than the existing results.

(b) The condition under which the population system is nonpersistent is estimated.

(c) We illustrate the conditions of the existence of the periodic solutions for the periodic system.

Some interesting topics deserve further investigation. In this paper, we consider the Lotka-Volterra cooperative system, the more general Lotka-Volterra system is also deserved to be studied. Moreover, one may consider the Lotka-Volterra system under regime switching.

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