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Introducing of an Orthogonally Relation for Stability of Ternary Cubic Homomorphisms and Derivations on C^*-Ternary Algebras

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Abstract. In this article, we introduce a kind of binary relation on a nonempty set with name of orthogonally relation which we develop for sequences, continuous maps, metric spaces, contraction maps, preserving maps and etc. All of the above concepts are generalized forms of ordinary case, so they are very important for extension and finding new results. we expect some of the concepts in the mathematics can be changed by orthogonally relation, such as functional equations and some of the theorem in the fixed point theorem method. In this research we illustrate one of the applications of orthogonally relation on ternary cubic homomorphism and ternary cubic derivations, so we prove the stability of orthogonally ternary cubic homomorphisms and orthogonally ternary cubic derivations on C^*-ternary algebras for the functional equation by using fixed point method. Also to create the stability, we choose a suitable control function and we show ability and validity of the proposed method for the functional analysis.

1. Introduction

The stability problem functional equations first had been raised by Ulam [27]. This problem solved by Hyers [14] in the framework of Banach spaces. In 1978, Th. M. Rassias [23] provided a generalization of the Heyrs theorem by proving the existence of unique linear mapping near approximate additive mapping. Lastly, Gajda [11] answered the question for another case of linear mapping, which was raised by Rassias. In 1982, J.M. Rassias [24] followed the innovative approach of the Rassias Theorem [23] in which he replaced the factor $||x||^p + ||y||^p$ by $||x||^p.||y||^q$ for $p, q \in \mathbb{R}$ with $p+q \neq 1$. Găvruta [12] obtained a generalized result of Th. M. Rassias theorem which allows the Cauchy difference to be controlled by a general unbounded function. For more details about the result concerning such problems, the reader to ([1–3, 8, 9, 25, 26]).

Park et al. proved stability homomorphisms and derivations in Banach algebras, Banach ternary algebras, C^*-algebras, Lie C^*-algebras, C^*-ternary algebras (see [7, 19–21]). Consider the functional equation

$$\begin{aligned} &f(x+y+2z) + f(x+y-2z) + f(2x) + f(2y) = \\ &\quad 2[f(x+y) + 2f(x+z) + 2f(x-z) + 2f(y+z) + 2f(y-z)]. \end{aligned} \tag{1}$$

The cubic function $f(x) = cx^3$ is a solution of this functional equation, and so one usually is said the above functional equation to be cubic [17]. Let $\mathcal{A}, \mathcal{B}$ are two ternary algebras. A mapping $f : \mathcal{A} \to \mathcal{B}$ is called a

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ternary cubic homomorphism if it is a cubic mapping (1) and satisfies
$f([x, y, z]) = [f(x), f(y), f(z)]$ for all $x, y, z \in \mathcal{A}$.

A mapping $f : \mathcal{A} \to \mathcal{A}$ is called a ternary cubic derivation if f is a cubic mapping (1) satisfies
$f([x, y, z]) = [f(x), y^3, z^3] + [x^3, f(y), z^3] + [x^3, y^3, f(z)]$ for all $x, y.z \in \mathcal{A}$.

In 2003, Cădariu and Radu applied the fixed point methods to the investigation of Jensen functional equations [4] (see also [5, 6, 13, 15, 16, 18, 22]).

We start our work with the following definitions, which can be considered as the main definition of our paper.

Definition 1.1. *Let $X \neq \emptyset$ and $\perp \subseteq X \times X$ be an binary relation. If $\perp$ satisfies the following condition*

$$\exists x_0; (\forall y; y \perp x_0) \ \ or \ \ (\forall y; x_0 \perp y),$$

it is called an orthogonally set (briefly O-set). We denote this O-set by $(X, \perp)$.

Definition 1.2. *Let $(X, \perp)$ be an O-set. A sequence $\{x_n\}_{n\in\mathbb{N}}$ is called orthogonally sequence (briefly O-sequence) if*

$$(\forall n; x_n \perp x_{n+1}) \ \ or \ \ (\forall n; x_{n+1} \perp x_n).$$

Definition 1.3. *Let $(X, \perp)$ be an O-set. Then $f : X \to X$ is $\perp-$ preserving if for each $x, y \in X$, $x \perp y$ then $f(x) \perp f(y)$.*

Definition 1.4. *Let $(X, \perp, d)$ be an orthogonally metric space ($(X, \perp)$ is an O-set and (X, d) is a metric space). Then $f : X \to X$ is $\perp-$continuous in $a \in X$ if for each O-sequence $\{a_n\}_{n\in\mathbb{N}}$ in X if $a_n \to a$, then $f(a_n) \to f(a)$. Also f is $\perp-$continuous on X if f is $\perp-$continuous on each $a \in X$.*

It is easy to see that every continuous mapping is $\perp-$continuous.

Definition 1.5. *Let $(X, \perp, d)$ be an orthogonally metric space, then X is orthogonally complete (briefly O-complete) if every Cauchy O-sequence is convergent.*

It is easy to see that every complete metric space is O-complete and the converse is not true.

Definition 1.6. *Let $(X, \perp, d)$ be an orthogonally metric space and $0 < \lambda < 1$. A mapping $f : X \to X$ is an orthogonality contraction with Lipschitz constant λ if*

$$d(fx, fy) \leq \lambda d(x, y) \quad if \ x \perp y.$$

Let H be a Hilbert space. Suppose that $f : H \to \mathbb{C}$ is a mapping satisfying

$$f(x) = ||x||^2 \tag{2}$$

for all $x \in X$. It is natural that this equation is a quadratic functional equation. On the other hand by considering $x \perp y$ with $< x, y >= 0$ for $x, y \in H$, it is easy to see that the above function $f : H \to \mathbb{C}$ is an orthogonally additive functional equation, that is $f(x+y) = f(x)+f(y)$ if $x \perp y$. This means that orthogonality may change a functional equation.

Recently, Eshaghi and Ramezani in [10] proved a fixed point theorem in O-sets as follows:

Theorem 1.7. *Let $(X, d, \perp)$ be an O-complete metric space (not necessarily complete metric space) and $0 < \lambda < 1$, Let $T : X \to X$ be a $\perp$-preserving, $\perp$-contraction with Lipschitz constant λ and $\perp$-continuous, then T has a unique fixed point x^* in X. Also, T is a Picard operator such that, $\lim_{n\to\infty} T^n(x) = x^*$ for all $x \in X$.*

Example 1.8. *Let $X = [0,1)$ and let the metric on X be the Euclidian metric. Define $x\perp y$ if $xy \in \{x, y\}$ for all $x, y \in X$. Let $f : X \to X$ be a mapping defined by*

$$f(x) = \begin{cases} \frac{x}{2} & if \ \ x \in \mathbb{Q} \cap X \\ 0 & if \ \ x \in \mathbb{Q}^c \cap X. \end{cases}$$

It is easy to see that X is O-complete (not complete), f is $\perp$-continuous (not continuous on X), $\perp$-λ-contraction for $\lambda = \frac{1}{2}$ (not contraction on X) and $\perp$-preserving on X. By our theorem, f has a unique fixed point. However, f is not a contraction on X and so by theorem Diaz and Margolis we cannot find any fixed point for f.

2. Main Results

Throughout this section, we suppose that $(A, \|.\|_1, \perp_1)$ with $a\perp_1 b$ if $ab^* = b^*a = 0$ and $(B, \|.\|_2, \perp_2)$ with $a\perp_2 b$ if $ab^* = b^*a = 0$ are two C^*-ternary algebras. Given a mapping $f : A \to B$, we set

$$\begin{aligned}\Delta f(x, y, z) =& f(x + y + 2z) + f(x + y - 2z) + f(2x) + f(2y) \\ & - 2[f(x + y) + 2f(x + z) + 2f(x - z) + 2f(y + z) + 2f(y - z)]\end{aligned}$$

for all $x, y, z \in A$.

Theorem 2.1. *Let $f : A \to B$ be a mapping for which there exists a control function $\varphi : A^3 \to [0,\infty)$ such that $\varphi(0, 0, 0) = 0$ and*

$$\|\Delta f(x, y, z)\|_2 \le \varphi(x, y, z), \tag{3}$$

$$\|f([x, y, z]) - [f(x), f(y), f(z)]\|_2 \le \varphi(x, y, z) \tag{4}$$

for all $x, y, z \in A$ that are mutually orthogonal. If there exists $L < 1$ such that

$$\varphi(x, y, z) \le 8L\varphi(\frac{x}{2}, \frac{y}{2}, \frac{z}{2}) \tag{5}$$

for all $x, y, z \in A$ that are mutually orthogonal. Then there exists a unique orthogonally C^*-ternary cubic homomorphism $H : A \to B$ such that

$$\|H(x) - f(x)\|_2 \le \frac{1}{8(1 - L)}\varphi(0, x, 0) \tag{6}$$

Proof. Putting $x = z = 0$ in (3) and by using of the $\varphi(0, 0, 0) = 0$, we get

$$\|f(2y) - 8f(y)\|_2 \le \varphi(0, y, 0)$$

for all $y \in A$. Hence we have

$$\|\frac{1}{8}f(2y) - f(y)\|_2 \le \frac{1}{8}\varphi(0, y, 0) \tag{7}$$

for all $y \in A$.

Consider the set

$$\Omega := \{g : \ g : A \to B, \ g(x) \perp_2 2^{-3}g(2x) \ or \ 2^{-3}g(2x) \perp_2 g(x), \forall x \in A, \ g(0) = 0\}. \tag{8}$$

For every $g, h \in \Omega$, define

$$d(g, h) = inf\{K \in (0,\infty) : \ \ \|g(x) - h(x)\|_2 \le K\varphi(0, x, 0), \ \ \forall x \in A\}. \tag{9}$$

Now, we put the orthogonality relation $\perp$ on Ω as follows: for all $g, h \in \Omega$

$$h \perp g \quad \Leftrightarrow \quad (h(x) \perp_2 g(x) \;\; or \;\; g(x) \perp_2 h(x) \;\; \forall x \in A). \tag{10}$$

Because A and B are C^*-ternary algebras so, we conclude that $g_0(x) = 0 \in \Omega$ and $\forall\ g(x) \in \Omega$ then $g(x) \perp g_0(x)$ so, $(\Omega, \perp)$ is an O-set. By Definition of (9) we find (Ω, d) is an orthogonally metric space. Also, if $\{g_n\}_{n\in\mathbb{N}}$ be a Cauchy O-sequence, for $\varepsilon > 0$ and enough small by concept of Cauchy sequence, we obtain $\|g_m(x) - g_n(x)\|_2 = 0\ \forall m, n > n_0, n_0 \in \mathbb{N}$ so, $\{g_n\}_{n\in\mathbb{N}}$ is convergent. Thus $(\Omega, d, \perp)$ is an O-complete generalized metric space.

Now, we consider the mapping $T : \Omega \to \Omega$ defined by $Tg(x) = 2^{-3}\ g(2x)$ for all $x \in A$ and $g \in \Omega$. Let $g, h \in \Omega$ and $g \perp h$ by definition Ω in (8), we can write

$$\begin{aligned} g \perp h \ &\Leftrightarrow g(x) \perp_2 h(x) \;\; or \;\; h(x) \perp_2 g(x), \forall x \in A \\ &\Longrightarrow g(2x) \perp_2 h(2x) \;\; or \;\; h(2x) \perp_2 g(2x), \forall x \in A \\ &\Longrightarrow 2^{-3}g(2x) \perp_2 2^{-3}h(2x) \;\; or \;\; 2^{-3}h(2x) \perp_2 2^{-3}g(2x), \forall x \in A \\ &\Longrightarrow Tg(x) \perp_2 Th(x) \;\; or \;\; Th(x) \perp_2 Tg(x), \forall x \in A \\ &\Longrightarrow Tg \perp Th. \end{aligned}$$

Thus T is a $\perp$-preserving.

For all $g, h \in \Omega$ with $g \perp h$ and $x \in A$,

$$\begin{aligned} d(g, h) < K \ &\Rightarrow \ \|g(x) - h(x)\|_2 \le K\varphi(0, x, 0) \\ &\Rightarrow \ \|2^{-3}g(2x) - 2^{-3}h(2x)\|_2 \le 2^{-3}\ K\ \varphi(0, 2x, 0) \\ &\Rightarrow \ \|2^{-3}g(2x) - 2^{-3}h(2x)\|_2 \le L\ K\ \varphi(0, x, 0) \\ &\Rightarrow \ d(Tg, Th) \le L\ K. \end{aligned}$$

Hence we see that

$$d(Tg, Th) \le L\ d(g, h) \tag{11}$$

for all $g, h \in \Omega$, that is, T is a strictly contractive self-mapping of Ω with the Lipschitz constant L. Now, we show that T is a $\perp$-continuous. To this end, let $\{g_n\}_{n\in\mathbb{N}}$ be an O-sequence with $g_n \perp g_{n+1}$ or $g_{n+1} \perp g_n$ for all $n \in \mathbb{N}$ in $(\Omega, d, \perp)$ which convergent to $g \in \Omega$ and let $\epsilon > 0$ be given. Then there exists $N \in \mathbb{N}$ and $K \in R^+$ with $K < \epsilon$ such that

$$\|g_n(x) - g(x)\|_2 \le K\varphi(0, x, 0)$$

for all $x \in A$ and $n \ge N$ and so

$$\|2^{-3}g_n(2x) - 2^{-3}g(2x)\|_2 \le 2^{-3}\ K\ \varphi(0, 2x, 0)$$

for all $x \in A$ and $n \ge N$. By inequality (5) and the definition of T, we get

$$\|Tg_n(x) - Tg(x)\|_2 \le L\ K\ \varphi(0, x, 0)$$

for all $x \in A$ and $n \ge N$. Hence

$$d(T(g_n), T(g)) \le LK < K < \epsilon$$

for all $n \ge N$. It follows that T is $\perp$-continuous. So according to the Theorem 1.7, T has a unique fixed point and $T^n(f)$ is convergent to unique fixed point of T, such as $H : A \to B$ that it can be defined by

$$H(x) = \lim_{n\to\infty} \frac{f(2^n x)}{8^n} \tag{12}$$

for all $x \in A$. It's easy to show that $T(H) = H$. In the follow by (12) and (7) we have,

$$\begin{aligned}
\|H(x) - f(x)\|_2 &= \| \lim_{n\to\infty} \frac{f(2^n x)}{8^n} - f(x)\|_2, \forall x \in A \\
&\leq \lim_{n\to\infty} \|\frac{f(2^n x)}{8^n} - \frac{f(2^{n-1}x)}{8^{n-1}}\|_2 + \lim_{n\to\infty} \|\frac{f(2^{n-1}x)}{8^{n-1}} - \frac{f(2^{n-2}x)}{8^{n-2}}\|_2 + \cdots + \lim_{n\to\infty} \|\frac{f(2x)}{8} - f(x)\|_2 \\
&\leq \lim_{n\to\infty} \frac{1}{8^{n-1}} \frac{1}{8} \varphi(0,\, 2^{n-1}x,\, 0) + \lim_{n\to\infty} \frac{1}{8^{n-2}} \frac{1}{8} \varphi(0,\, 2^{n-2}x,\, 0) + \cdots + \lim_{n\to\infty} \frac{1}{8} \varphi(0,\, 2^0 x,\, 0) \\
&= \lim_{n\to\infty} \frac{1}{8} \sum_{j=0}^{n-1} \frac{1}{8^j} \varphi(0,\, 2^j x,\, 0)
\end{aligned}$$

by using (5) rapidly we obtain,

$$\begin{aligned}
&\leqslant \lim_{n\to\infty} \frac{1}{8} \sum_{j=0}^{n-1} L^j \varphi(0,\, x,\, 0) \\
&= \frac{1}{8(1-L)} \varphi(0,\, x,\, 0).
\end{aligned}$$

Now, we show that H is an orthogonally cubic. From (5) we get

$$\lim_{n\to\infty} \frac{1}{8^n} \varphi(2^n x, 2^n y, 2^n z) = 0. \tag{13}$$

Then it follows from (12) and (3) that

$$\|\Delta H(x,y,z)\|_2 = \lim_{n\to\infty} \|\frac{\Delta f(2^n x, 2^n y, 2^n z)}{8^n}\|_2 \leq \lim_{n\to\infty} \frac{\varphi(2^n x, 2^n y, 2^n z)}{8^n} = 0.$$

for all $x, y, z \in A$ that are mutually orthogonal. This shows that H is orthogonally cubic. Also from (12) and (4) we have

$$\begin{aligned}
&\|H([x,y,z]) - [H(x), H(y), H(z)]\|_2 \\
&= \lim_{n\to\infty} \frac{1}{8^{3n}} \|f([2^n x, 2^n y, 2^n z]) - [f(2^n x), f(2^n y), f(2^n z)]\|_2 \\
&\leq \lim_{n\to\infty} \frac{1}{8^n} \varphi(2^n x, 2^n y, 2^n z) = 0
\end{aligned}$$

for all $x, y, z \in A$ that are mutually orthogonal. This shows that H is orthogonally cubic C^*-ternary homomorphism. □

Corollary 2.2. *Let $p \in (0,3)$ and $\epsilon \in [0,\infty)$. Suppose that $f : A \to B$ is a mapping such that*

$$\|\Delta f(x,y,z)\|_2 \leq \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p), \tag{14}$$

$$\|f([x,y,z]) - [f(x), f(y), f(z)]\|_2 \leq \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p) \tag{15}$$

for all $x, y, z \in A$ that are mutually orthogonal. Then there exists a unique orthogonally C^*-ternary cubic homomorphism $H : A \to B$ such that

$$\|H(x) - f(x)\|_2 \leq \frac{\epsilon}{8 - 2^p} \|x\|_1^p$$

for all $x \in X$.

Proof. According to the Theorem 2.1 we Set $\varphi : A^3 \to [0,\infty)$ such that $\varphi(x,y,z) = \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p)$ for all $x,y,z \in A$ that are mutually orthogonal. Then it follows that $\varphi(0,0,0) = 0$ and

$$\begin{aligned}\exists L < 1; \quad \varphi(x,y,z) &= \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p) \\ &= 2^p\epsilon(\|\frac{x}{2}\|_1^p + \|\frac{y}{2}\|_1^p + \|\frac{z}{2}\|_1^p) \\ &= 8(2^{p-3})\epsilon(\|\frac{x}{2}\|_1^p + \|\frac{y}{2}\|_1^p + \|\frac{z}{2}\|_1^p) \\ &= 8(2^{p-3})\varphi(\frac{x}{2},\frac{y}{2},\frac{z}{2}) \\ &= 8L\varphi(\frac{x}{2},\frac{y}{2},\frac{z}{2}); \quad L = 2^{p-3} < 1.\end{aligned}$$

Also,

$$\begin{aligned}\lim_{n\to\infty}\frac{1}{8^n}\varphi(2^nx,2^ny,2^nz) &= \lim_{n\to\infty}\frac{\epsilon}{8^n}(\|2^nx\|_1^p + \|2^ny\|_1^p + \|2^nz\|_1^p) \\ &= \lim_{n\to\infty}(2^{p-3})^n\epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p) \\ &= \lim_{n\to\infty}L^n\epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p) \to 0; \quad L = 2^{p-3} < 1.\end{aligned}$$

Thus, conditions of Theorem 2.1 are held so, there exists a unique orthogonally C^*-ternary cubic homomorphism $H : A \to B$ such that from (6) we have

$$\|H(x) - f(x)\|_2 \le \frac{1}{8(1-L)}\varphi(0,x,0) = \frac{\epsilon}{8-2^p}\|x\|_1^p.$$

□

Theorem 2.3. *Let $f : A \to A$ be a mapping for which there exists a function $\varphi : A^3 \to [0,\infty)$ such that*

$$\|\Delta f(x,y,z)\|_2 \le \varphi(x,y,z), \tag{16}$$

$$\|f([x,y,z]) - [f(x),y^3,z^3] - [x^3,f(y),z^3] - [x^3,y^3,f(z)]\|_2 \le \varphi(x,y,z) \tag{17}$$

for all $x,y,z \in A$ that are mutually orthogonal. If there exists $L < 1$ such that

$$\varphi(x,y,z) \le 8L\varphi(\frac{x}{2},\frac{y}{2},\frac{z}{2}) \tag{18}$$

for all $x,y,z \in A$ that are mutually orthogonal. Then there exists a unique orthogonally C^*-ternary cubic derivation mapping $D : A \to A$ such that

$$\|D(x) - f(x)\|_2 \le \frac{1}{8(1-L)}\varphi(0,x,0) \tag{19}$$

for all $x \in X$.

Proof. By the reasoning as that in the proof Theorem 2.1, there exists a unique orthogonally cubic ternary derivation mapping $D : A \to A$ satisfying (19). The mapping $D : A \to A$ is given by $D(x) := \lim_{n\to\infty} 2^{-3n}f(2^nx)$ for all $x \in A$. It follows from (16),

$$\begin{aligned}&\|D([x,y,z]) - [D(x),y^3,z^3] - [x^3,D(y),z^3] - [x^3,y^3,D(z)]\|_2 \\ &= \lim_{n\to\infty}\frac{1}{8^{3n}}\|f(2^n[x,y,z]) - [f(2^nx),y^3,z^3] - [x^3,f(2^ny),z^3] - [x^3,y^3,f(2^nz)]\|_2 \\ &\le \lim_{n\to\infty}\frac{1}{8^n}\varphi(2^nx,2^ny,2^nz) = 0\end{aligned}$$

for all $x,y,z \in A$ that are mutually orthogonal. This show that D is orthogonally C^*-ternary cubic derivation. □

Theorem 2.4. *Let $p \in (0,3)$ and $\epsilon \in [0,\infty)$. Suppose that $f : A \to A$ is a mapping such that*

$$\|\Delta f(x,y,z)\|_2 \leq \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p),$$

$$\|f([x,y,z]) - [f(x),y^3,z^3] - [x^3,f(y),z^3] - [x^3,y^3,f(z)]\|_2 \leq \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p)$$

for all $x,y,z \in A$ that are mutually orthogonal. Then there exists a unique orthogonally C^*-ternary cubic derivation $D : A \to A$ such that

$$\|D(x) - f(x)\|_2 \leq \frac{\epsilon}{8-2^p}\|x\|_1^p$$

for all $x \in X$.

Proof. Set $\varphi(x,y,z) = \epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p)$ for all $x,y,z \in A$ Then it follows that $\varphi(0,0,0) = 0$ and

$$\frac{\varphi(2^n x, 2^n y, 2^n z)}{8} = (2^n)^{p-3}\epsilon(\|x\|_1^p + \|y\|_1^p + \|z\|_1^p) \to 0$$

as $n \to \infty$. Let $L = 2^{p-3}$ in Theorem 2.3. Then we get the desired result. □

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