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Regularized Level Set Models Using Fuzzy Clustering for Medical Image Segmentation

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Abstract. Level set methods are a kind of general numerical analysis tools that are specialized for describing and controlling implicit interface dynamically. It receives widespread attention in medical image computing and analysis. There have been a lot of level set models designed and regularized for medical image segmentation. For the sake of simplicity and clarity, we merely concentrate on our recent works of regularizing level set methods with fuzzy clustering in this paper. It covers two most famous level set models, namely Hamilton-Jacobi functional and Mumford-Shah functional, for variational segmentation and region competition respectively. The strategies of fuzzy regularization are elaborated in detail and their applications in medical image segmentation are demonstrated with examples.

1. Introduction

Level set methods are a computational framework defined for computing and analyzing dynamic interface with partial differential equations (PDEs) [1]. One of the salient points is to represent a dynamic interface as the zero level set of a higher-dimensional function. The following motion of the dynamic interface can be solved according to a velocity field that possibly comes from interface geometry and external forces [2]. It is possible to restore or capture the implicit interface anytime as the zero level set. Different from active contour methods, level set methods are convenient to accommodate topological merging and breaking, which is often necessary for computer vision and image processing, in particular segmentation.

There should one or several interfaces (e.g., contours in two dimensions and surfaces in three dimensions) defined and adapted for image segmentation.It is possible to settle the initial interfaces artificially or by referring to some kinds of statistic information of an image. Then the interfaces could be directed towards the targeted contours or surfaces adaptively by taking the image information into account. The information could be image intensity, gradient, texture and even other kinds of more advanced statistical features [3].

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Coming to medical image segmentation, level set methods confront a series of new challenges, for example, partial volume effect, intertwining physiological organs and inhomogeneous pathological tissues [4],[5]. In other words, it is usually more difficult than the issues of general image analysis [6]. Therefore, the general level set models (LSMs) have to be regularized in accordance with different medical applications [5],[7]. For example, it is possible to introduce wavelet analysis into level set methods for noise suppression and multi-scale level set segmentation [7]. In this paper, we concentrate on our recent works of regularizing level set models with fuzzy clustering for medical image segmentation [4],[5],[8],[9].

The remaining parts of this paper are organized as follows. Section 1 elaborates the related level set models and the strategies of fuzzy regularization in detail. Some of selected medical applications will be introduced in Section 2. The following sections are for discussion and conclusion.

2. Level Set Models with Fuzzy Regularization

One of the most salient features of level set methods is the expression of dynamic interface implicitly by embedding it into a higher-dimensional Lipschitz function:

$$\phi(R,t) = \begin{cases} < 0, & \text{for } R \in \Omega^- \\ = 0, & \text{for } R \in \Omega \\ < 0, & \text{for } R \in \Omega^+ \end{cases} \tag{1}$$

where R denotes a spatial vector, namely (x,y) in two dimensions or (x,y,z) in three dimensions, Φ is an interface separating the domain Ω and evolves following the increment t. It is convenient to recover the implicit interface in a lower dimension by checking $\Phi(R, t = T) = 0$.

2.1. Level Set Models for Image Segmentation

The evolution of a dynamic interface in level set methods is totally determined by geometrical PDEs. The classical Hamilton-Jacobi functional characterizes the interface evolution as

$$\begin{cases} \frac{\partial \phi}{\partial t} + F \cdot \left|\nabla \phi\right| = 0 \\ \phi(R, t = 0) = \phi_0(R) \end{cases} \tag{2}$$

where ∇ denotes geometric gradients, $\phi_0(R)$ the beginning interface, and F the velocity field that is possibly comprised of interface smoothness and curvature, image intensity and gradient, and/or artificial balloon forces. The famous geodesic active contour model for image segmentation is as

$$\frac{\partial \phi}{\partial t} = (\beta\kappa + v)g\left|\nabla\phi\right| \tag{3}$$

where v denotes a constant balloon force advancing the dynamic interface, κ is the mean curvature, β controls the interface to expand ($\beta < 0$) or shrink ($\beta > 0$), and g is an image-dependent indication function. It is of vital importance in Hamilton-Jacobi functional for segmentation: The indication function should be positive, but approach zero in the boundary of interested objects.

The Mumford-Shah functional follows a totally different framework:

$$F^{MS}(u, \Phi) = \mu \cdot Length(\Phi) + \lambda \int \left|\omega - u\right|^2 dR + \int_{\Omega\backslash\Phi} \left|\nabla u\right|^2 dR \tag{4}$$

where u is a piecewise smooth function. This model can be optimized by a level set formulation for homogeneity-based image segmentation:

$$\begin{cases} \frac{\partial \phi}{\partial t} = \delta(\phi)[\mu \cdot div(\frac{\nabla\phi}{|\nabla\phi|}) - \lambda_1(\omega - c_1)^2 + \lambda_2(\omega - c_2)^2] \\ \phi(R, t = 0) = \phi_0(R) \end{cases} \tag{5}$$

where μ, λ_1 and λ_2 are pre-defined controlling parameters, c_1 and c_2 characterize region homogeneity.

2.2. Fuzzy Clustering

It is necessary for medical image segmentation to introduce fuzzy membership functions because physiological tissues are often intertwining and not homogeneous. Fuzzy clustering follows a cost function for optimization

$$J = \sum_{n=1}^{N} \sum_{m=1}^{C} \mu_{mn}^{l} \left\| i_n - v_m \right\|^2 \tag{6}$$

where N is the amount of image pixels in, C the desired number of clusters v_m, and l controls fuzziness. The membership functions μ_{mn} are subject to:

$$\sum_{m=1}^{C} \mu_{mn} = 1; \qquad 0 \leq \mu_{mn} \leq 1; \qquad \sum_{n=1}^{N} \mu_{mn} > 0 \tag{7}$$

They are updated iteratively together with the centroids v_m:

$$\mu_{mn} = \frac{\left\| i_n - v_m \right\|^{-2/(l-1)}}{\sum_{k=1}^{C} \left\| i_n - v_k \right\|^{-2/(l-1)}}; \tag{8}$$

$$v_i = \frac{\sum_{n=1}^{N} \mu_{mn}^{l} i_n}{\sum_{n=1}^{N} \mu_{mn}^{l}} \tag{9}$$

The optimization is achieved when similar objects have large membership values and the different ones have small values.

2.3. Regularized Level Set Models

As mentioned above, LSMs are a kind of general analyzing tools for image segmentation. The Hamilton-Jacobi functional is suitable for picking up any selected local object, while the Mumford-Shah functional is competitive for region segmentation without edge. However, it is noteworthy that the performance of the Hamilton-Jacobi level set models (HJ-LSMs) often suffers from weak boundary and that of the Mumford-Shah level set models (MS-LSMs) is sensitive to inhomogeneity. Meanwhile, the latter is not able to select the specific objects of interest.

In order to enhance HJ-LSMs against weak boundary, a new HJ-LSM was proposed in [4] by taking advantage of fuzzy clustering for image segmentation

$$\begin{cases} \xi(g, \phi) = \lambda \delta(\phi) div(g \frac{\nabla \phi}{|\nabla \phi|}) + g \cdot G(S_k) \cdot \delta(\phi) \\ \phi_0(x, y) = -4\varepsilon[0.5 - (S_k \geq b_0)] \end{cases} \tag{10}$$

where λ, ε and b_o are controlling constants, $S_k\{S_k(x, y) = \mu_{nk}\}$ is the component of interest in fuzzy clustering, $\delta()$is a Dirac function or the derivative of the Heaviside function, g is a standard object indication function [1], and $G()$ is a balloon force

$$G(S_k) = 1 - 2S_k \tag{11}$$

It is contributive to image segmentation in that, no matter outside, inside or partially, the initial interface evolves in accordance with a variable pulling or pushing force at each spatial position. As a consequence, this fuzzy-regularized HJ-LSM is advantageous in that it is initialized nearby the object of interest and in particular, controlled by a self-adaptive balloon force.

Coming to the Mumford-Shah functional, a new model was proposed in [8] for selective segmentation. It is also able to deal with inhomogeneity in some sense:

$$\begin{cases} \frac{\partial \phi}{\partial t} = \delta(\phi)[\alpha E \cdot G + (1 - \alpha) R] \\ \phi_0 = 2(\sum \mu_s > b_0) - 1 \end{cases} \tag{12}$$

where α is a coordinating constant, μ_s denote the selected fuzzy object of interest, G is the self-adaptive balloon force as in (11), E is an enhanced object indication function

$$E = e^{-10max(\eta \cdot g_i,(1-\eta)g_\mu)} \tag{13}$$

and R is the force of fuzzy region competition

$$R = \sum\nolimits_{s \in S} \mu_s - \sum\nolimits_{(j \in J) \cap (j \notin S)} \mu_j \tag{14}$$

Benefitted from preparatory fuzzy clustering, this new MS-LSM is able to detect and track the arbitrary combination of selected components of interest.

The above fuzzy MS-LSM (12) is similar to the unified LSM proposed in [5]

$$\frac{\partial \phi}{\partial t} = \alpha \delta(\phi) E \cdot (\kappa + \lambda G) + (1 - \alpha) \delta(\phi) R \tag{15}$$

where κ denotes the curvature of the dynamic interface. The only difference is that (15) is limited to a specific probability of interest but (12) accommodates the arbitrary combination of interested probabilities.

3. Selected Applications in Medical Image Segmentation

Medical image segmentation is different from the general analysis of computer vision. It is featured with intertwining tissues, inhomogeneous organs and weak boundaries between them. Without fuzzy regularization, neither HJ-LSMs nor MS-LSMs can obtain the acceptable performance of segmentation. Till now the three fuzzy LSMs in the preceding section have been successfully applied to different issues of medical image segmentation. Two of them are briefly introduced here for demonstration.

3.1. *Liver Tumor Segmentation*

Considering the diversified shape, intensity and texture, it remains challenging to segment liver and tumor from contrast-enhanced CT images. HJ-LSMs are usually susceptible to the weak boundaries of liver and tumors, while MS-LSMs may suffer from the images with low contrast and field inhomogeneity. The unified LSM (15) provides a comprehensive and flexible platform to overcome the abovementioned drawbacks. As shown in Figure 1, it is able to detect liver tumors despite their weak boundaries and low contrast.

This unified model has been evaluated on four 3D CT scans with 10 tumors. As pointed out in [11], the fuzzy classification supervised method achieved a state-of-the-art error rate 32.6%. In contrast, this method attained the error rate 26.3%, and thus was competitive at the time of publication [5].

3.2. *Segmenting Magnetic Resonance Elastography*

Magnetic resonance elastography (MRE) is an emerging technique for medical imaging. Instead of physiological anatomy, it quantifies soft tissue elasticity. However, the interpretation of elastograms is often confusing and cumbersome [10]. The experience-dependent practice often suffers from intrapersonal and interpersonal variability. A new LSM with alternating global and local region competitions is thus proposed for MRE elastogram analysis in [9].

As shown in Figure 2, both the magnitude image and the reconstructed elastogram are inhomogeneous and have weak boundaries. It is pretty difficult to attain acceptable segmentation in either of them. However, after enhancing with fuzzy clustering, the hybrid LSM is able to remodel the elastogram with piecewise constant level sets by referring to the corresponding magnitude image. The resulting elastograms are obviously friendlier to quantification and interpretation.

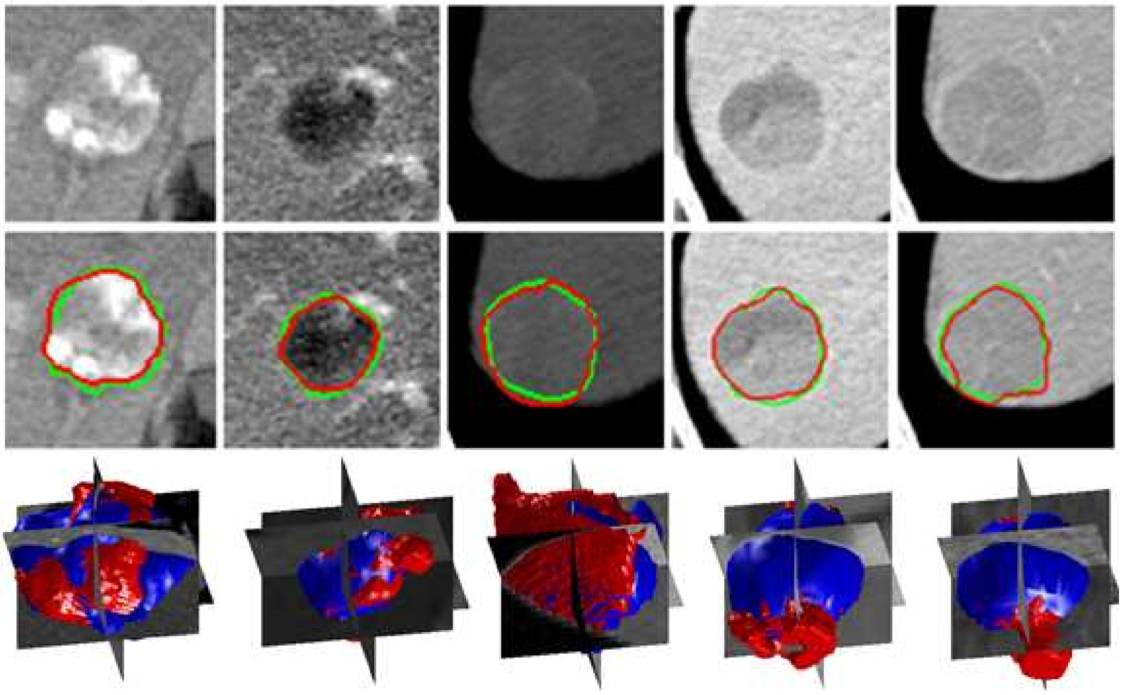

Figure 1: Liver tumor segmentation using a unified LSM regularized with fuzzy clustering.

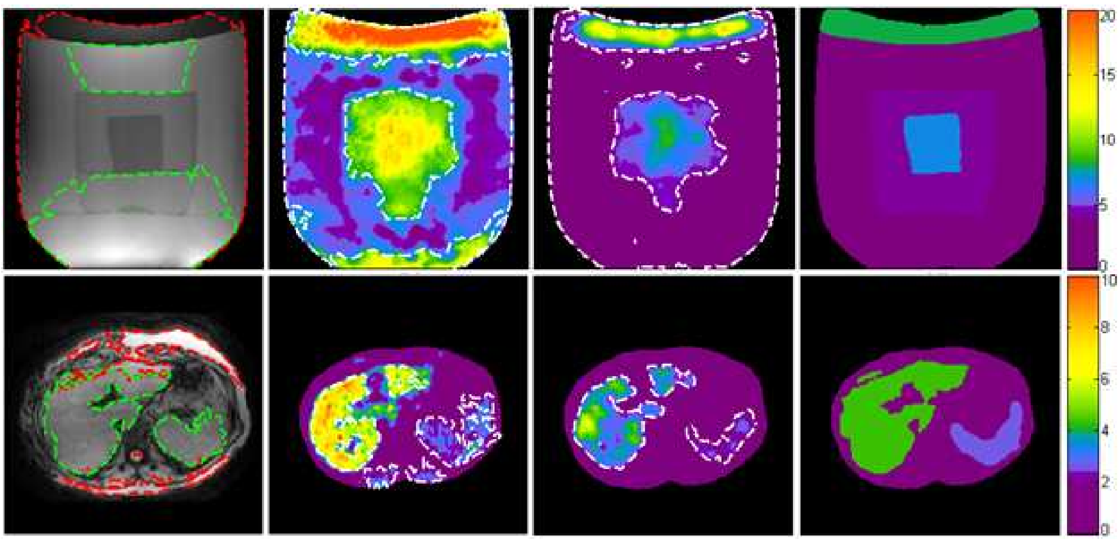

Figure 2: Segmenting and remodeling MRE with a hybrid LSM.

4. Discussion and Conclusion

Level set methods are universally applicable to image segmentation, but confront unique challenges in medical image segmentation. A series of new strategies are proposed to enhance LSMs by using fuzzy clustering in order to cope with the challenges. They have been quantitatively or qualitatively evaluated against some advanced methods including but not limited to [11]-[13]. Overall speaking, the advantages are benefitted from: (1) fuzzy clustering makes the initial interface nearby the objects of interest; (2) the fuzzified object indication function is better to characterize ambiguous physiological and pathological tissues; (3) it leads to a balloon force with locally-adaptive pulling or pushing effects; (4) the probabilities from fuzzy clustering are contributive to enhance region and even combinational competitions. Fortunately, level set methods provide a flexible while powerful framework to integrate them together for optimal medical image segmentation.

Some of the regularized LSMs have been publically shared in the Matlab Central. They are freely accessible via:

- http://www.mathworks.com/matlabcentral/fileexchange/31068-spatial-fuzzy-clustering-and-level-set-segmentation
- http://www.mathworks.com/matlabcentral/fileexchange/59171-selective-level-set-segmentation-using-fuzzy-region-competition

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