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Hesitant fuzzy linguistic two-sided matching decision making

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Abstract. This paper combines the theory of hesitant fuzzy linguistic term sets (HFLTSs) with two-sided matching decision making (TSMDM). The related definitions of HFLTSs and two-sided matchings (TSMs) are introduced. Then, the problem of TSMDM with HFLTSs is presented. For solving this problem, a model of TSMDM with HFLTSs is developed. The AHP method is used to determine the important degrees of agents of each side. On this base, the model of TSMDM can be changed into a double-goal model with HFLTSs. Then, the double-goal model with HFLTSs is changed into the double-goal model with scores through using the proposed score function. Furthermore, the double-goal model can be changed into a single-goal model by using the linear weighting technique once again. The scheme of TSM can be obtained through solving the single-goal model. At last, an example with sensitive analysis is provided for the illustration of the presented approach of TSM.

1. Introduction

The two-sided matching decision making (TSMDM) is about how to match the agents of two sides according to their own preferences. There exist a large number of TSMDM problems in reality, such as taxi dispatching and stable marriage [1, 2], college admissions [3], job matchings [4], and staff designation [5]. Hence, the TSMDM is a hot theme with far-ranging actual backgrounds.

Gale and Shapley [6] first study the problems of college admissions and the stability marriage. From then on, various different concepts, theories and algorithms have been presented with respect to TSMDM. For instance, Castillo and Dianat [7] study the truncation strategies in a centralized matching clearing house. Xu et al. [8] propose the matching algorithms for one-to-one two-sided dynamic service markets. Chen et al. [9] point out that the generalized median stable matchings are existed for the problem of many-to-many TSM. Liang et al. [10] give a new approach for solving the multi-objective satisfied and stable TSMDM problem.

The previous researches improve the theory of TSMDM, and develop the different algorithms for TSMDM with various formats of information, and expand the actual application background. However,

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a lot of real-world TSMDM problems are in qualitative environment. It is unrealistic for the agents to provide the exact preference values. In this situation, the linguistic variables [11] are suitable to express the fuzzy preferences of agents, such as high, medium, and low. Moreover, in many different situations, the problems are also defined with uncertainty; the agents cant effortlessly give one linguistic term as their preferences [12]. The HFLTS [12] adds to the elasticity of expressing linguistic information. It allows the agents to employ several linguistic terms for evaluating one linguistic variable. The theory of HFLTSs has been applied to MADM [13, 14], but very seldom to TSMDM. For that reason, this paper will investigate the problem of TSMDM from the view point of HFLTSs.

The structure is arranged bellow: Section 2 introduces the concepts of HFLTS and TSM. Section 3 formulates the problem of TSMDM with HFLTSs. Section 4 presents the method for TSMDM with HFLTSs. Section 5 gives an instance. Section 6 concludes this paper.

2. HFLTS and TSM

2.1. HFLTS

Definition 1 [15]. Let S={$s_0, s_1, ..., s_g$} be an ordered linguistic term set, where cardinality $g + 1$ is usually odd.

Definition 2 [12]. A HFLT on set S={$s_0, s_1, ..., s_g$} (noted as H_g) is an ordered subset of consecutive linguistic terms in S.

Definition 3 [12]. The envelope of H_S (noted as env(H_S)) is a linguistic interval, i.e., env(H_S)=[H_S^-, H_S^+] , where $H_S^-(H_S^+)$ is the lower (upper) bound.

Definition 4 [12]. The empty HFLTS on set S={$s_0, s_1, ..., s_g$} (noted as H_S^E) is defined as $H_S^E = \Phi$, and the full HFLTS (noted as H_S^F) is defined as $H_S^F = S$. Any other HFLTS is consisted of at least one linguistic term in S

Definition 5 [13]. Let $s_l \in S = \{s_0, s_1, ..., s_g\}$, then Ind(s_l)=l, and let H_S be a HFLT on S, then Ind(H_S) represents the set of indexes of the linguistic terms in S.

Definition 6. Let $l \in [0, \mathrm{g}]$, then $\mathrm{Ind}^{-1} l = s_l$; and let H_S be a HFLT on S, then Ind^{-1} (Ind(H_S))=H_S.

Similar to Ref. [16], the operations on HFLTs can be given as follows.

Definition 7. Let H_S, H_S^1 and H_S^2 be the HFLTs on the linguistic term set S, then some operations on the HFLTs H_S, H_S^1 and H_S^2 are defined as follows:

(1) $H_S^k = \cup_{\gamma\in H_S}\{\mathrm{Ind}^{-1}[(\mathrm{Ind}(\gamma))^k]\}], 0 < k \leq 1$;

(2) $kH_S = \cup_{\gamma\in H_S}\left\{\mathrm{Ind}^{-1}\left[\mathrm{g}\left(1-(1-\mathrm{Ind}(\gamma)/\mathrm{g})^k\right)\right]\right\}, 0 < k \leq 1$;

(3) $H_S^1 + H_S^2 = \cup_{\gamma_1\in H_S^1,\gamma_2\in H_S^2}\left\{\mathrm{Ind}^{-1}\left[\mathrm{g}\left(\mathrm{Ind}(\gamma_1)/\mathrm{g} + \mathrm{Ind}(\gamma_1)/\mathrm{g} - \mathrm{Ind}(\gamma_1)/\mathrm{g}\mathrm{Ind}(\gamma_2)/\mathrm{g}\right)\right]\right\}$;

(4) $H_S^1 H_S^2 = \cup_{\gamma_1\in H_S^1,\gamma_2\in H_S^2}\{\mathrm{Ind}^{-1}[\mathrm{Ind}(\gamma_1)\mathrm{Ind}(\gamma_2)]\}$.

Definition 8. Let H_S=[H_S^-, H_S^+] be the HFLTs on the linguistic term set S, then a novel score function of HFLTs H_S is defined below:

$$S(H_S) = \sum_{k=1}^{\mathrm{len}(H_S)} \varpi_k \mathrm{Ind}^{-1}[\mathrm{Ind}(H_S^-) + k - 1] \tag{1}$$

where $\mathrm{len}(H_S) = \mathrm{Ind}(H_S^+) - \mathrm{Ind}(H_S^-) + 1$.

In Eq.1, the associated weight ϖ_k could be given as follows [17]:

$$\varpi_k = Q\left(\frac{k}{\mathrm{len}(H_S)}\right) - Q\left(\frac{k-1}{\mathrm{len}(H_S)}\right), k = 1, ..., \mathrm{len}(H_S) \tag{2}$$

where Q is a non-decreasing function, which can be expressed by

$$Q(r) = \begin{cases} 0, & r < \underline{r} \\ \frac{r-\underline{r}}{\bar{r}-\underline{r}}, & \underline{r} \leq r \leq \bar{r} \\ 1, & r > \bar{r} \end{cases} \tag{3}$$

with $r, \underline{r}, \bar{r} \in [0, 1]$. Here, the parameter pair $(\underline{r}, \bar{r})$ is predefined.

2.2. TSM

In the general TSM problem, let $\partial = \{\partial_1, \partial_2, ..., \partial_p\}$ (or $\wp = \{\wp_1, \wp_2, ..., \wp_q\}$) be the set of agents on side ∂ (or $\wp$), where ∂_i (or $\wp_j$) represents the ith (or jth) agent on side ∂ (or $\wp$). In addition, we assume $q \geq p \geq 2$. And suppose $P = \{1, \ldots, p\}, Q = \{1, \ldots, q\}$.

Definition 9 [18]. A TSM Υ is a one-to-one mapping $\Upsilon : \partial \cup \wp \to \partial \cup \wp$, which meets the following conditions: (1) $\Upsilon(\partial_i) \in \wp$, (2) $\Upsilon(\wp_j) \in \partial \cup \{\wp_j\}$, (3) $\Upsilon(\partial_i) = \wp_j$ iff $\Upsilon(\wp_j) = \partial_i$.

Definition 10 [18]. The TSM Υ can be expressed by $\Upsilon = \Upsilon_{mp} \cup \Upsilon_{sp}$, where Υ_{mp} (or Υ_{sp}) represents the set of matching pair (or single pair).

3. Problem of TSMDM with HFLTSs

In the considered TSM problem with HFLTSs, let $S = \{s_0, s_1, ..., s_g\}$ be the predefined linguistic term set, where g is even. Let $H_S^{\partial} = [h_{S,ij}^{\partial}]_{p\times q}$ be the HFLTS matrix on side ∂, thereinto $h_{S,ij}^{\partial}$ represents the HFLTS preference of agent ∂_i towards $\wp_j$, $h_{S,ij}^{\partial} \subseteq S$; $H_S^{\wp} = [h_{S,ij}^{\wp}]_{p\times q}$ be the HFLTS matrix on side $\wp$, thereinto $h_{S,ij}^{\wp}$ represents the HFLTS preference of agent $\wp_j$ towards ∂_i , $h_{S,ij}^{\wp} \subseteq S$. Let $E = \{1/9, 1/8, ..., 1/2, 1, 2, ..., 9\}$ be the set of scale 1-9. Let $D^{\partial} = [d_{ik}^{\partial}]_{p\times p}$ be the reciprocal judgment matrix of side ∂, where d_{ik}^{∂} represents the important degree of agent ∂_i towards ∂_k, $d_{ik}^{\partial} \in E$; $D^{\wp} = [d_{rj}^{\wp}]_{q\times q}$ be the reciprocal judgment matrix of side $\wp$, where $d_{rj}^{\wp}$ represents the important degree of agent $\wp_r$ towards $\wp_j$, $d_{rj}^{\wp} \in E$.

In summary, the problem that needs to be studied is how to determine the reasonable TSM scheme Υ^* on the basis of $H_S^{\partial} = [h_{S,ij}^{\partial}]_{p\times q}$, $H_S^{\wp} = [h_{S,ij}^{\wp}]_{p\times q}$, $D^{\partial} = [d_{ik}^{\partial}]_{p\times p}$ and $D^{\wp} = [d_{rj}^{\wp}]_{q\times q}$.

4. Approach for TSMDM with HFLTSs

4.1. Development of TSMDM model with HFLTSs

First, let $m_{ij} = \begin{cases} 1, & \Upsilon(\partial_i) = \wp_j \\ 0, & \Upsilon(\partial_i) \neq \wp_j \end{cases}$. Then, the matching matrix $M = [m_{ij}]_{p\times q}$ can be built. Based on the HFLTS matrices $H_S^{\partial} = [h_{S,ij}^{\partial}]_{p\times q}$ and $H_S^{\wp} = [h_{S,ij}^{\wp}]_{p\times q}$, and matching matrix $M = [m_{ij}]_{p\times q}$, the following TSMDM model (M-1) is established.

$$\text{(M-1)}\begin{cases} \max \quad O_{\partial_i} = \sum_{j=1}^{q} h_{S,ij}^{\partial} m_{ij}, i \in P \\ \max \quad O_{\wp_j} = \sum_{i=1}^{p} h_{S,ij}^{\wp} m_{ij}, j \in Q \\ \text{s.t.} \quad \sum_{j=1}^{q} m_{ij} = 1, i \in P; \sum_{i=1}^{p} m_{ij} \leq 1, j \in Q; m_{ij} \in \{0,1\}, i \in P, j \in Q \end{cases}$$

In model (M-1), max O_{∂_i} and max $O_{\wp_j}$ represent maximizing the satisfaction degree of agent ∂_i towards $\wp_j$, and that of agent $\wp_j$ towards ∂_i , respectively.

4.2. Solution of TSMDM model with HFLTSs

Let w_i^{∂} be the important degree of agent ∂_i on side ∂, which meets $0 \leq w_i^{\partial} \leq 1$ and $\sum_{i=1}^{p} w_i^{\partial} = 1$, then w_i^{∂} can be calculated by using the AHP method based on matrix $D^{\partial} = [d_{ik}^{\partial}]_{p\times p}$; Similarly, let $w_j^{\wp}$ be the important degree of agent $\wp_j$ on side $\wp$, which meets $0 \leq w_j^{\wp} \leq 1$ and $\sum_{j=1}^{q} w_j^{\wp} = 1$, then $w_j^{\wp}$ can be calculated by using the AHP method based on matrix $D^{\wp} = [d_{rj}^{\wp}]_{q\times q}$. Moreover, by using the linear weighted method, model (M-1) is transformed into model (M-2):

$$\text{(M-2)}\begin{cases} \max \quad O_\partial = \sum_{i=1}^{p}\sum_{j=1}^{q} h_{S,ij}^{\partial\partial} m_{ij} \\ \max \quad O_\wp = \sum_{i=1}^{p}\sum_{j=1}^{q} h_{S,ij}^{\wp\wp} m_{ij} \\ \text{s.t.} \quad \sum_{j=1}^{q} m_{ij} = 1, i \in P; \sum_{i=1}^{p} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q \end{cases}$$

In model (2), $h_{S,ij}^{\partial\partial} = w_i^\partial h_{S,ij}^\partial$ and $h_{S,ij}^{\wp\wp} = w_j^\wp h_{S,ij}^\wp$, which can be calculated by Definition 7.

In order to solve model (M-2), the HFLTSs $h_{S,ij}^{\partial\partial}$ and $h_{S,ij}^{\wp\wp}$ are changed into scores $s_{S,ij}^{\partial\partial}$ and $s_{S,ij}^{\wp\wp}$ by Eqs. (1)-(3). Hence, model (M-2) is transformed into model (M-3):

$$\text{(M-3)}\begin{cases} \max \quad O_\partial = \sum_{i=1}^{p}\sum_{j=1}^{q} s_{S,ij}^{\partial\partial} m_{ij} \\ \max \quad O_\wp = \sum_{i=1}^{p}\sum_{j=1}^{q} s_{S,ij}^{\wp\wp} m_{ij} \\ \text{s.t.} \quad \sum_{j=1}^{q} m_{ij} = 1, i \in P; \sum_{i=1}^{p} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q \end{cases}$$

Furthermore, let ω_∂ and $\omega_\wp$ be the weights of O_∂ and $O_\wp$ respectively, such that $\omega_\partial, \omega_\wp \in [0,1]$, $\omega_\partial + \omega_\wp = 1$, then model (M-3) can be transformed into the model (M-4):

$$\text{(M-4)}\begin{cases} \max \quad O = \sum_{i=1}^{p}\sum_{j=1}^{q} s_{S,ij}^{\partial\wp} m_{ij} \\ \text{s.t.} \quad \sum_{j=1}^{q} m_{ij} = 1, i \in P; \sum_{i=1}^{p} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q \end{cases}$$

where $s_{S,ij}^{\partial\wp} = \omega_\partial s_{S,ij}^{\partial\partial} + \omega_\wp s_{S,ij}^{\wp\wp}$, which can be calculated by Definition 8.

Remark 1. In model (M-4), ω_∂ and $\omega_\wp$ can be regarded as the important degrees of the agents of side ∂ and $\wp$ respectively, which are determined by the matching intermediary based on the statuses of the agents of two sides. If the statuses of the agents of two sides are considered as the same, then $\omega_\partial = \omega_\wp$; otherwise $\omega_\partial \neq \omega_\wp$. In this case, ω_∂ and $\omega_\wp$ can be determined by comparing the objectives O_∂ and $O_\wp$. For example, if the related important degree of objective O_∂ comparing with $O_\wp$ is θ , then we obtain $\omega_\partial = \frac{\theta}{1+\theta}$, $\omega_\wp = \frac{1}{1+\theta}$.

By solving model (M-4), the optimum matching matrix $M^* = [m_{ij}^*]_{p\times q}$ can be obtained. Based on the matching matrix $M^* = [m_{ij}^*]_{p\times q}$, the TSM scheme Υ^* can be determined.

4.3. Determination of TSMDM algorithm with HFLTSs

In summary, an algorithm is developed. The process of the algorithm is given, as described below.

Step 1. Develop TSMDM model (M-1) on the basis of the HFLTS matrices $H_S^\partial = [h_{S,ij}^\partial]_{p\times q}$ and $H_S^\wp = [h_{S,ij}^\wp]_{p\times q}$, and matching matrix $M = [m_{ij}]_{p\times q}$.

Step 2. Determine the important degree w_i^∂ (or $w_j^\wp$) based on the reciprocal judgment matrix $D^\partial = [d_{ik}^\partial]_{p\times p}$ (or $D^\wp = [d_{rj}^\wp]_{q\times q}$) by using the AHP method.

Step 3. Change model (M-1) into model (M-2).

Step 4. Change model (M-2) into model (M-3) through using Eqs. (1)-(3).

Step 5. Change model (M-3) into model (M-4).

Step 6. Determine the TSM scheme Υ^* .

5. Example

Assume a domestic venture-capital company intends to invest a biological medicine firm in Nan Chang. To operate smoothly, the director plans to schedule the experienced employees to vacancy posts in the new firm. Every post is matched with an employee, and every employee is matched with one post. There exists four vacancy posts, i.e., one buyer (∂_1), one material handler (∂_2), one production schemer (∂_3), and one quality checker (∂_4). Through primary screening, six experienced employees ($\wp_1$, $\wp_2$, , and $\wp_6$) who possess multi-skills apply for four vacancy posts. The supervisors assess six employees from four aspects: personality characteristics, technical ability, past experience, and interpersonal skill. Six employees assess posts from three aspects: salary and benefit, advancement space, and working environment. Furthermore, set S is pre-given bellow: S ={ s_0 =N, s_1 =VL, s_2 =L, s_3 =M, s_4 =H, s_5 =VH, s_6 =P }, and the HFLTS matrices $H_S^\partial = [h_{S,ij}^\partial]_{4\times6}$ and $H_S^\wp = [h_{S,ij}^\wp]_{4\times6}$ are provided in Tables 1 and 2; the reciprocal judgment matrices $D^\partial = [d_{ik}^\partial]_{4\times4}$ and $D^\wp = [d_{rj}^\wp]_{6\times6}$ are given in Tables 3 and 4. In order to promote the level of operating efficiency, the intermediary who is engaged in human resource allocation is hired to show the reasonable TSM scheme Υ^* .

Table 1: The HFLTS matrix $H_S^\partial = [h_{S,ij}^\partial]_{4\times6}$

$h_{S,ij}^\partial$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
∂_1	$\{s_2,s_3\}$	$\{s_1,s_2\}$	$\{s_2,s_3,s_4\}$	$\{s_1,s_2,s_3\}$	$\{s_3,s_4\}$	$\{s_4,s_5\}$
∂_2	$\{s_3,s_4\}$	$\{s_1,s_2,s_3\}$	$\{s_2,s_3,\}$	$\{s_4,s_5\}$	$\{s_2,s_3,s_4\}$	$\{s_2,s_3\}$
∂_3	$\{s_4,s_5\}$	$\{s_3,s_4,s_5\}$	$\{s_3,s_4,\}$	$\{s_2,s_3\}$	$\{s_1,s_2\}$	$\{s_2,s_3\}$
∂_4	$\{s_2,s_3,s_4\}$	$\{s_2,s_3\}$	$\{s_2,s_3,\}$	$\{s_1,s_2\}$	$\{s_1,s_2,s_3\}$	$\{s_1,s_2\}$

Table 2: The HFLTS matrix $H_S^\wp = [h_{S,ij}^\wp]_{4\times6}$

$h_{S,ij}^\wp$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
∂_1	$\{s_1,s_2,s_3\}$	$\{s_2,s_3\}$	$\{s_3,s_4\}$	$\{s_2,s_3\}$	$\{s_1,s_2\}$	$\{s_2,s_3\}$
∂_2	$\{s_3,s_4\}$	$\{s_2,s_3\}$	$\{s_1,s_2,\}$	$\{s_2,s_3,s_4\}$	$\{a_1,s_2,s_3\}$	$\{s_2,s_3\}$
∂_3	$\{s_1,s_2\}$	$\{s_2.s_3,s_4\}$	$\{s_1,s_2,s_3,\}$	$\{s_1,s_2\}$	$\{s_3,s_4\}$	$\{s_1.s_2,s_3\}$
∂_4	$\{s_2,s_3\}$	$\{s_1,s_2\}$	$\{s_2,s_3,\}$	$\{s_3,s_4\}$	$\{s_2,s_3\}$	$\{s_3,s_4\}$

Table 3: The reciprocal judgment matrix $D^\partial = [d_{ik}^\partial]_{4\times4}$

d_{ik}^∂	∂_1	∂_2	∂_3	∂_4
∂_1	1	2	4	1
∂_2	1/2	1	2	1/3
∂_3	1/4	1/2	1	1/5
∂_4	1	3	5	1

To solve the above problem, the proposed TSMDM method is used, and the procedure is given as follows.

Step 1. Based on the HFLTS matrices $H_S^\partial = [h_{S,ij}^\partial]_{4\times6}$ and $H_S^\wp = [h_{S,ij}^\wp]_{4\times6}$, and matching matrix $M = [m_{ij}]_{4\times6}$, TSMDM model (M1) is developed.

Table 4: The reciprocal judgment matrix $D^{\wp} = [d^{\wp}_{rj}]_{6\times6}$

$d^{\wp}_{rj}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
$\wp_1$	1	2	1/2	1	3	2
$\wp_2$	1/2	1	1/4	1/2	2	1
$\wp_3$	2	4	1	2	5	3
$\wp_4$	1	2	1/2	1	3	2
$\wp_5$	1/3	1/2	1/5	1/3	1	1
$\wp_6$	1/2	1	1/3	1/2	1	1

$$
\text{(M-1)}\begin{cases}
\max \quad O_{\partial_i} = \sum\limits_{j=1}^{6} h^{\partial}_{S,ij} m_{ij}, i \in P \\
\max \quad O_{\wp_j} = \sum\limits_{i=1}^{4} h^{\wp}_{S,ij} m_{ij}, j \in Q \\
\text{s.t.} \quad \sum\limits_{j=1}^{6} m_{ij} = 1, i \in P; \sum\limits_{i=1}^{4} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q
\end{cases}
$$

where $P = \{1, 2, 3, 4\}$, $Q = \{1, 2, ..., 6\}$.

Step 2. Based on the reciprocal judgment matrix $D^{\partial} = [d^{\partial}_{ik}]_{4\times4}$, the important degree w^{∂}_i can be obtained by using the AHP method. The results are displayed below: $\lambda^{\partial}_{\max} = 4.0155$, $CR^{\partial} = 0.0058 < 0.01$, $w^{\partial}_1 = 0.35, w^{\partial}_2 = 0.16, w^{\partial}_3 = 0.08, w^{\partial}_4 = 0.41$. Based on the reciprocal judgment matrix $D^{\wp} = [d^{\wp}_{rj}]_{6\times6}$, the important degree $w^{\wp}_j$ also can be obtained by using the AHP method. The results are displayed as follows: $\lambda^{\wp}_{\max} = 6.0497$, $CR^{\wp} = 0.0079 < 0.01$, $w^{\wp}_1 = 0.19, w^{\wp}_2 = 0.10, w^{\wp}_3 = 0.35, w^{\wp}_4 = 0.19, w^{\wp}_5 = 0.07, w^{\wp}_6 = 0.10$.

Step 3. By using the linear weighted method and Definition 7, model (M-1) is transformed into model (M-2),i.e.,

$$
\text{(M-2)}\begin{cases}
\max \quad O_{\partial} = \sum\limits_{i=1}^{4}\sum\limits_{j=1}^{6} h^{\partial\partial}_{S,ij} m_{ij} \\
\max \quad O_{\wp} = \sum\limits_{i=1}^{4}\sum\limits_{j=1}^{6} h^{\wp\wp}_{S,ij} m_{ij} \\
\text{s.t.} \quad \sum\limits_{j=1}^{6} m_{ij} = 1, i \in P; \sum\limits_{i=1}^{4} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q
\end{cases}
$$

where $h^{\partial\partial}_{S,ij} = w^{\partial}_i h^{\partial}_{S,ij}$ and $h^{\wp\wp}_{S,ij} = w^{\wp}_j h^{\wp}_{S,ij}$.

Step 4. Suppose $(a; b) = (0 : 3; 0 : 8)$, then the following associated weight ϖ_k can be obtained by Eqs. (2) and (3): $\varpi_1 = Q\left(\frac{1}{2}\right) - Q(0) = \frac{2}{3}$, $\varpi_2 = Q(1) - Q\left(\frac{1}{2}\right) = \frac{1}{3}$ if $\text{len}(H_S) = 2$, and $\varpi_1 = Q\left(\frac{1}{3}\right) - Q(0) = \frac{1}{15}$, $\varpi_2 = Q\left(\frac{2}{3}\right) - Q\left(\frac{1}{3}\right) = \frac{2}{3}$, $\varpi_3 = Q(1) - Q\left(\frac{2}{3}\right) = \frac{4}{15}$, if $\text{len}(H_S) = 3$. Then, the index matrices $\text{Ind}(H^{\partial\partial}_S) = [\text{Ind}(h^{\partial\partial}_{S,ij})]_{4\times6}$ and $\text{Ind}(H^{\wp\wp}_S) = [\text{Ind}(h^{\wp\wp}_{S,ij})]_{4\times6}$ can be changed into score matrices $S^{\partial\partial}_S = [s^{\partial\partial}_{S,ij}]_{4\times6}$ and $S^{\wp\wp}_S = [s^{\wp\wp}_{S,ij}]_{4\times6}$ by Eq. (1). Furthermore, model (M-2) is changed into model (M-3), i.e.,

$$
\text{(M-3)}\begin{cases}
\max \quad O_{\partial} = \sum\limits_{i=1}^{4}\sum\limits_{j=1}^{6} s^{\partial\partial}_{S,ij} m_{ij} \\
\max \quad O_{\wp} = \sum\limits_{i=1}^{4}\sum\limits_{j=1}^{6} s^{\wp\wp}_{S,ij} m_{ij} \\
\text{s.t.} \quad \sum\limits_{j=1}^{6} m_{ij} = 1, i \in P; \sum\limits_{i=1}^{4} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q
\end{cases}
$$

Step 5. Suppose $\theta = 3/2$, then $\omega_\partial = 0.6, \omega_\wp = 0.4$. Then, by using the linear weighted method and Definition 7, model (M-3) is changed into model (M-4), i.e.,

$$\text{(M-4)}\begin{cases} \max \quad O = \sum_{i=1}^{4}\sum_{j=1}^{6} s_{S,ij}^{\partial\wp} m_{ij} \\ \text{s.t.} \quad \sum_{j=1}^{6} m_{ij} = 1, i \in P; \sum_{i=1}^{4} m_{ij} \le 1, j \in Q; m_{ij} \in \{0,\ 1\}, i \in P, j \in Q \end{cases}$$

where $s_{S,ij}^{\partial\wp} = 0.6 s_{S,ij}^{\partial\partial} + 0.4 s_{S,ij}^{\wp\wp}$. The comprehensive score matrix $[s_{S,ij}^{\partial\wp}]_{4\times 6}$ is displayed in Table5.

Table 5: The comprehensive score matrix $[s_{S,ij}^{\partial\wp}]_{4\times 6}$

$s_{S,ij}^{\partial\wp}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	$\wp_5$	$\wp_6$
∂_1	0.44	0.24	0.8	0.42	0.59	0.92
∂_2	0.42	0.2	0.25	0.57	0.3	0.22
∂_3	0.27	0.28	0.3	0.18	0.1	0.13
∂_4	0.71	0.44	0.57	0.36	0.43	0.29

Step 6. Through solving model (M-4), the matching matrix $M^* = [m_{ij}^*]_{4\times 6}$ is determined. Based on the matching matrix $M^* = [m_{ij}^*]_{4\times 6}$, the TSM scheme Υ^* can be obtained, i.e., $\Upsilon^* = \Upsilon^*_{mp} \cup \Upsilon^*_{sp}$, where $\Upsilon^*_{mp} = \{(\partial_1, \wp_6), (\partial_2, \wp_4), (\partial_3, \wp_3), (\partial_4, \wp_1)\}$, $\Upsilon^*_{sp} = \{(\wp_2, \wp_2), (\wp_5, \wp_5)\}$.

Next, we analyze how the weights ω_∂ and $\omega_\wp$ influence the TSM scheme Υ^*. The comparison analysis of the influence of weights ω_∂ and $\omega_\wp$ towards the TSM scheme Υ^* is shown in Table 6. From Table 6, we know that the TSM scheme Υ^* may be changed when the weight vector is changed from (0.7, 0.3) to (0.6, 0.4) and from (0.5, 0.5) to (0.4, 0.6). In many other cases, the obtained TSM scheme Υ^* doesnt change. Therefore, weights ω_∂ and $\omega_\wp$ play a huge role in determining the TSM scheme.

Table 6: The comparison analysis

Weight vector $(\omega_\partial, \omega_\wp)$	Υ^*_{mp}	Υ^*_{sp}
(1, 0); (0.9, 0.1); (0.8, 0.2); (0.7, 0.3)	$\{(\partial_1, \wp_6), (\partial_2, \wp_4), (\partial_3, \wp_2), (\partial_4, \wp_1)\}$	$\{(\wp_3, \wp_3), (\wp_5, \wp_5)\}$
(0.6, 0.4); (0.5, 0.5)	$\{(\partial_1, \wp_6), (\partial_2, \wp_4), (\partial_3, \wp_3), (\partial_4, \wp_1)\}$	$\{(\wp_2, \wp_2), (\wp_5, \wp_5)\}$
(0.4, 0.6); (0.3, 0.7); (0.2, 0.8); (0.1, 0.9); (0, 1)	$\{(\partial_1, \wp_6), (\partial_2, \wp_4), (\partial_3, \wp_2), (\partial_4, \wp_1)\}$	$\{(\wp_3, \wp_3), (\wp_5, \wp_5)\}$

6. Conclusion

This paper proposes an approach for solving TSMDM problem with HFLTSs. The TSMDM model with HFLTSs is firstly developed. Then the AHP method is used to determine the important degrees of agents of each side. Moreover, the TSMDM model can be changed into a double-goal model with HFLTSs. The double-goal model with HFLTSs is changed into a single-goal model with scores through using the presented score function and the linear weighted method. The reasonable TSM scheme can be determined through model solution. An example is introduced to clarify the validity of the presented approach.

Compared with the existing research, the main contribution is as follows: (1) The theory of HFLTSs was combined with TSMDM, which are seldom considered in previous research; (2) The operations and novel score function of HFLTSs are presented, which are new ideas; (3) The proposed method enriches the theory and method for hesitant fuzzy linguistic TSMDM.

The limitation is that it only discussed the TSMDM problem with complete HFLTSs preliminarily. Therefore, the following two aspects could be further concerned. First, the related theory of stable matching with HFLTSs should be studied. Second, the TSMDM problem with incomplete HFLTSs information should be further investigated.

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