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Adaptive Fuzzy Iterative Learning Control of Robotic Systems with Time-delay Outputs and Input Dead Zone via Observer

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Abstract. This paper presents an adaptive fuzzy iterative learning control method for the output tracking problem of robotic systems with unknown time delay output and input dead-zone. A state observer is designed to estimate unmeasurable velocity variables. By introducing boundary layer function, the identical initial condition for most iterative learning control schemes is relaxed. By combining appropriate Lyapunov-Krasovskii functional and fuzzy logic systems approximation technique, the proposed control scheme can guarantee that the output tracking converges to the desired reference trajectory within an error tolerance and all the closed-loop signals remain bounded.

1. Introduction

The past decades have witnessed a great deal of research efforts that aim at the development of iterative learning control (ILC) for systems repeatedly running over a limited time interval. It has been proven that ILC scheme is the most effective and suitable control strategy for repeatable control tasks due to its ability of achieving perfect tracking through learning mechanism. Robotic systems are generally used in repetitive tasks, so ILC can be applied to enhance tracking performance.

Generally, ILC can be classified into two types: traditional ILC [1] and adaptive iterative learning control (AILC) [2] according to the stability analysis method. Traditional ILC demands for global Lipschitz condition and takes contraction mapping theorem instead of Lyapunov method as stability analysis tool, which makes it difficult to cooperate with the mainstream methods of control theory.Then the so-called AILC method is pro-posed To break through the shortcomings of traditional ILC. AILC enables us to make use of neural networks or fuzzy logic systems (FLS) as approximators to estimate non-smooth nonlinear uncertainties.

In control community, the importance of dead-zone cannot be overemphasized any more, because it usually results in undesirable inaccuracies and even instability [3]. For control systems with dead-zone,

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many works have been reported [4]. However, to the best of our knowledge, there are few literatures studied from the viewpoint of AILC to solve the control problem of robotic manipulator with input dead-zone up to now.

In the field of control, state feedback is very powerful for control systems when the states are accessible for feedback. However, in a great number of control systems, only the output, instead of full state information, is accessible for feedback. For output tracking control, observer is one of the most effective scheme, which estimates the system states on-line. Up to now, there has been great development for various observer design methods. However, only a few results are related to AILC [5-7]. The method of designing an AILC using only output information is an interesting and challenging issue.

Motivated by aforementioned observations, we consider the observer-based AILC problem for robotic manipulator systems with unknown time delay output and input dead zone in this paper. As far as we know, no works has been reported deal with such problem using AILC method. In the proposed AILC scheme, FLS is utilized to construct the iterative learning controller.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation

We consider the n degrees-of-freedom rigid robots which are described by

$$M(q_k(t))\ddot{q}_k(t) + C(q_k(t), \dot{q}_k(t))\dot{q}_k(t) + G(q_k(t)) + H(q_{k,\tau}) = u_k(t) + d_k(t) \tag{1}$$

where $t \in [0, T]$ is the time and $k \in \mathbb{Z}_+$ denotes iteration number $\mathbb{Z}_+$ is the set of positive integers. $q_k(t) = [q_{1,k}(t), \ldots, q_{n,k}(t)]^T \in R^n$ and the signals $q_k(t)$, $\dot{q}_k(t)$ and $\ddot{q}_k(t)$ are the joint position, velocity and acceleration vectors, respectively. $M(q_k(t)) \in R^{n\times n}$ is the inertia matrix, $C(q_k(t), \dot{q}_k(t)) \in R^n$ is a vector resulting from Coriolis and centrifugal forces, and $G(q_k(t)) \in R^n$ is the vector resulting from the gravitational forces. $u_k(t) \in R^n$ is the control input vector. $d_k(t) \in R^n$ is the vector containing the unknown external disturbances. $q_{k,\tau} \triangleq [q_{1,k}(t - \tau_1(t)), \ldots, q_{n,k}(t - \tau_n(t))]^T$, where $\tau_i(t)$ is un-known time-varying delay with the upper bound $\tau_{\max}$, $i = 1, 2, \ldots, n$. $H(\cdot)$ is a bounded unknown smooth functions of time-delay position. It is well known that the inertia matrix $M(q_k(t))$ is positive definite and bounded, i.e.

$$0 < m_1 I_n \le M(q_k(t)) \le m_2 I_n \tag{2}$$

for all $q_k(t)$ with some $m_1, m_2 > 0$ where I_n is the $n \times n$ identity matrix. Then the dynamic formulation (1) can be rewritten as

$$\ddot{q}_k(t) = -M^{-1}(q_k)C(q_k, \dot{q}_k)\dot{q}_k(t) - M^{-1}(q_k)G(q_k) - M^{-1}(q_k)H(q_{k,\tau}) + M^{-1}(q_k)u_k(t) + M^{-1}(q_k)d_k(t) \tag{3}$$

Define the state variable at the k-th iteration as $x_{1,k}(t) = q_k(t)$, $x_{2,k}(t) = \dot{q}_k(t)$, $x_k(t) = \left[x_{1,k}^T(t), x_{2,k}^T(t)\right]^T$ and choose the output variable as $y_k(t) = q_k(t)$, denote $f(q_k, \dot{q}_k) = -M^{-1}(q_k)C(q_k, \dot{q}_k)\dot{q}_k(t) - M^{-1}(q_k)G(q_k)$ and $g(x_{1,k}) \triangleq M^{-1}(x_{1,k})$. Then we can rewrite the robotic system as

$$\begin{cases} \dot{x}_{1,k}(t) = x_{2,k}(t); \dot{x}_{2,k}(t) = f(x_k) - g(x_{1,k}) H(y_{k,\tau}) + g(x_{1,k}) u_k(t) + g(x_{1,k}) d_k(t) \\ y_k(t) = Cx_k(t), t \in [0, T]; y_k(t) = 0, t \in [-\tau_{\max}, 0) \end{cases} \tag{4}$$

where $y_{k,\tau} \triangleq [y_{k,\tau_1}, \ldots, y_{k,\tau_n}]^T = [y_{1,k}(t - \tau_1(t)), \ldots, \ y_{n,k}(t - \tau_n(t))]^T \in R^n$, $C = [I_n, O]^T \in R^{2n\times n}$, O is the $n \times n$ zero matrix. In the rest parts, when no confusions arise the variable t will be omit-ted. The velocity variables are assumed to be unmeasurable and only the joint position is available for measurement.

Remark 2.1. *Time delay exists in extensive physical systems. Time delay may degrade the control performance, and even leads to instability.*

The design objective of this paper is to design an observer-based AILC scheme for robotic manipulator (1) to steer the output y_k track a reference signal y_d over $[0, T]$ as $k \to \infty$, while guaranteeing that all the system signals remain bounded.

Define the desired reference trajectory $x_d = \left[y_d^T, \dot{y}_d^T\right]^T$. To facilitate control design, we make following reasonable assumptions.

Assumption 1. The desired signal $y_d(t)$ and $\dot{y}_d(t)$ are continuous, bounded and available.

Assumption 2. The unknown time delays $\tau_i(t)$ satisfy: $0 \le \tau_i(t) \le \tau_{\max}$, $\dot{\tau}_i(t) \le \kappa < 1$, $i = 1, 2, \cdots, n$, where κ is an unknown positive constant.

Assumption 3. The unknown smooth continuous function $H(.)$ satisfies

$$\|H(.)\| \le \sum_{j=1}^{n} \rho_j(.) \tag{5}$$

where $\rho_j(.)$ is unknown positive smooth function.

Assumption 4. The unknown external disturbance $\|d_k(t)\| \le D_1$ with D_1 as an unknown constant.

2.2. Dead zone nonlinearity

In this paper, we consider the dead-zone characteristic in the control input, which is described by [8]

$$u_k = D(v_k) = \begin{cases} m(t)(v_k - b_r), \text{for } v_k \ge b_r \\ 0, \text{ for } b_l v_k < b_r \\ m(t)(v_k - b_l), \text{for } v_k \le b_l \end{cases} \tag{6}$$

where $v_k(t)$ is the input and $u_k(t)$ is the output, $b_r \ge 0$ and $b_l \le 0$ are unknown constants, $m(t) > 0$ is unknown time-varying slope. The assumption on dead-zone parameters is as follows:

Assumption 5. There exist unknown constants $b_{r\min}$, $b_{r\max}$, $b_{l\min}$, $b_{l\max}$, $m_{\min}$, $m_{\max}$, such that $b_{r\min} \le b_r \le b_{r\max}$, $b_{l\min} \le b_l \le b_{l\max}$ and $m_{\min} \le m(t) \le m_{\max}$.

From a practical purpose, we can re-define the dead-zone nonlinearity as

$$u_k(t) = D(v_k) = m(t)v_k(t) - d_1(v_k(t)) \tag{7}$$

with

$$d_1(v_k(t)) = \begin{cases} m(t)b_r \text{ for } v_k(t) \ge b_r \\ m(t)v_k(t) \text{ for } b_l v_k(t) < b_r \\ m(t)b_l \text{ for } v_k(t) \le b_l \end{cases} \tag{8}$$

It is obvious that $d_1(v_k(t))$ is bounded.

2.3. Fuzzy logic systems

A FLS includes four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier [9]. For more details of FLS, readers may refer to [9]. According to [9], the fuzzy logic system can be expressed as

$$y(x) = W^T \phi(x) \tag{9}$$

Lemma 1. Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists an FLS such that

$$\sup\left|f(x) - W^T\phi(x)\right| \le \varepsilon \tag{10}$$

The FLS (9) is a universal approximator, namely, it can approximate any continuous function on a compact set. FLS has been widely used in the control design due to its perfect approximation ability.

3. State observer and adaptive fuzzy iterative output feedback controller design

3.1. Observer design

For simplification of expression, we denote $d_2(x_{1,k}, t) = g(x_{1,k})(d_k(t) + d_1(v_k(t)))$ and $g(x_{1,k})m(t)$ by $g_m(x_{1,k}) \triangleq g(x_{1,k})m(t)$. It is clear that $m_{\min}I_n/m_2 \le g_m(x_{1,k}) \le m_{\max}I_n/m_1$. Then we can rewrite the system (1) as

$$\dot{x}_k = Ax_k + K_0 y_k + B\left[f(x_k) - g(x_{1,k})H(y_{k,\tau}) + g_m(x_{1,k})v_k + d_2(x_{1,k}, t)\right] \tag{11}$$

with $A = \begin{bmatrix} -K_1 & I_n \\ -K_2 & O \end{bmatrix}_{2n\times 2n}$, $K_1 = diag\{k_{11}, \ldots, k_{1n}\}$ and $K_2 = diag\{k_{21}, \ldots, k_{2n}\}$ are diagonal matrices, $K_o = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_{2n\times n}$, $B = \begin{bmatrix} O \\ I_n \end{bmatrix}_{2n\times n}$. K_1 and K_2 should be chosen suitably so that A is strict Hurwitz. Then, given a matrix $Q > 0$, there exists a matrix $P > 0$ that satisfies:

$$A^T P + PA + \frac{n+3}{\lambda}PP^T < -Q \tag{12}$$

In order to estimate the states of system (11), design an observer as

$$\dot{\hat{x}}_k = A\hat{x}_k + K_o y_k + B(\Psi_k - v_{rk}), \hat{y}_k = \hat{x}_{1,k} \tag{13}$$

where $\Psi_k \in R^n$, v_{rk} is defined as the robust term which will be designed later.

Define $z_k \triangleq [z_{1,k}, z_{2,k}, \ldots, z_{2n,k}] = x_k - \hat{x}_k$. Choose a positive function as $V_{z_k} = z_k^T P z_k$. Recalling Assumption 2 and using Youngs inequality, taking the time derivative of V_{z_k} yields

$$\dot{V}_{z_k} \le z_k^T\left(A^T P + PA + \frac{n+2}{\lambda}\right)z_k + 2z_k^T PB\left[f(x_k) + g_m(x_{1,k})v_k - \Psi_k\right] + \frac{\lambda}{m_1}\sum_{j=1}^{n}\rho_j^2\left(y_{k,\tau_j}\right) + \lambda D_0^2 + \lambda v_{rk}^2 \tag{14}$$

where D_0 denote the upper bound of $d_2(x_{1,k}, t)$.

To deal with time-delay term, define the following Lyapunov-Krasovskii functional

$$V_{U_k}(t) = \frac{\lambda}{m_1(1-\kappa)}\sum_{j=1}^{n}\int_{t-\tau_j(t)}^{t}\rho_j^2\left(y_{j,k}(\sigma)\right)\mathrm{d}\sigma \tag{15}$$

Taking the time derivative of (15) and considering (14), it results in

$$\begin{aligned}\dot{V}_{z_k} + \dot{V}_{U_k} \le & z_k^T\left(A^T P + PA + \frac{n+2}{\lambda}\right)z_k + 2z_k^T PB\left[f(x_k) + g_m(x_{1,k})v_k - \Psi_k\right] \\ & + \frac{\lambda}{m_1(1-\kappa)}\sum_{j=1}^{n}\rho_j^2\left(y_{j,k}\right) + \lambda D_0^2 + \lambda v_{rk}^2\end{aligned} \tag{16}$$

To deal with time-varying uncertainties $f(x_k)$ and $g_m(x_{1,k})$, we apply the fuzzy approximation technique to approximate $f(x_k)$ and $g_m(x_{1,k})$ on the compact sets $\Omega_f = \{x_k\} \subset R^{2n}$ and $\Omega_g = \{x_{1,k}\} \subset R^n$, respectively

$$f(x_k) = \begin{bmatrix} W_{f1}^{*T}(t)\phi_{f1}(x_k) \\ \vdots \\ W_{fn}^{*T}(t)\phi_{fn}(x_k) \end{bmatrix} + \begin{bmatrix} \varepsilon_{f1}(x_k) \\ \vdots \\ \varepsilon_{fn}(x_k) \end{bmatrix} = \left[\left\{W_f^*(t)\right\}^T \bullet \left\{\phi_f(x_k)\right\}\right] + \varepsilon_f(x_k) \tag{17}$$

$$g_m(x_{1,k}) = \begin{bmatrix} \bar{W}_{g11}^{*T}(t)\bar{\phi}_{g11}(x_{1,k}) & \cdots & \bar{W}_{g1n}^{*T}(t)\bar{\phi}_{g1n}(x_{1,k}) \\ \vdots & \vdots & \vdots \\ \bar{W}_{gn1}^{*T}(t)\bar{\phi}_{gn1}(x_{1,k}) & \cdots & \bar{W}_{gnn}^{*T}(t)\bar{\phi}_{gnn}(x_{1,k}) \end{bmatrix} = \left[\left\{\bar{W}_g^*(t)\right\}^T \bullet \left\{\bar{\phi}_g(x_{1,k})\right\}\right] \tag{18}$$

where, $W_{fi}^{*}(t), \phi_{fi}(x_k) \in R^{l_{fi}}$, $i = 1, \cdots, n$; $W_{gij}^{*}, \phi_{gij}(x_{1,k}) \in R^{l_{gij}}$, $\bar{W}_{gij}^{*} = \left[W_{gij}^{*T}(t), \varepsilon_{gij}(x_{1,k})\right]^T$, $\bar{\phi}_{gij}(x_{1,k}) = \left[\phi_{gij}(x_{1,k}), 1\right]^T$, $i = 1, \cdots, n$, $j = 1, \cdots, n$. Here we employ GL matrix operator [10].

Consequently, we can determine that

$$\Psi_k = \left\{\hat{W}_{f,k}(t)\right\}^T \bullet \left\{\phi_f(\hat{x}_k)\right\} + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \bullet \left\{\bar{\phi}_g(x_{1,k})\right\}\right] v_k \tag{19}$$

Then we can have

$$2z_k^T PB\left[f(x_k) - \hat{f}(\hat{x}_k)\right] = 2z_k^T PB\left[\left\{W_f^*(t)\right\}^T \bullet \left\{\tilde{\phi}_f(x_k, \hat{x}_k)\right\} + \varepsilon_f(x_k) - \left\{\tilde{W}_{f,k}\right\}^T \bullet \left\{\phi_f(\hat{x}_k)\right\}\right] \tag{20}$$

where $\tilde{W}_{fk} = \hat{W}_{fk} - W_f^*$ and $\tilde{\phi}_f(x_k, \hat{x}_k) = \phi_f(x_k) - \phi_f(\hat{x}_k)$, denote $\delta_{fk} = \left\{W_f^*(t)\right\}^T \bullet \left\{\tilde{\phi}_f(x_k, \hat{x}_k)\right\} + \varepsilon_f(x_k)$ which is bounded by $\left\|\delta_{fk}\right\| \le \delta^*$. Using Youngs inequality and substituting (19) back into (16) and applying (20) we have

$$\begin{aligned}\dot{V}_{z_k} + \dot{V}_{U_k} \le &- \lambda_{\min}(Q)\|z_k\|^2 - 2z_k^T PB\left[\left\{\tilde{W}_{f,k}\right\}^T \bullet \left\{\phi_f(\hat{x}_k)\right\}\right] - 2z_k^T PB\left[\left\{\tilde{\bar{W}}_{g,k}(t)\right\}^T \bullet \left\{\bar{\phi}_g(x_{1,k})\right\}\right] v_k \\ &+ \frac{\lambda}{m_1(1-\kappa)} \sum_{j=1}^{n} \rho_j^2(y_k) + \lambda D_1^2 + \lambda v_{rk}^2 + \lambda \delta^{*2}\end{aligned} \tag{21}$$

3.2. *Adaptive fuzzy iterative learning controller design*

Define errors as $e_{1,k} = \left[e_{1,k}^1, \ldots, e_{1,k}^n\right]^T = \hat{x}_{1,k} - y_d$, $e_{2,k} = \left[e_{2,k}^1, \ldots, e_{2,k}^n\right]^T = \hat{x}_{2,k} - \dot{y}_d$, $e_k = \left[e_{1,k}^T, e_{2,k}^T\right]^T$.

Assumption 6. $z_{i,k}(0) = 0$, $i = 1, 2, \ldots, n$.

Assumption 7. The initial tracking errors $e_{i,k}(0)$ at each iteration are assumed to be bounded, but not necessarily zero, small or fixed.

Define a tracking error variable as $e_{sk} = [e_{sk,1}, \ldots, e_{sk,n}]^T = [\Lambda\ I_n]\, e_k$, where $\Lambda = diag\{\lambda_1, \ldots, \lambda_n\}$ and $\lambda_1, \cdots, \lambda_n$ are chosen such that the polynomial $H_i(s) = s + \lambda_i$ is Hurwitz. It is clear that if e_{sk} approaches zero as $k \to \infty$, then $\|e_k\|$ will converge to the origin asymptotically.

Based on Assumption 7, there exist known constants ε_1^i and ε_2^i, such that, $\left|e_{1,k}^i(0)\right| \le \varepsilon_1^i$ and $\left|e_{2,k}^i(0)\right| \le \varepsilon_2^i$, $i = 1, 2, \cdots n$. Define an auxiliary function $s_k = [s_{1,k}, \ldots, s_{n,k}]^T$ as

$$s_{i,k} = e_{sk,i} - \eta_i(t)\mathrm{sat}\left(\frac{e_{sk,i}}{\eta_i(t)}\right),\ \eta_i(t) = \varepsilon_i e^{-Kt} \tag{22}$$

where $\varepsilon_i = \lambda_i \varepsilon_1^i + \varepsilon_2^i$, $K > 0$. The saturation function sat(.) is defined as

$$\mathrm{sat}\left(\frac{e_{sk,i}}{\eta_i(t)}\right) = \mathrm{sgn}(e_{sk,i}) \min\left\{\left|e_{sk,i}/\eta_i(t)\right|, 1\right\} \tag{23}$$

According to initial condition, we can know that $\left|e_{sk,i}(0)\right| \le \eta_i(0)$, thus $s_{i,k}(0) = 0$ is satisfied for any $k \in N$. Define a Lyapunov function as $V_{s_k} = \frac{1}{2}s_k^T s_k$. Taking the derivative of V_{s_k} with respective to time, it yields

$$\dot{V}_{s_k} = s_k^T\left[\Lambda(K_1 z_{1,k} + e_{2,k}) + K_2 z_{1,k} + K e_{sk} + \left\{\hat{W}_{f,k}(t)\right\}^T \bullet \left\{\phi_f(\hat{x}_k)\right\} + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \bullet \left\{\bar{\phi}_g(x_{1,k})\right\}\right] v_k - v_{rk} - \ddot{y}_d\right] - K s_k^T s_k \tag{24}$$

where $\eta(t) = [\eta_1(t), \ldots, \eta_n(t)]^T$, $\mathrm{sgn}(s_k) = [\mathrm{sgn}(s_{1,k}), \ldots, \mathrm{sgn}(s_{n,k})]^T$ and utilizing the relation $s_{i,k}(-K e_{sk,i} + K \times \eta_i(t)\,\mathrm{sgn}(s_{i,k})) = -K s_{i,k}^2$.

Lemma 2 [8]. For any constant $\eta > 0$ and any variable $p \in R$,

$$\lim_{p\to 0}\left[\tanh^2(p/\eta)\right]/p \ = 0 \tag{25}$$

Choose the Lyapunov function as $V_k = V_{z_k} + V_{U_k} + V_{s_k}$. For convenience of expression, denote $\Xi(y_k) \triangleq \frac{\lambda}{m_1(1-\kappa)}\sum_{j=1}^{n}\rho_j^2(y_k) + \lambda D_0^2 + v_{rk}^2 + \lambda\delta^{*2}$. Employing the hyperbolic tangent function and combining (21) and (24) we can obtain

$$\begin{aligned}\dot{V}_k \le & -\lambda_{\min}(Q)\|z_k\|^2 - 2z_k^T PB\left[\left\{\tilde{W}_{f,k}\right\}^T \cdot \left\{\phi_f(\hat{x}_k)\right\}\right] - 2z_k^T PB\left[\left\{\tilde{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]v_k + s_k^T\left[\Lambda(K_1 z_{1,k} + e_{2,k})\right.\\ & + K_2 z_{1,k} + Ke_{sk} + \left\{\hat{W}_{f,k}(t)\right\}^T \cdot \left\{\phi_f(\hat{x}_k)\right\} + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]v_k - v_{rk} - \ddot{y}_d + \frac{1}{n}b\mathrm{T}anh(s_k/\eta(t))\,s_k^{-1}\Xi(y_k)\Big]\\ & + \frac{1}{n}\sum_{i=1}^{n}\left[1 - b\tanh^2(s_{i,k}/\eta_i(t))\right]\Xi(y_k) - Ks_k^T s_k\end{aligned} \tag{26}$$

where we define $\mathrm{T}anh(s_k/\eta(t)) = diag\left[\tanh^2(s_{i,k}/\eta_i(t))\right]$, $i = 1, \cdots, n$. Hence, $b\tanh^2(s_k/\eta(t))\,\Xi(y_k)/s_k$ is de-fined at $s_k = 0$ and the problem of possible singularity is solved. Apparently, $b\mathrm{T}anh(s_k/\eta(t))\,s_k^{-1}\Xi(y_k)$ is well-defined and continuous over compact set $\Omega_\Xi = \{\hat{x}_k, x_d, y_k\} \subset R^{5n}$, so it can be approximated by a FLS to arbitrary accuracy as

$$b\mathrm{T}anh(s_k/\eta(t))\,s_k^{-1}\Xi(y_k)/n \ = \left[\bar{W}_{\Xi 1}^{*T}\bar{\phi}_{\Xi 1}(Z_k), \cdots, \bar{W}_{\Xi n}^{*T}\bar{\phi}_{\Xi n}(Z_k)\right]^T = \left\{\bar{W}_\Xi^*\right\}^T \cdot \left\{\bar{\phi}_\Xi(Z_k)\right\} \tag{27}$$

where, $Z_k = \left[\hat{x}_k^T, x_d^T, y_k^T\right]^T$, $W_{\Xi i}^*, \phi_{\Xi i}(Z_k) \in R^{l_{\Xi i}}$, $\bar{W}_{\Xi i}^* = \left[W_{\Xi i}^{*T}, \varepsilon_{\Xi i}(Z_k)\right]^T$ and $\bar{\phi}_{\Xi i}(Z_k) = \left[\phi_{\Xi i}^T(Z_k), 1\right]^T$, $i = 1, \cdots, n$. Then we can arrive at

$$\begin{aligned}\dot{V}_k \le & -\lambda_{\min}(Q)\|z_k\|^2 - 2z_k^T PB\left[\left\{\tilde{W}_{f,k}\right\}^T \cdot \left\{\phi_f(\hat{x}_k)\right\}\right] - 2z_k^T PB\left[\left\{\tilde{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]v_k - s_k^T\left[\left\{\tilde{\bar{W}}_{\Xi,k}\right\}^T\right.\\ & \left.\cdot \left\{\bar{\phi}_\Xi(Z_k)\right\}\right] + s_k^T\left[\Lambda(K_1 z_{1,k} + e_{2,k}) + K_2 z_{1,k} + Ke_{sk} + \left\{\hat{W}_{f,k}(t)\right\}^T \cdot \left\{\phi_f(\hat{x}_k)\right\} + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]v_k\right.\\ & \left. - v_{rk} - \ddot{y}_d + \left\{\hat{\bar{W}}_\Xi\right\}^T \cdot \left\{\bar{\phi}_\Xi(Z_k)\right\}\right] + \frac{1}{n}\sum_{i=1}^{n}\left[1 - b\tanh^2(s_{i,k}/\eta_i(t))\right]\Xi(y_k) - Ks_k^T s_k\end{aligned} \tag{28}$$

For simplicity, denote $\Upsilon_k = -\Lambda(K_1 z_{1,k} + e_{2,k}) - K_2 z_{1,k} - Ke_{sk} - \left\{\hat{W}_{f,k}(t)\right\}^T \cdot \left\{\phi_f(\hat{x}_k)\right\} + \ddot{y}_d - \left\{\hat{\bar{W}}_\Xi\right\}^T \cdot \left\{\bar{\phi}_\Xi(Z_k)\right\}$. Then, we can design the output tracking controller as

$$v_k = \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]\left[\delta I_n + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]^T\left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]\right]^{-1}\Upsilon_k \tag{29}$$

where δ is a small positive constant. Design v_{rk} as

$$\begin{aligned}v_{rk} = & \delta\left[\delta I_n + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]^T\left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]\right]^{-1}\Upsilon_k \times\\ & \tanh\left(\delta s_k^T\left[\delta I_n + \left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]^T\left[\left\{\hat{\bar{W}}_{g,k}(t)\right\}^T \cdot \left\{\bar{\phi}_g(x_{1,k})\right\}\right]\right]^{-1}\Upsilon_k/\Delta_k\right)\end{aligned} \tag{30}$$

where $\Delta_k = q/k^l$, with l and q being constants and $q(\in R) > 0$, $l(\in Z_+) \ge 2$. $\Delta_k$is a convergent series sequence. For subsequent analysis, we will use the following properties.

Property 1 [11]. For any $\Delta_k > 0$ and $x \in R$, the inequality $|x| - x\tanh(x/\Delta_k) \le \theta\Delta_k$ is established, where θ is a positive constant and $\theta = e^{-(\theta+1)}$ or $\theta = 0.2785$.

Property 2 [12]. $\lim\limits_{k\to\infty} \sum\limits_{j=1}^{k} \Delta_j < 2q$.

Using the matrix equality $GG^T\left[\delta I_n + GG^T\right]^{-1} = I_n - \delta\left[\delta I_n + GG^T\right]^{-1}$ and recalling property 1, we can know

$$\begin{aligned}\dot{V}_k \le & -\lambda_{\min}(Q)\|z_k\|^2 - 2z_k^T PB\left[\left\{\tilde{W}_{f,k}\right\}^T \bullet \left\{\phi_f(\hat{x}_k)\right\}\right] - 2z_k^T PB\left[\left\{\tilde{\bar{W}}_{g,k}(t)\right\}^T \bullet \left\{\bar{\phi}_g(x_{1,k})\right\}\right] v_k \\ & - s_k^T\left[\left\{\tilde{\bar{W}}_{\Xi,k}\right\}^T \bullet \left\{\bar{\phi}_\Xi(Z_k)\right\}\right] + \theta\Delta_k + \frac{1}{n}\sum_{i=1}^{n}\left[1 - b\tanh^2\left(s_{i,k}/\eta_i(t)\right)\right]\Xi(y_k) - Ks_k^T s_k \end{aligned} \tag{31}$$

The adaptive learning algorithms are designed as follows

$$\left\{\hat{W}_{f,k}(t)\right\} = \left\{\hat{W}_{f,k-1}(t)\right\} + 2q_1 z_k^T PB \bullet \left\{\phi_f(\hat{x}_k)\right\};\ \left\{\hat{W}_{f,0}(t)\right\} = 0, t \in [0,T] \tag{32}$$

$$\left\{\hat{\bar{W}}_{g,k}(t)\right\} = \left\{\hat{\bar{W}}_{g,k-1}(t)\right\} + 2q_2 z_k^T PBv_k \bullet \left\{\bar{\phi}_g(x_{1,k})\right\};\ \left\{\hat{W}_{g,0}(t)\right\} \ne 0, t \in [0,T] \tag{33}$$

$$\begin{cases}(1-\gamma)\left\{\dot{\hat{\bar{W}}}_{\Xi,k}\right\} = -\gamma\left\{\hat{\bar{W}}_{\Xi,k}\right\} + \gamma\left\{\hat{\bar{W}}_{\Xi,k-1}\right\} + q_3 s_k^T \bullet \left\{\bar{\phi}_\Xi(Z_k)\right\} \\ \left\{\hat{\bar{W}}_{\Xi,k}(0)\right\} = \left\{\hat{\bar{W}}_{\Xi,k-1}(T)\right\};\ \left\{\hat{\bar{W}}_{\Xi,0}(t)\right\} = 0, t \in [0,T]\end{cases} \tag{34}$$

where $q_1, q_2, q_3 > 0$ and $0 < \gamma < 1$ are design parameters.

4. Analysis of Stability and Convergence

For stability analysis, we using the following property.

Lemma 3 [8]. Consider the set Ω_{s_k} defined by $\Omega_{s_k} := \left\{s_{i,k}\middle|\left|s_{i,k}\right| \le m_\eta \eta_i(t), i = 1,\ldots,n\right\}$. Then for any $s_{i,k} \notin \Omega_{s_k}$, the following inequality is satisfied.

$$1 - b\tanh^2\left(s_{i,k}/\eta_i(t)\right) < 0 \tag{35}$$

where $b > 1$, $m_\eta = \ln\left(\sqrt{b/(b-1)} + \sqrt{1/(b-1)}\right)$.

Theorem 4.1. *Considering the manipulator plant (1) and dead-zone model (6) under Assumption 1-Assumption 7, design the state observer (13) and AILC scheme (29) and (30) with parameter learning algorithms (32)-(34), the following properties can be guaranteed: (i) the boundedness of all the system signals; (ii) the convergence of observer estimation error* $z_k(t)$ *and tracking error* $e_{sk}(t)$*, i.e.,* $\lim\limits_{k\to\infty}\int_0^T \|z_k\|^2 d\sigma = 0$ *and* $\lim\limits_{k\to\infty}\|e_{sk}(t)\| = \left(1 + m_\eta\right)\left\|\eta(t)\right\|$

Proof: According to Lemma 3, two cases are considered for subsequent analysis of stability.

Case 1. $s_{i,k} \in \Omega_{s_k}, i = 1,\ldots,n.$

If $s_{i,k} \in \Omega_{s_k}$, then $\left|s_{i,k}\right| \le m_\eta\eta_i(t)$ is satisfied. We discuss in three cases. 1) If $s_{i,k} = 0$, we know $e_{sk,i}$ is bounded by $\eta_i(t)$, i.e., $\left|e_{sk,i}\right| \le \eta_i(t)$; 2) If $s_{i,k} > 0$ we know $s_{i,k} = e_{sk,i} - \eta_i(t)$, then from $\left|s_{i,k}\right| \le m_\eta\eta_i(t)$ we have $s_{i,k} = e_{sk,i} - \eta_i(t) \le m_\eta\eta_i(t)$ which further leads to $0 < e_{sk,i} \le \left(1 + m_\eta\right)\eta_i(t)$; 3) Similarly, if $s_{i,k} < 0$ we have $s_{i,k} = e_{sk,i} + \eta_i(t) \ge -m_\eta\eta_i(t)$ which implies $0 > e_{sk,i} \ge -\left(1 + m_\eta\right)\eta_i(t)$. Summarizing above discussion we know that $\left|e_{sk,i}\right| \le \left(1 + m_\eta\right)\eta_i(t)$ holds. Then it is easy to see the finiteness of $\hat{x}_{i,k}$ since $x_d(t)$ is bounded in L_T^∞-norm. From the updating law (32)-(34), we know that $\hat{W}_{f,k}(t)$ $\hat{\bar{W}}_{g,k}(t)$ and $\hat{\bar{W}}_{\Xi,k}(t)$ are also bounded. Finally, the boundedness of z_k and x_k in L_T^∞-norm can be deduced. Obviously, the finiteness of v_k is proved. Therefore, all the signals of the closed-loop system are bounded in L_T^∞-norm.

Case 2. $s_{i,k} \notin \Omega_{s_k}, i = 1,\ldots,n.$

It is clear that $\frac{1}{n}\sum_{i=1}^{n}\left[1-b\tanh^{2}\left(s_{i,k}/\eta_i(t)\right)\right]\Xi(y_k)$ can be removed in (31). Therefore, it follows from (31) that

$$\begin{aligned}&2z_k^TPB\left\{\tilde{W}_{f,k}\right\}^T\bullet\left\{\phi_f(\hat{x}_k)\right\}+s_k^T\left[\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}^T\bullet\left\{\bar{\phi}_\Xi(Z_k)\right\}\right]+2z_k^TPB\left\{\tilde{\tilde{W}}_{g,k}(t)\right\}^T\bullet\left\{\bar{\phi}_g(x_{1,k})\right\}v_k\\&\leq-\dot{V}_k-\lambda_{\min}(Q)\|z_k\|^2+\theta\Delta_k-Ks_k^Ts_k\end{aligned}\tag{36}$$

To carry out stability analyze, define the Lyapunov-like CEF:

$$\begin{aligned}E_k(t)=&\frac{1}{2q_1}\int_0^t tr\left\{\left[\left\{\tilde{W}_{f,k}\right\}^T\bullet\left\{\tilde{W}_{f,k}\right\}\right]\right\}d\sigma+\frac{1}{2q_2}\int_0^t tr\left\{\left[\left\{\tilde{\tilde{W}}_{g,k}\right\}^T\bullet\left\{\tilde{\tilde{W}}_{g,k}\right\}\right]\right\}d\sigma\\&+\frac{\gamma}{2q_3}\int_0^t tr\left\{\left[\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}^T\bullet\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}\right]\right\}d\sigma+\frac{(1-\gamma)}{2q_3}tr\left\{\left[\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}^T\bullet\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}\right]\right\}\end{aligned}\tag{37}$$

The subsequent derivation includes five parts.

1) Difference of $E_k(t)$

Recalling adaptive learning law (32)-(34), we can obtain the difference of $E_k(t)$ by using similar technique in [8] as

$$\Delta E_k(t)=E_k(t)-E_{k-1}(t)\leq V_k(0)+\theta\Delta_k t+\frac{(1-\gamma)}{2q_3}tr\left\{\left[\left\{\tilde{\tilde{W}}_{\Xi,k}(0)\right\}^T\bullet\left\{\tilde{\tilde{W}}_{\Xi,k}(0)\right\}\right]-\left[\left\{\tilde{\tilde{W}}_{\Xi,k-1}\right\}^T\bullet\left\{\tilde{\tilde{W}}_{\Xi,k-1}\right\}\right]\right\}\tag{38}$$

From Assumption 2, 6 and 7, It is obvious that $V_k(0)=0$. Let $t=T$ in (38), according to $\hat{\hat{W}}_{\Xi,k}(0)=\hat{\hat{W}}_{\Xi,k-1}(T)$, $\hat{\hat{W}}_{\Xi,1}(0)=0$, we can have

$$\begin{aligned}\Delta E_k(T)&\leq-V_k(T)-K\int_0^T s_k^Ts_k d\sigma-\lambda_{\min}(Q)\int_0^T\|z_k\|^2d\sigma+\theta\Delta_kT\\&\leq-K\int_0^T s_k^Ts_k d\sigma-\lambda_{\min}(Q)\int_0^T\|z_k\|^2d\sigma+\theta\Delta_kT\end{aligned}\tag{39}$$

2) The finiteness of $E_1(T)$

Let k=1 in (37). Recalling parameter adaptive learning laws, we can obtain that the derivative of $\dot{E}_1$ satisfies

$$\begin{aligned}\dot{E}_1(t)\leq&-\dot{V}_1-\lambda_{\min}(Q)\|z_1\|^2-Ks_1^Ts_1+\theta\Delta_1+\frac{1}{2q_1}tr\left\{\left[\left\{W_f^*\right\}^T\bullet\left\{W_f^*\right\}\right]\right\}+\frac{1}{2q_2}tr\left\{\left[\left\{\hat{W}_{g,0}\right\}^T\bullet\left\{\hat{W}_{g,0}\right\}\right.\right.\\&\left.\left.+\left\{W_g^*\right\}^T\bullet\left\{W_g^*\right\}-2\left\{W_g^*\right\}^T\bullet\left\{\hat{W}_{g,0}\right\}\right]\right\}+\frac{\gamma}{2q_3}tr\left\{\left[\left\{W_\Xi^*\right\}^T\bullet\left\{W_\Xi^*\right\}\right]\right\}\end{aligned}\tag{40}$$

Denote $c_{\max}=\max_{t\in[0,T]}\left\{\frac{1}{2q_1}tr\left\{\left[\left\{W_f^*\right\}^T\bullet\left\{W_f^*\right\}\right]\right\}+\frac{1}{2q_2}tr\left\{\left[\left\{\hat{W}_{g,0}\right\}^T\bullet\left\{\hat{W}_{g,0}\right\}+\left\{W_g^*\right\}^T\bullet\left\{W_g^*\right\}-2\left\{W_g^*\right\}^T\bullet\left\{\hat{W}_{g,0}\right\}\right]\right\}+\frac{\gamma}{2q_3}tr\left\{\left[\left\{W_\Xi^*\right\}^T\bullet\left\{W_\Xi^*\right\}\right]\right\}\right\}$. Integrating (40) over $[0,t]$ leads to:

$$E_1(t)-E_1(0)\leq-V_1(t)+V_1(0)-\lambda_{\min}(Q)\int_0^t\|z_1\|^2d\sigma-\int_0^t Ks_1^Ts_1d\sigma+\theta\Delta_1t+t\cdot c_{\max}\tag{41}$$

According to $\hat{\hat{W}}_{\Xi,1}(0)=0$, we obtain

$$E_1(t)\leq t\cdot c_{\max}+\theta\Delta_1t+\frac{(1-\gamma)}{2q_3}tr\left\{\left[\left\{W_\Xi^*\right\}^T\bullet\left\{W_\Xi^*\right\}\right]\right\},t\in[0,T]\tag{42}$$

which implies the finiteness of $E_1(t)$ on $[0,T]$. Letting $t=T$, we can obtain the finiteness of $E_1(T)$ as

$$E_1(T) \leq T \cdot (c_{\max} + \theta\Delta_1) + \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{W_\Xi^*\right\}^T \cdot \left\{W_\Xi^*\right\}\right]\right\} \tag{43}$$

Applying (39) repeatedly, we have

$$\begin{aligned} E_k(T) &= E_1(T) + \sum_{j=2}^{k} \Delta E_j(T) \leq -K \sum_{j=2}^{k} \int_0^T s_j^T s_j \mathrm{d}\sigma - \lambda_{\min}(Q) \sum_{j=2}^{k} \int_0^T \left\|z_j\right\|^2 \mathrm{d}\sigma + T \cdot c_{\max} \\ &+ \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{W_\Xi^*\right\}^T \cdot \left\{W_\Xi^*\right\}\right]\right\} + \theta T \sum_{j=1}^{k} \Delta_k \leq T \cdot c_{\max} + \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{W_\Xi^*\right\}^T \cdot \left\{W_\Xi^*\right\}\right]\right\} + \theta T \sum_{j=1}^{k} \Delta_k \end{aligned} \tag{44}$$

According to the Property 2 of we can have $\theta T \sum_{j=1}^{k} \Delta_k \leq \lim_{k\to\infty} \theta T \sum_{j=1}^{k} \Delta_k \leq 2\theta T q$, which leads to the boundedness of $E_k(T)$.

3) The finiteness of $E_k(t)$

Next we will use induction method to prove the boundedness of $E_k(t)$. Separate $E_k(t)$ into two parts.

$$E_k^1(t) = \frac{1}{2q_1} \int_0^t tr\left\{\left[\left\{\tilde{W}_{f,k}\right\}^T \cdot \left\{\tilde{W}_{f,k}\right\}\right]\right\} \mathrm{d}\sigma + \frac{1}{2q_2} \int_0^t tr\left\{\left[\left\{\tilde{\tilde{W}}_{g,k}\right\}^T \cdot \left\{\tilde{\tilde{W}}_{g,k}\right\}\right]\right\} \mathrm{d}\sigma + \frac{\gamma}{2q_3} \int_0^t tr\left\{\left[\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}^T \cdot \left\{\tilde{\tilde{W}}_{\Xi,k}\right\}\right]\right\} \mathrm{d}\sigma \tag{45}$$

$$E_k^2(t) = \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{\tilde{\tilde{W}}_{\Xi,k}\right\}^T \cdot \left\{\tilde{\tilde{W}}_{\Xi,k}\right\}\right]\right\} \tag{46}$$

The boundedness of $E_k^1(T)$ and $E_k^2(T)$ is guaranteed for all iterations. Hence, $\forall k \in N$, there exist two constants M_1 and M_2 which satisfy

$$E_k^1(t) \leq E_k^1(T) \leq M_1 < \infty, E_k^2(T) \leq M_2 \tag{47}$$

On the other hand, from (38) and $E_{k+1}^2(0) = E_k^2(T)$, we obtain

$$\Delta E_{k+1}(t) < \theta\Delta_k t + M_2 - E_k^2(t) \tag{48}$$

Combining (47) and (48) results in

$$E_{k+1}(t) = E_k(t) + \Delta E_{k+1}(t) \leq M_1 + M_2 + \theta\Delta_k t \tag{49}$$

As we have known that $E_1(t)$ is bounded, consequently $E_k(t)$ is finite.

4) Learning convergence property

According to (44) and taking the limitation, we have

$$\lim_{k\to\infty} \sum_{j=2}^{k} K \int_0^T s_j^T s_j \mathrm{d}\sigma \leq T \cdot c_{\max} + \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{W_\Xi^*\right\}^T \cdot \left\{W_\Xi^*\right\}\right]\right\} + 2q\theta T \tag{50}$$

$$\lim_{k\to\infty} \sum_{j=2}^{k} \lambda_{\min}(Q) \int_0^T \left\|z_j\right\|^2 \mathrm{d}\sigma \leq T \cdot c_{\max} + \frac{(1-\gamma)}{2q_3} tr\left\{\left[\left\{W_\Xi^*\right\}^T \cdot \left\{W_\Xi^*\right\}\right]\right\} + 2q\theta T \tag{51}$$

Using the convergence theorem of the sum of series, we know $\lim_{k\to\infty} \int_0^T s_k^T s_k \mathrm{d}\sigma = 0$, $\lim_{k\to\infty} \int_0^T \|z_k\|^2 \mathrm{d}\sigma = 0$. It is obvious $\lim_{k\to\infty} \int_0^T \left\|y_k - \hat{y}_k\right\|^2 \mathrm{d}\sigma \leq \lim_{k\to\infty} \int_0^T \|z_k\|^2 \mathrm{d}\sigma = 0$, $\forall t \in [0,T]$. Besides, $\lim_{k\to\infty} \int_0^T s_k^T s_k \mathrm{d}\sigma = 0$ is equivalent to

$\lim_{k\to\infty}\int_0^T \|s_k\|\mathrm{d}\sigma = 0$ which means that $\lim_{k\to\infty}\int_0^T \|e_{sk}\|\mathrm{d}\sigma \leq \int_0^T \left\|\eta(\sigma)\right\|\mathrm{d}\sigma$ and $\lim_{k\to\infty}\|e_{sk}\| \leq \left\|\eta(\sigma)\right\|$. Furthermore, we can get the boundedness of $s_k(t)$ and z_k in L_T^2-norm, which further means that $x_k(t)$ and $\hat{x}_k(t)$ are bounded. From the boundedness of $E_k(t)$ we can draw the conclusion that $\hat{W}_{f,k}$, $\hat{W}_{g,k}$ and $\hat{W}_{\Xi,k}$ are bounded. Based on foregoing reasoning, we can arrive at that $v_k(t)$ is bounded.

Summarizing the discussions above, we can conclude that, the proposed control algorithm can guarantee that all system signal are bounded for two cases, and $\lim_{k\to\infty}\int_0^T \|z_k\|^2\mathrm{d}\sigma = 0$ and $\lim_{k\to\infty}\int_0^T \|e_{sk}\|^2\mathrm{d}\sigma \leq \int_0^T \left\|\eta(\sigma)\right\|^2\mathrm{d}\sigma$. Ulteriorly, $e_{s\infty}(t)$ is bounded by $\lim_{k\to\infty}\|e_{sk}(t)\| \leq \left(1+m_\eta\right)\left\|\eta(t)\right\|$, $\forall t \in [0,T]$.

This concludes the proof. □

References

[1] S. Arimoto, S. Kawamura and F. Miyazaki, Bettering operation of robots by learning, Journal of Robotic Systems, l (1987), 123–140.

[2] J. X. Xu and Y. Tan. A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties, IEEE Trans. Autom. Contr., 47 (2002), 1940–1945.

[3] G. Tao and P. V. Kokotovic, Adaptive control of plants with unknown deadzones, IEEE Transactions on Automatic Control, 39 (1994), 59–68.

[4] T.P. Zhang and S.S. Ge, Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form, Automatica, 44 (2008) 1895–1903.

[5] J.X. Xu and Jing Xu, Observer Based Learning Control for a Class of Nonlinear Systems With Time-Varying Parametric Uncertainties, IEEE Trans. Autom. Contr., 49 (2004), 275–281.

[6] Y.C. Wang and C.J. Chien, An observer-based model reference adaptive iterative learning controller for MIMO nonlinear sys-tems, in 2014 11th IEEE International Conference on Control & Automation (ICCA), Taichung, Taiwan, June 2014, pp.1168–1173.

[7] Y.C. Wang and C.J. Chien, An observer based adaptive iterative learning control for robotic systems, in 2011 IEEE Interna-tional Conference on Fuzzy Systems, Taipei, China, June, 2011, pp.2876–2881.

[8] J. M. Wei, Y. A. Hu and M. M. Sun, Adaptive iterative learning control for a class of nonlinear time -varying systems with unknown delays and input dead-zone, IEEE/CAA Journal of Automatica Sinica, 1(2014), 302–314.

[9] L. X. Wang, Adaptive fuzzy systems and control, Prentice Hall, Englewood Cliffs, 1994.

[10] S.S. Ge, C.C Hang. and L.C. Woon, Adaptive neural network control of robot manipulators in task space. IEEE Trans. Industrial Electronics, 44(1997), 746–752.

[11] M M POLYCARPOU. Stable adaptive neural control scheme for nonlinear systems, IEEE Trans. Autom. Contr., 41(1996), 447–451.

[12] J. M. Wei, Y. A. Zhang, J. M. Liu, Observer-based adaptive neural iterative learning control for a class of time-varying nonlinear systems, J. Shanghai Jiao Tong Univ. (Sci.) 22 (2017), 303–312