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Bounds on the Domination Number of a Digraph and its Reverse

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Abstract. Let D be a digraph. A dominating set of D is the subset S of $V(D)$ such that each vertex in $V(D) - S$ is an out-neighbor of a vertex in S. The minimum cardinality of a dominating set of G is denoted by $\gamma(D)$. We let D^- denote the reverse of D.

In [Discrete Math. **197/198** (1999) 179–183], Chartrand, Harary and Yue proved that every connected digraph D of order $n \geq 2$ satisfies $\gamma(D) + \gamma(D^-) \leq \frac{4n}{3}$ and characterized the digraphs D attaining the equality. In this paper, we pose a reduction of the determining problem for $\gamma(D) + \gamma(D^-)$ using the total domination concept. As a corollary of such a reduction and known results, we give new bounds for $\gamma(D) + \gamma(D^-)$ and an alternative proof of Chartrand-Harary-Yue theorem.

1. Introduction

All graphs and digraphs considered in this paper are finite and simple. In particular, no digraph has two arcs with same initial vertex and same terminal vertex (but a digraph may contain a directed cycle of order 2).

Let G be a graph or a digraph. Let $V(G)$ denote the vertex set of G. If G is a graph, let $E(G)$ denote the edge set of G; if G is a digraph, let $A(G)$ denote the arc set of G.

Let G be a graph. For $x \in V(G)$, let $N_G(x)$ and $d_G(x)$ denote the *neighborhood* and the *degree* of x, respectively; thus $N_G(x) = \{y \in V(G) : xy \in E(G)\}$ and $d_G(x) = |N_G(x)|$. Let $\delta(G)$ denote the *minimum degree* of G. For $n \geq 3$, let P_n and C_n denote the *path* and the *cycle* of order n, respectively.

Let D be a digraph. For $x \in V(D)$, let $N_D^+(x)$, $N_D^-(x)$, $d_D^+(x)$ and $d_D^-(x)$ denote the *out-neighborhood*, the *in-neighborhood*, the *out-degree* and *in-degree* of x, respectively; thus $N_D^+(x) = \{y \in V(D) : (x,y) \in A(D)\}$, $N_D^-(x) = \{y \in V(D) : (y,x) \in A(D)\}$, $d_D^+(x) = |N_D^+(x)|$ and $d_D^-(x) = |N_D^-(x)|$. Set $\delta^+(D) = \min\{d_D^+(x) : x \in V(D)\}$, $\delta^-(D) = \min\{d_D^-(x) : x \in V(D)\}$ and $\delta^{\pm}(D) = \min\{\delta^+(D), \delta^-(D)\}$. Let D^- denote the reverse of D; thus D^- is the digraph on $V(D)$ such that $A(D^-) = \{(x,y) : (y,x) \in A(D)\}$. A digraph D is connected if the graph obtained from D by replacing any arcs by edges is connected. For $n \geq 3$, let $\overrightarrow{P_n}$ and $\overrightarrow{C_n}$ denote the *directed path* and the *directed cycle* of order n, respectively; thus $\overrightarrow{P_n}$ is the digraph with $V(\overrightarrow{P_n}) = \{u_1, u_2, \dots, u_n\}$ and $E(\overrightarrow{P_n}) = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\}$, and $\overrightarrow{C_n} = \overrightarrow{P_n} + (u_n, u_1)$.

Let G be a graph or a digraph. A set $S \subseteq V(G)$ is a *dominating set* of G if $(\bigcup_{x\in S} N_G(x)) \cup S = V(G)$ or $(\bigcup_{x\in S} N_G^+(x)) \cup S = V(G)$ according as G is a graph or a digraph. The minimum cardinality of a dominating

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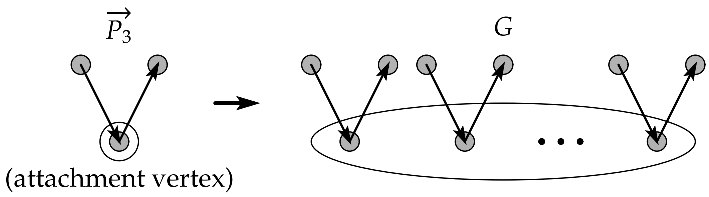

Figure 1: The digraph $\overrightarrow{P_3}$ and a digraph G belonging to $\mathcal{G}(\mathcal{H}_1)$

set of G, denoted by $\gamma(G)$, is called the *domination number* of G. The domination number is a classical invariant in graph theory, and it has been widely studied (see the books [8, 9] and, for example, [6, 14–16] for the domination in digraphs). In particular, the domination number of digraphs can be applied to the solution for various problems: answering skyline query, routing in networks, the choice problem of hotels, etc. (see [19]).

Let again G be a graph or a digraph of order n. Since $V(G)$ is a dominating set of G, the inequality $\gamma(G) \leq n$ trivially holds. The inequality for graphs can be dramatically improved if G is connected: every connected graph G of order $n \geq 2$ satisfies $\gamma(G) \leq \frac{n}{2}$ (see [18]). Here we consider a similar problem for digraphs (i.e., the estimation problem for the domination number of connected digraphs). Since a connected digraph D of order at least two has an arc $(x, y) \in A(D)$, the set $V(D) - \{y\}$ is a dominating set of D. Thus the following proposition holds.

Proposition 1.1 *Let D be a connected digraph of order $n \geq 2$. Then $\gamma(D) \leq n - 1$.*

The gap between the trivial inequality $\gamma(D) \leq n$ and the inequality in Proposition 1.1 is very small, but Proposition 1.1 is best possible. For $n \geq 2$, let D_n be the digraph with $V(D_n) = \{x_i, y : 1 \leq i \leq n-1\}$ and $A(D_n) = \{(x_i, y) : 1 \leq i \leq n-1\}$. Then D_n is connected and $\gamma(D) = |V(D)| - 1$, and hence Proposition 1.1 is best possible. On the other hand, the domination number of the reverse D_n^- of D_n is very small (indeed, $\gamma(D_n^-) = 1$ clearly holds). Thus we expect that, in general, if the domination number of a digraph D is large, then the domination number of its reverse D^- tend to be small. Chartrand, Harary and Yue [3] studied the value $\gamma(D) + \gamma(D^-)$ for digraphs D from such a motivation.

Let $\mathcal{H}$ be a set of connected graphs or a set of connected digraphs. For each $H \in \mathcal{H}$, we will fix a vertex $v \in V(H)$ and call v the *attachment vertex* of H (for example, see the next paragraph). Let $\mathcal{G}(\mathcal{H})$ be the set of connected graphs G or the set of connected digraphs G, according as the elements of $\mathcal{H}$ are graphs or digraphs, such that

(H1) $H_1, \ldots, H_m$ are vertex-disjoint graphs or digraphs, and H_i is a copy of an element of $\mathcal{H}$, and

(H2) G is obtained from $\bigcup_{1 \leq i \leq m} H_i$ by adding some edges or some arcs which join attachment vertices.

For $G \in \mathcal{G}(\mathcal{H})$, since G is connected, the subgraph or the subdigraph of G induced by the attachment vertices is also connected.

We let $\mathcal{H}_1 = \{\overrightarrow{P_3}\}$ and define the attachment vertex of $\overrightarrow{P_3}$ as the vertex v of $\overrightarrow{P_3}$ with $d^+_{\overrightarrow{P_3}}(v) = d^-_{\overrightarrow{P_3}}(v) = 1$ (see Figure 1). Chartrand et al. [3] proved the following theorem.

Theorem A ([3]) *Let D be a connected digraph of order $n \geq 2$. Then $\gamma(D) + \gamma(D^-) \leq \frac{4n}{3}$. Furthermore, if $\gamma(D) + \gamma(D^-) = \frac{4n}{3}$, then $D \in \{\overrightarrow{C_3}\} \cup \mathcal{G}(\mathcal{H}_1)$.*

Recently, the domination number of the reverse of a digraph has been focused on. For example, Hao and Qian [10] continued the study of $\gamma(D) + \gamma(D^-)$ for digraphs without small directed cycles. Furthermore, the difference of $\gamma(D)$ and $\gamma(D^-)$ was studied in [7, 17].

In this paper, we suggest an approach to estimate the value $\gamma(D) + \gamma(D^-)$ using the total domination concept. In Section 2, we show that the value $\gamma(D) + \gamma(D^-)$ is equal to the total domination number of a special bipartite graph. This, together with known results concerning total domination, leads to many upper bounds for $\gamma(D) + \gamma(D^-)$. Our main results in this paper are following:

- We give an alternative proof of Theorem A in Section 3.
- We show that $\gamma(D) + \gamma(D^-) \leq \frac{8|V(D)|}{7}$ for every connected digraph D satisfying $\delta^{\pm}(D) \geq 1$ with finite exceptions, and characterize the digraphs with the equality (Theorem 4.1 in Section 4).
- We give upper bounds on $\gamma(D) + \gamma(D^-)$ for a digraph with large $\delta^{\pm}(D)$ (Theorem 5.1 in Section 5).

2. Reduction to a total domination problem in bipartite graphs

Let G be a graph without isolated vertices, and let $X \subseteq V(G)$. A set $S \subseteq V(G)$ is a *total X-dominating set* of G if $X \subseteq \bigcup_{v \in S} N_G(v)$. The minimum cardinality of a total X-dominating set of G is denoted by $\gamma_t(G; X)$. The integer $\gamma_t(G) := \gamma_t(G; V(G))$ is called the *total domination number* of G.

Lemma 2.1 *Let G be a bipartite graph with the bipartition (X, Y), and suppose that G has no isolated vertices. Then $\gamma_t(G) = \gamma_t(G; X) + \gamma_t(G; Y)$.*

Proof. Let S_X and S_Y be a total X-dominating set and a total Y-dominating set of G, respectively. Then $S_X \cup S_Y$ is a total dominating set of G. Thus $\gamma_t(G) \leq \gamma_t(G; X) + \gamma_t(G; Y)$.

Let S be a total $V(G)$-dominating set of G. Then $S \cap Y$ and $S \cap X$ are a total X-dominating set and a total Y-dominating set of G, respectively. Thus $\gamma_t(G) \geq \gamma_t(G; X) + \gamma_t(G; Y)$. □

For a digraph D, let $G(D)$ be the graph such that

$$V(G(D)) = \{x^+, x^- : x \in V(D)\}$$

and

$$E(G(D)) = \{x^+x^- : x \in V(D)\} \cup \{x^+y^- : (x, y) \in A(D)\},$$

and set $X_D^+ = \{x^+ : x \in V(D)\}$ and $X_D^- = \{x^- : x \in V(D)\}$. Then $G(D)$ is a bipartite graph with the ordered bipartition $\mathcal{X}_D := (X_D^+, X_D^-)$ and $\delta(G(D)) = \delta^{\pm}(D)+1$. In particular, $G(D)$ has no isolated vertices. Furthermore, $G(D)$ is connected if and only if D is connected. Let $M_D := \{x^+x^- : x \in V(D)\}$. Note that M_D is a perfect matching of $G(D)$.

Lemma 2.2 *Let D be a digraph. Then $\gamma(D) = \gamma_t(G(D); X_D^-)$.*

Proof. Let S be a total X_D^--dominating set of $G(D)$, and set $S_0 = \{x \in V(D) : x^+ \in S\}$. Note that $|S_0| \leq |S|$. Fix a vertex $y \in V(D) - S_0$. Since $y^+ \notin S$, there exists a vertex $x^+ \in S$ such that $x^+ \neq y^+$ and $x^+y^- \in E(G(D))$. In particular, there exists a vertex $x \in S_0$ such that $(x, y) \in A(D)$. Since y is arbitrary, S_0 is a dominating set of D. Consequently, $\gamma(D) \leq \gamma_t(G(D); X_D^-)$.

Let S' be a dominating set of D, and set $S_0' = \{x^+ : x \in S'\}$. Fix a vertex $y^- \in X_D^-$. Then there exists a vertex $x \in S'$ such that either $x = y$ or $(x, y) \in A(D)$. In either case, we have $x^+ \in S_0'$ and $x^+y^- \in E(G(D))$. Since y^- is arbitrary, S_0' is a total X_D^--dominating set of G. Consequently, $\gamma(D) \geq \gamma_t(G(D); X_D^-)$. □

By similar argument in the proof of Lemma 2.2, we also obtain the following lemma.

Lemma 2.3 *Let D be a digraph. Then $\gamma(D^-) = \gamma_t(G(D); X_D^+)$.*

By Lemmas 2.1–2.3, the following theorem holds.

Theorem 2.4 *Let D be a digraph. Then $\gamma(D) + \gamma(D^-) = \gamma_t(G(D))$.*

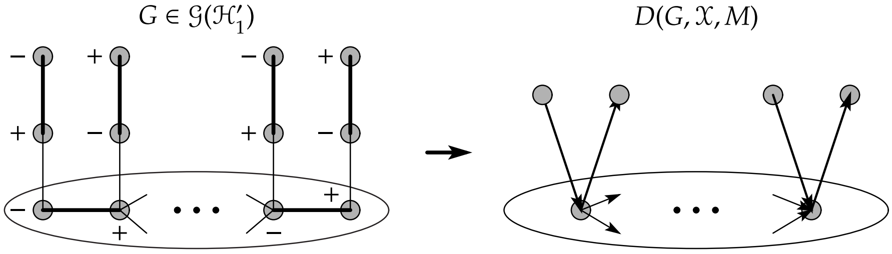

Figure 2: Perfect matching M (bold lines) of a graph G in $\mathcal{G}(\mathcal{H}'_1)$

Let G be a bipartite graph with an ordered bipartition $\mathcal{X} = (X_1, X_2)$, and suppose that G has a perfect matching $M = \{x_1^1 x_2^1, \ldots, x_1^n x_2^n\}$ where $X_i = \{x_i^1, \ldots, x_i^n\}$. Let $D(G, \mathcal{X}, M)$ be the digraph such that

$$V(D(G, \mathcal{X}, M)) = \{x^1, \ldots, x^n\}$$

and

$$A(D(G, \mathcal{X}, M)) = \{(x^i, x^j) : x_1^i x_2^j \in E(G), i \neq j\}.$$

By the definition of $D(G, \mathcal{X}, M)$, we obtain the following observation.

Observation 2.5 (i) *A digraph D is isomorphic to $D(G(D), \mathcal{X}_D, M_D)$.*

(ii) *A bipartite graph G with an ordered bipartition $\mathcal{X}$ having a perfect matching M is isomorphic to $G(D(G, \mathcal{X}, M))$.*

3. An alternative proof of Theorem A

Let $\mathcal{H}'_1 = \{P_3\}$, and define the attachment vertex of P_3 as a leaf of P_3. Then the following theorem holds.

Theorem B ([2, 4]) *Let G be a connected graph of order $n \geq 3$. Then $\gamma_t(G) \leq \frac{2n}{3}$. Furthermore, if $\gamma_t(G) = \frac{2n}{3}$, then $G \in \{C_3, C_6\} \cup \mathcal{G}(\mathcal{H}'_1)$.*

Now we prove Theorem A by Theorem B and some results in Section 2.

Proof of Theorem A. Let D and n be as in Theorem A. Since $G(D)$ is connected and $|V(G(D))| = 2n \geq 4$, it follows from Theorems 2.4 and B that

$$\gamma(D) + \gamma(D^-) = \gamma_t(G(D)) \leq \frac{2|V(G(D))|}{3} = \frac{4n}{3}. \tag{3.1}$$

Assume that $\gamma(D) + \gamma(D^-) = \frac{4n}{3}$. Then (3.1) forces $\gamma_t(G(D)) = \frac{2|V(G(D))|}{3}$. Since $G(D)$ is bipartite, it follows from Theorem B that $G(D) \in \{C_6\} \cup \mathcal{G}(\mathcal{H}'_1)$. For any ordered bipartition $\mathcal{X}$ of C_6 and any perfect matching M of C_6, we have $D(C_6, \mathcal{X}, M) \simeq \overrightarrow{C_3}$; for any bipartite graph $G \in \mathcal{G}(\mathcal{H}'_1)$, any ordered bipartition $\mathcal{X}$ of G and any perfect matching M of G, we can verify that $D(G, \mathcal{X}, M) \in \mathcal{G}(\mathcal{H}_1)$ (see Figure 2). This together with Observation 2.5(i) implies $D \simeq D(G(D), \mathcal{X}_D, M_D) \in \{\overrightarrow{C_3}\} \cup \mathcal{G}(\mathcal{H}_1)$. □

4. Digraphs D with $\delta^{\pm}(D) \geq 1$

Let $H_{1,1}$, $H_{1,2}$ and H_2 be the digraphs depicted in Figure 3. Let $\mathcal{H}_2 = \{W_1, W_2, W_3, W_4\}$ where W_i is the digraph depicted in Figure 4. We define the attachment vertex of W_i as the vertex of W_i enclosed with a circle. In this section, we show the following theorem.

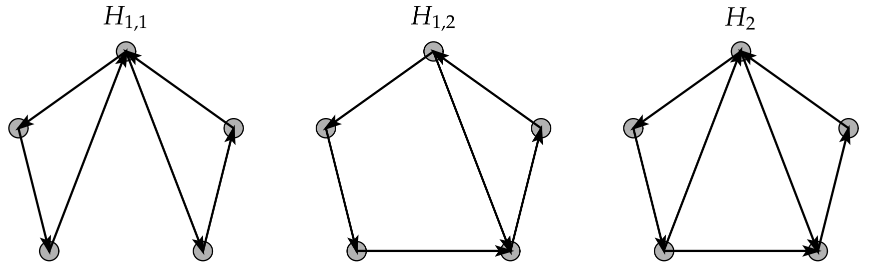

Figure 3: Digraphs $H_{1,1}$, $H_{1,2}$ and H_2

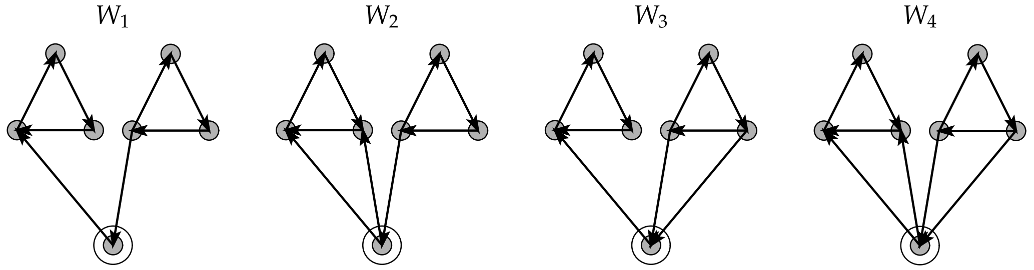

Figure 4: Digraphs W_i with the attachment vertex

Theorem 4.1 *Let D be a connected digraph of order n with $\delta^{\pm}(D) \geq 1$. Then either $D \in \{\overrightarrow{C_3}, \overrightarrow{C_5}, H_{1,1}, H_{1,2}, H_2\}$ or $\gamma(D) + \gamma(D^-) \leq \frac{8n}{7}$. Furthermore, if $n \geq 8$ and $\gamma(D) + \gamma(D^-) = \frac{8n}{7}$, then $D \in \mathcal{G}(\mathcal{H}_2)$.*

Let H'_1 and H'_2 be the graphs depicted in Figure 5. Let $\mathcal{H}'_2 = \{W'_1, W'_2\}$ where W'_i is the graph depicted in Figure 6. We define the attachment vertex of W'_i as the vertex of W'_i enclosed with a circle. Henning [11] proved the following theorem.

Theorem C ([11]) *Let G be a connected graph of order n with $\delta(G) \geq 2$. Then either $G \in \{C_3, C_5, C_6, C_{10}, H'_1, H'_2\}$ or $\gamma_t(G) \leq \frac{4n}{7}$. Furthermore, if $n \geq 15$ and $\gamma_t(G) = \frac{4n}{7}$, then $G \in \mathcal{G}(\mathcal{H}'_2)$.*

Proof of Theorem 4.1. Let D and n be as in Theorem 4.1. Since $G(D)$ is a connected bipartite graph with $\delta(G(D)) \geq 2$ and $|V(G(D))| = 2n$, it follows from Theorems 2.4 and C that either

$$G(D) \in \{C_6, C_{10}, H'_1, H'_2\} \tag{4.1}$$

or

$$\gamma(D) + \gamma(D^-) = \gamma_t(G(D)) \leq \frac{4|V(G(D))|}{7} = \frac{8n}{7}. \tag{4.2}$$

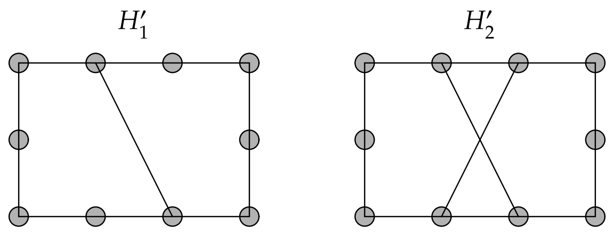

Figure 5: Graphs H'_i

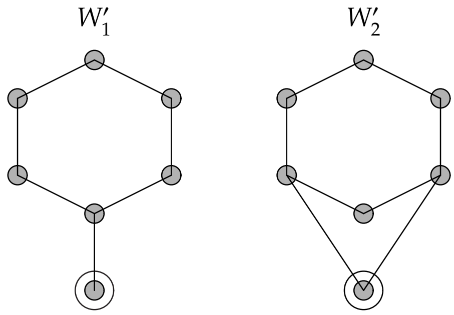

Figure 6: Graphs W_i' with the attachment vertex

If (4.2) holds, then the first statement of the theorem holds. Thus, for the moment, we may assume that (4.1) holds. Let $G \in \{C_6, C_{10}, H_1', H_2'\}$, and let $\mathcal{X}$ be an ordered bipartition of G and M be a perfect matching of G. Then we can check the following: If $G = C_6$, then $D(G, \mathcal{X}, M) = \overrightarrow{C_3}$; if $G = C_{10}$, then $D(G, \mathcal{X}, M) = \overrightarrow{C_5}$; if $G = H_1'$, then $D(G, \mathcal{X}, M) \in \{H_{1,1}, H_{1,2}\}$; if $G = H_2'$, then $D(G, \mathcal{X}, M) = H_2$ (see Figure 7). In either case, $D(G, \mathcal{X}, M) \in \{\overrightarrow{C_3}, \overrightarrow{C_5}, H_{1,1}, H_{1,2}, H_2\}$. Hence $D \simeq D(G(D), \mathcal{X}_D, M_D) \in \{\overrightarrow{C_3}, \overrightarrow{C_5}, H_{1,1}, H_{1,2}, H_2\}$ by Observation 2.5(i). This completes the proof of the first statement of the theorem.

Assume that $n \geq 8$ (i.e., $|V(G(D))| = 2n \geq 16$) and $\gamma(D) + \gamma(D^-) = \frac{8n}{7}$. Then (4.2) forces $\gamma_t(G(D)) = \frac{4|V(G(D))|}{7}$. It follows from Theorem C that $G(D) \in \mathcal{G}(\mathcal{H}_2')$. For any bipartite graph $G \in \mathcal{G}(\mathcal{H}_2')$, any ordered bipartition $\mathcal{X}$ of G and any perfect matching M of G, we can verify that $D(G, \mathcal{X}, M) \in \mathcal{G}(\mathcal{H}_2)$ (see Figure 8). This together with Observation 2.5(i) implies $D \simeq D(G(D), \mathcal{X}_D, M_D) \in \mathcal{G}(\mathcal{H}_2)$.

This completes the proof of Theorem 4.1. □

5. Digraphs D with large $\delta^{\pm}(D)$

There are many results concerning the total domination number of graphs with large minimum degree, as follows.

Theorem D ([1]) *Let G be a connected graph of order n with $\delta(G) \geq 3$. Then $\gamma_t(G) \leq \frac{n}{2}$.*

Theorem E ([20]) *Let G be a connected graph of order n with $\delta(G) \geq 4$. Then $\gamma_t(G) \leq \frac{3n}{7}$.*

Theorem F ([5]) *Let G be a connected graph of order n with $\delta(G) \geq 5$. Then $\gamma_t(G) \leq \frac{2453n}{6500}$.*

Theorem G ([12]) *Let $d \geq 2$ be an integer, and let G be a connected graph of order n with $\delta(G) \geq d$. Then $\gamma_t(G) \leq \frac{(1+\ln d)n}{d}$.*

By Theorem 2.4 and above results, we obtain the following theorem.

Theorem 5.1 *Let $d \geq 2$ be an integer, and let D be a connected digraph of order n with $\delta^{\pm}(D) \geq d$. Then*

$$\gamma(D) + \gamma(D^-) \leq \begin{cases} n & (d = 2) \\ \frac{6n}{7} & (d = 3) \\ \frac{2453n}{3250} & (d = 4) \\ \frac{2(1+\ln(d+1))n}{d+1} & (d \geq 5). \end{cases}$$

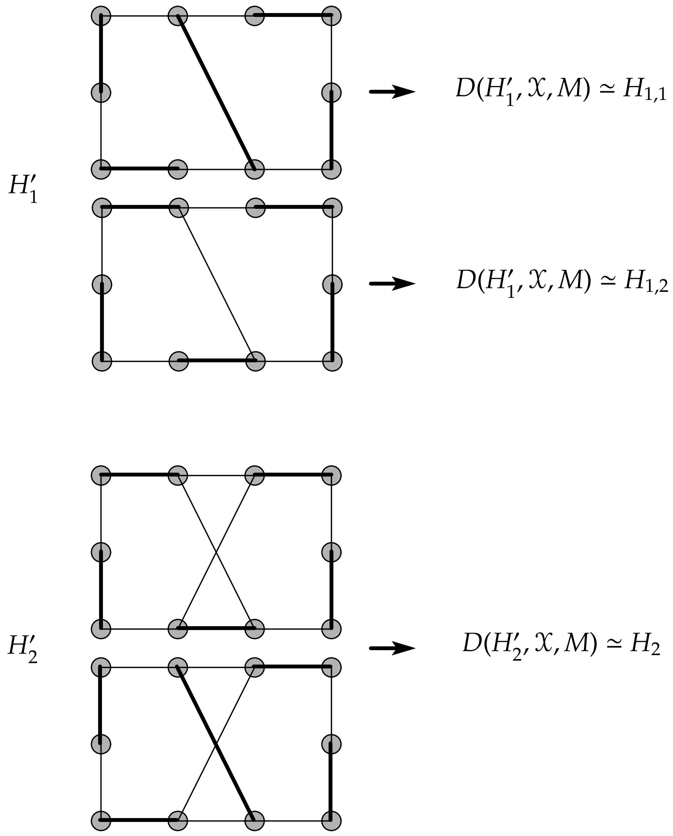

Figure 7: Perfect matchings M (bold lines) of graphs H'_1 and H'_2

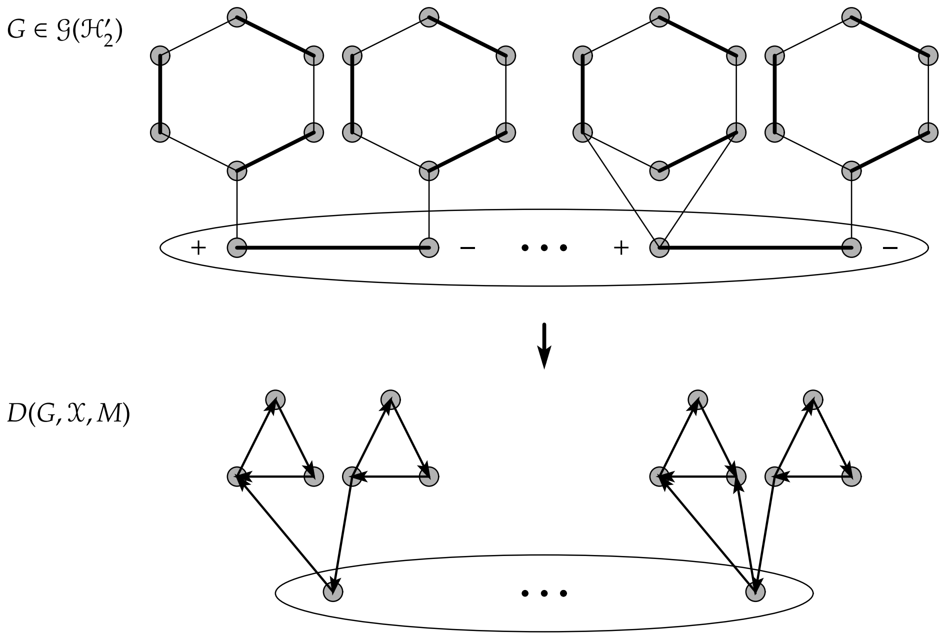

Figure 8: Perfect matching M (bold lines) of a graph G in $\mathcal{G}(\mathcal{H}'_2)$

Henning and Yeo [13] characterized the set $\mathcal{H}^*$ of the connected graphs G of order n with $\delta(G) \geq 3$ and $\gamma_t(G) = \frac{n}{2}$. The set $\mathcal{H}^*$ contains infinitely many bipartite graphs having perfect matchings. In particular, for any bipartite graph $G \in \mathcal{H}^*$ with an ordered bipartition $\mathcal{X}$ having a perfect matching M, it follows from Observation 2.5(ii) that $D(G,\mathcal{X},M)$ is a connected digraph with $\delta^{\pm}(D(G,\mathcal{X},M)) \geq 2$ and

$$\begin{aligned}\gamma(D(G,\mathcal{X},M)) + \gamma(D(G,\mathcal{X},M)^-) &= \gamma_t(G(D(G,\mathcal{X},M)))\\ &= \gamma_t(G)\\ &= \frac{|V(G)|}{2}\\ &= |V(D(G,\mathcal{X},M))|.\end{aligned}$$

Hence Theorem 5.1 for the case $d = 2$ is best possible. Furthermore, by the similar strategy in the proof of Theorems A and 4.1, we can characterize the connected digraphs D of order n with $\delta^{\pm}(D) \geq 2$ and $\gamma(D) + \gamma(D^-) = n$. However, since the bipartite graphs in $\mathcal{H}^*$ have many perfect matchings, it seems that the characterization of such digraphs D is not easy. We leave the characterization problem as an exercise for the readers.

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