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Fekete-Szegö Inequality for Analytic and Bi-univalent Functions Subordinate to Chebyshev Polynomials

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Abstract. In the present study, a new subclass of analytic and bi-univalent functions by means of Chebyshev polynomials is introduced. Certain coefficient bounds for functions belong to this subclass are obtained. Furthermore, the Fekete-Szegö problem in this subclass is solved.

1. Introduction

The classical Chebyshev polynomials of degree n of the first and second kinds, which are denoted respectively by $T_n(t)$ and $U_n(t)$, have generated a great deal of interest in recent years. These orthogonal polynomials, in a real variable t and a complex variable z, have played an important role in applied mathematics, numerical analysis and approximation theory. For this reason, Chebyshev polynomials have been studied extensively, see [8, 10, 16]. In the study of differential equations, the Chebyshev polynomials of the first and second kinds are the solution to the Chebyshev differential equations

$$(1-t^2)y'' - ty' + n^2y = 0 \tag{1}$$

and

$$(1-t^2)y'' - 3ty' + n(n+2)y = 0, \tag{2}$$

respectively. The roots of the Chebyshev polynomials of the first kind are used as nodes in polynomial interpolation and the monic Chebyshev polynomials minimize all norms among monic polynomials of a given degree. For a brief history of Chebyshev polynomials of the first and second kinds and their applications, the reader is referred to [19, 22].

A classical result of Fekete and Szegö [13] determines the maximum value of $|a_3 - \eta a_2^2|$, as a non-linear functional of the real parameter η, for the class of normalized univalent functions

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots .$$

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There are now several results of this type in the literature, each of them dealing with $|a_3 - \eta a_2^2|$ for various classes of functions defined in terms of subordination (see e.g., [1, 20]). Motivated by the earlier work of Dziok et al. [10], the main focus of this work is to utilize the Chebyshev polynomials expansions to solve Fekete-Szegö problem for certain subclass of bi-univalent functions (see, for example, [5–7, 14]).

This paper is divided into three sections with this introduction being the first. In Section 2, we define the class of analytic and bi-univalent functions $\mathscr{B}_\Sigma(\lambda, \mu, t)$ using the generating function for the Chebyshev polynomials of the second kind, and we also discuss some other definitions and results. Section 3 is devoted to solve problems concerning the coefficients of functions in the class $\mathscr{B}_\Sigma(\lambda, \mu, t)$. Section 4 is the main part of the paper, we find the sharp bounds of functionals of Fekete-Szegö type.

2. Definitions and Preliminaries

Let $\mathscr{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{3}$$

which are *analytic* in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Further, by $\mathscr{S}$ we shall denote the class of all functions in $\mathscr{A}$ which are *univalent* in $\mathbb{U}$.

Given two functions $f, g \in \mathscr{A}$. The function $f(z)$ is said to be *subordinate* to $g(z)$ in $\mathbb{U}$, written $f(z) \prec g(z)$, if there exists a Schwarz function $\omega(z)$, analytic in $\mathbb{U}$, with

$$\omega(0) = 0 \text{ and } |\omega(z)| < 1 \text{ for all } z \in \mathbb{U},$$

such that $f(z) = g(\omega(z))$ for all $z \in \mathbb{U}$. Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence (see [17] and [23]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

The Koebe one-quarter theorem [9] asserts that the image of $\mathbb{U}$ under each univalent function f in $\mathscr{S}$ contains a disk of radius $\frac{1}{4}$. According to this, every function $f \in \mathscr{S}$ has an *inverse map* f^{-1}, defined by

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{U}),$$

and

$$f\left(f^{-1}(w)\right) = w \quad \left(|w| < r_0(f); r_0(f) \geq \tfrac{1}{4}\right).$$

In fact, the inverse function is given by

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{4}$$

A function $f \in \mathscr{A}$ is said to be *bi-univalent* in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(w)$ are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (3). For a brief history and some intriguing examples of functions and characterization of the class Σ, see Srivastava et al. [21] and Frasin and Aouf [11], see also [2–4, 12, 15, 18].

The Chebyshev polynomials of the first and second kinds are orthogonal for $t \in [-1, 1]$ and defined as follows:

Definition 2.1. *The Chebyshev polynomials of the first kind are defined by the following three-terms recurrence relation:*

$$T_0(t) = 1,$$
$$T_1(t) = t,$$
$$T_{n+1}(t) := 2tT_n(t) - T_{n-1}(t).$$

The first few of the Chebyshev polynomials of the first kind are

$$T_2(t) = 2t^2 - 1,\ T_3(t) = 4t^3 - 3t,\ T_4(t) = 8t^4 - 8t^2 + 1, \cdots . \tag{5}$$

The generating function for the Chebyshev polynomials of the first kind, $T_n(t)$, is given by:

$$F(z,t) = \frac{1 - tz}{1 - 2tz + z^2} = \sum_{n=0}^{\infty} T_n(t) z^n \quad (z \in \mathbb{U}).$$

Definition 2.2. *The Chebyshev polynomials of the second kind are defined by the following three-terms recurrence relation:*

$$\begin{aligned} &U_0(t) = 1, \\ &U_1(t) = 2t, \\ &U_{n+1}(t) := 2tU_n(t) - U_{n-1}(t). \end{aligned}$$

The first few of the Chebyshev polynomials of the second kind are

$$U_2(t) = 4t^2 - 1,\ U_3(t) = 8t^3 - 4t,\ U_4(t) = 16t^4 - 12t^2 + 1, \cdots . \tag{6}$$

The generating function for the Chebyshev polynomials of the second kind, $U_n(t)$, is given by:

$$H(z,t) = \frac{1}{1 - 2tz + z^2} = \sum_{n=0}^{\infty} U_n(t) z^n \quad (z \in \mathbb{U}).$$

The Chebyshev polynomials of the first and second kinds are connected by the following relations:

$$\frac{dT_n(t)}{dt} = nU_{n-1}(t);\ T_n(t) = U_n(t) - tU_{n-1}(t);\ 2T_n(t) = U_n(t) - U_{n-2}(t).$$

Definition 2.3. *For $\lambda \geq 1, \mu \geq 0$ and $t \in (1/2, 1)$, a function $f \in \Sigma$ given by (3) is said to be in the class $\mathscr{B}_\Sigma(\lambda, \mu, t)$ if the following subordinations hold for all $z, w \in \mathbb{U}$:*

$$(1 - \lambda)\frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) \prec H(z,t) := \frac{1}{1 - 2tz + z^2} \tag{7}$$

and

$$(1 - \lambda)\frac{g(w)}{w} + \lambda g'(w) + \mu w g''(w) \prec H(w,t) := \frac{1}{1 - 2tw + w^2}, \tag{8}$$

where the function $g(w) = f^{-1}(w)$ is defined by (4).

Remark 2.4. 1. *For $\lambda = 1$ and $\mu = 0$, we have the class $\mathscr{B}_\Sigma(1, 0, t) := \mathscr{B}_\Sigma(t)$ of functions $f \in \Sigma$ given by (3) and satisfying the following subordination conditions for all $z, w \in \mathbb{U}$:*

$$f'(z) \prec H(z,t) = \frac{1}{1 - 2tz + z^2}$$

and

$$g'(w) \prec H(w,t) = \frac{1}{1 - 2tw + w^2}.$$

This class of functions have been introduced and studied by Altinkaya and Yalçin [5].

2. *For $\mu = 0$, we have the class $\mathscr{B}_\Sigma(\lambda, 0, t) := \mathscr{B}_\Sigma(\lambda, t)$ of functions $f \in \Sigma$ given by (3) and satisfying the following subordination conditions for all $z, w \in \mathbb{U}$:*

$$(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) \prec H(z,t) = \frac{1}{1-2tz+z^2}$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) \prec H(w,t) = \frac{1}{1-2tw+w^2}.$$

This class of functions have been introduced and studied by Bulut et al. [7].

3. Coefficient Bounds for the Function Class $\mathscr{B}_\Sigma(\lambda, \mu, t)$

We begin with the following result involving initial coefficient bounds for the function class $\mathscr{B}_\Sigma(\lambda, \mu, t)$.

Theorem 3.1. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, \mu, t)$. Then*

$$|a_2| \le \frac{2t\sqrt{2t}}{\sqrt{\left|(1+\lambda+2\mu)^2 - 4t^2\left[(\lambda+2\mu)^2 - 2\mu\right]\right|}} \tag{9}$$

and

$$|a_3| \le \frac{4t^2}{(1+\lambda+2\mu)^2} + \frac{2t}{1+2\lambda+6\mu}. \tag{10}$$

Proof. Let $f \in \mathscr{B}_\Sigma(\lambda, \mu, t)$. From (7) and (8), we have

$$(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) = 1 + U_1(t)w(z) + U_2(t)w^2(z) + \cdots \tag{11}$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) + \mu w g''(w) = 1 + U_1(t)v(w) + U_2(t)v^2(w) + \cdots, \tag{12}$$

for some analytic functions

$$w(z) = c_1 z + c_2 z^2 + c_3 z^3 + \cdots \qquad (z \in \mathbb{U}),$$

and

$$v(w) = d_1 w + d_2 w^2 + d_3 w^3 + \cdots \quad (w \in \mathbb{U}),$$

such that $w(0) = v(0) = 0$, $|w(z)| < 1$ $(z \in \mathbb{U})$ and $|v(w)| < 1$ $(w \in \mathbb{U})$.
It follows from (11) and (12) that

$$(1-\lambda)\frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) = 1 + U_1(t)c_1 z + \left[U_1(t)c_2 + U_2(t)c_1^2\right]z^2 + \cdots$$

and

$$(1-\lambda)\frac{g(w)}{w} + \lambda g'(w) + \mu w g''(w) = 1 + U_1(t)d_1 w + \left[U_1(t)d_2 + U_2(t)d_1^2\right])w^2 + \cdots.$$

A short calculation shows that

$$(1+\lambda+2\mu)\,a_2 = U_1(t)c_1, \tag{13}$$

$$(1+2\lambda+6\mu)\,a_3 = U_1(t)c_2 + U_2(t)c_1^2, \tag{14}$$

and

$$-(1+\lambda+2\mu)\,a_2 = U_1(t)d_1, \tag{15}$$

$$(1+2\lambda+6\mu)\,(2a_2^2-a_3) = U_1(t)d_2 + U_2(t)d_1^2. \tag{16}$$

From (13) and (15), we have

$$c_1 = -d_1, \tag{17}$$

and

$$2\,(1+\lambda+2\mu)^2\,a_2^2 = U_1^2(t)\left(c_1^2+d_1^2\right). \tag{18}$$

By adding (14) to (16), we get

$$2\,(1+2\lambda+6\mu)\,a_2^2 = U_1(t)\,(c_2+d_2) + U_2(t)\left(c_1^2+d_1^2\right). \tag{19}$$

By using (18) in (19), we obtain

$$\left[2\,(1+2\lambda+6\mu) - \frac{2U_2(t)}{U_1^2(t)}(1+\lambda+2\mu)^2\right]a_2^2 = U_1(t)\,(c_2+d_2)\,. \tag{20}$$

It is fairly well known [9] that if $|w(z)| < 1$ and $|v(w)| < 1$, then

$$|c_j| \le 1 \text{ and } |d_j| \le 1 \text{ for all } j \in \mathbb{N}. \tag{21}$$

By considering (6) and (21), we get from (20) the desired inequality (9).
Next, by subtracting (16) from (14), we have

$$2\,(1+2\lambda+6\mu)\,a_3 - 2(1+2\lambda+6\mu)a_2^2 = U_1(t)\,(c_2-d_2) + U_2(t)\left(c_1^2-d_1^2\right). \tag{22}$$

Further, in view of (17), it follows from (22) that

$$a_3 = a_2^2 + \frac{U_1(t)}{2(1+2\lambda+6\mu)}\,(c_2-d_2)\,. \tag{23}$$

By considering (18) and (21), we get from (23) the desired inequality (10). This completes the proof of Theorem 3.1. □

Taking $\lambda = 1$ and $\mu = 0$ in Theorem 3.1, we get the following corollary.

Corollary 3.2. *[7] Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(t)$. Then*

$$|a_2| \le \frac{t\sqrt{2t}}{\sqrt{1-t^2}},$$

and

$$|a_3| \le t^2 + \frac{2}{3}t.$$

For Corollary 3.2, it's worthy to mention that Altinkaya and Yalçin [5] have obtained a remarkable result for the coefficient $|a_2|$, as shown in the following corollary.

Corollary 3.3. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(t)$. Then*

$$|a_2| \leq \frac{t\sqrt{2t}}{\sqrt{1+2t-t^2}}.$$

Taking $\mu = 0$ in Theorem 3.1, we get the following corollary.

Corollary 3.4. *[7] Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, t)$. Then*

$$|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{|(1+\lambda)^2 - 4t^2\lambda^2|}}$$

and

$$|a_3| \leq \frac{4t^2}{(1+\lambda)^2} + \frac{2t}{1+2\lambda}.$$

4. Fekete-Szegö Inequality for the Function Class $\mathscr{B}_\Sigma(\lambda, \mu, t)$

Now, we are ready to find the sharp bounds of Fekete-Szegö functional $a_3 - \eta a_2^2$ defined for $f \in \mathscr{B}_\Sigma(\lambda, \mu, t)$ given by (3).

Theorem 4.1. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, \mu, t)$. Then for some $\eta \in \mathbb{R}$,*

$$|a_3 - \eta a_2^2| \leq \begin{cases} \frac{2t}{1+2\lambda+6\mu}, & |\eta - 1| \leq M \\ \frac{8|\eta-1|t^3}{\left|(1+\lambda+2\mu)^2 - 4t^2\left[(\lambda+2\mu)^2 - 2\mu\right]\right|}, & |\eta - 1| \geq M \end{cases} \tag{24}$$

where

$$M := \frac{\left|(1+\lambda+2\mu)^2 - 4t^2\left[(\lambda+2\mu)^2 - 2\mu\right]\right|}{4(1+2\lambda+6\mu)t^2}.$$

Proof. Let $f \in \mathscr{B}_\Sigma(\lambda, \mu, t)$. By using (20) and (23) for some $\eta \in \mathbb{R}$, we get

$$\begin{aligned} a_3 - \eta a_2^2 &= (1-\eta)\left[\frac{U_1^3(t)\,(c_2+d_2)}{2(1+2\lambda+6\mu)U_1^2(t) - 2(1+\lambda+2\mu)^2 U_2(t)}\right] + \frac{U_1(t)\,(c_2-d_2)}{2(1+2\lambda+6\mu)} \\ &= U_1(t)\left[\left(h(\eta) + \frac{1}{2(1+2\lambda+6\mu)}\right)c_2 + \left(h(\eta) - \frac{1}{2(1+2\lambda+6\mu)}\right)d_2\right], \end{aligned}$$

where

$$h(\eta) = \frac{U_1^2(t)\,(1-\eta)}{2\left[(1+2\lambda+6\mu)U_1^2(t) - (1+\lambda+2\mu)^2 U_2(t)\right]}.$$

Then, in view of (6), we easily conclude that

$$|a_3 - \eta a_2^2| \leq \begin{cases} \frac{2t}{1+2\lambda+6\mu}, & |h(\eta)| \leq \frac{1}{2(1+2\lambda+6\mu)} \\ 4|h(\eta)|t, & |h(\eta)| \geq \frac{1}{2(1+2\lambda+6\mu)} \end{cases}$$

This proves Theorem 4.1. □

We end this section with some corollaries concerning the sharp bounds of Fekete-Szegö functional $a_3 - \eta a_2^2$ defined for $f \in \mathscr{B}_\Sigma(\lambda, \mu, t)$ given by (3).

Taking $\eta = 1$ in Theorem 4.1, we get the following corollary.

Corollary 4.2. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, \mu, t)$. Then*

$$|a_3 - a_2^2| \leq \frac{2t}{1 + 2\lambda + 6\mu}.$$

Taking $\lambda = 1$ and $\mu = 0$ in Theorem 4.1, we get the following corollary.

Corollary 4.3. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(t)$. Then for some $\eta \in \mathbb{R}$,*

$$|a_3 - \eta a_2^2| \leq \begin{cases} \frac{2}{3}t, & |\eta - 1| \leq \frac{1-t^2}{3t^2} \\ \frac{2|\eta-1|t^3}{1-t^2}, & |\eta - 1| \geq \frac{1-t^2}{3t^2} \end{cases}$$

Taking $\eta = 1$ in Corollary 4.3, we get the following corollary.

Corollary 4.4. *Let the function $f(z)$ given be (3) be in the class $\mathscr{B}_\Sigma(t)$. Then*

$$|a_3 - a_2^2| \leq \frac{2}{3}t.$$

Taking $\mu = 0$ in Theorem 4.1, we get the following corollary.

Corollary 4.5. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, t)$. Then for some $\eta \in \mathbb{R}$,*

$$|a_3 - \eta a_2^2| \leq \begin{cases} \frac{2t}{1+2\lambda}, & |\eta - 1| \leq \frac{\left|(1+\lambda)^2 - 4t^2\lambda^2\right|}{4(1+2\lambda)t^2} \\ \frac{8|\eta-1|t^3}{\left|(1+\lambda)^2 - 4t^2\lambda^2\right|}, & |\eta - 1| \geq \frac{\left|(1+\lambda)^2 - 4t^2\lambda^2\right|}{4(1+2\lambda)t^2} \end{cases} \tag{25}$$

Taking $\eta = 1$ in Corollary 4.5, we get the following corollary.

Corollary 4.6. *Let the function $f(z)$ given by (3) be in the class $\mathscr{B}_\Sigma(\lambda, t)$. Then*

$$|a_3 - a_2^2| \leq \frac{2t}{1 + 2\lambda}.$$

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References

[1] T. Al-Hawary, B. A. Frasin, M. Darus, Fekete-Szegö problem for certain classes of analytic functions of complex order defined by the Dziok-Srivastava operator, Acta Mathematica Vietnamica 39.2 (2014) 185–192.

[2] Ş. Altinkaya, S. Yalçin, Initial coefficient bounds for a general class of bi-univalent functions, International Journal of Analysis, Article ID 867871 (2014), 4 pages.

[3] Ş. Altinkaya, S. Yalçin, Coefficient bounds for a subclass of bi-univalent functions, TWMS J. P. and App. Math. 6.2 (2015).

[4] Ş. Altinkaya, S. Yalçin, Coefficient estimates for two new subclasses of bi-univalent functions with respect to symmetric points, Journal of function spaces, Article ID 145242 (2015), 5 pages.

[5] Ş. Altinkaya, S. Yalçin, Estimates on coefficients of a general subclass of bi-univalent functions associated with symmetric q-derivative operator by means of the Chebyshev polynomials, Asia Pacific Journal of Mathematics, 4.2 (2017) 90-99.

[6] Ş. Altinkaya, S. Yalçin, On the Chebyshev polynomial coefficient problem of some subclasses of bi-univalent functions, Gulf Journal of Mathematics (2017) 1464-1473.

[7] S. Bulut, N. Magesh, V. K. Balaji, Initial bounds for analytic and biunivalent functions by means of chebyshev polynomials, Analysis 11.1 (2017) 83–89.

[8] E. H. Doha, The first and second kind Chebyshev coefficients of the moments of the general-order derivative of an infinitely differentiable function, Int. J. of Comput. Math. 51 (1994) 21-35.

[9] P. L. Duren, Univalent functions, Grundlehren der Mathematischen Wissenschaften 259, Springer-Verlag, New York, 1983.

[10] J. Dziok, R. K. Raina, J. Sokol, Application of Chebyshev polynomials to classes of analytic functions, Comptes Rendus Mathematique 353.5 (2015) 433–438.

[11] B. A. Frasin, M. K. Aouf, New subclasses of bi-univalent functions, Appl. Math. Lett. 24.9 (2011) 1569-1573.

[12] B. A. Frasin, Tariq Al-Hawary, Initial Maclaurin coefficients bounds for new subclasses of bi-univalent functions, Theory and App. of Math. & Computer Science 5.2 (2015) 186–193.

[13] M. Fekete, G. Szegö, Eine Bermerkung über ungerade schlichte Funktionen, Journal of the London Math. Soc. 1.2 (1933) 85-89.

[14] H. Ö. Güney, G. Murugusundaramoorthy, K. Vijaya, Coefficient bounds for subclasses of biunivalent functions associated with the Chebyshev polynomials, Journal of Complex Analysis (2017) 11 pages.

[15] N. Magesh, J. Yamini, Coefficient bounds for a certain subclass of bi-univalent functions, Int. Math. Forum 8.27 (2013) 1337-1344.

[16] J. C. Mason, Chebyshev polynomial approximations for the L-membrane eigenvalue problem, SIAM J. Appl. Math. 15 (1967) 172-186.

[17] S. S. Miller, P. T. Mocanu, Differential Subordination: theory and applications, CRC Press, New York, 2000.

[18] S. Porwal, M. Darus, On a new subclass of bi-univalent functions, J. Egypt. Math. Soc. 21.3 (2013) 190-193.

[19] B. Simon, Orthogonal polynomials on the unit circle, American Mathematical Society, 2009.

[20] H. M. Srivastava, A. K. Mishra, M. K. Das, The Fekete-Szegö problem for a subclass of close-to-convex functions, Complex Variables and Elliptic Equations 44.2 (2001) 145–163.

[21] H. M. Srivastava, A. K. Mishra, P. Gochhayat, Certain subclasses of analytic and bi-univalent functions, Appl. Math. Lett. 23 (2010) 1188-1192.

[22] G. Szegö, Orthogonal polynomials, American Mathematical Society, New York, 1939.

[23] F. Yousef, A. A. Amourah, M. Darus, Differential sandwich theorems for p-valent functions associated with a certain generalized differential operator and integral operator, Italian Journal of Pure and Applied Mathematics 36 (2016) 543–556.