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Sandwich Results for Subclasses of Multivalent Meromorphic Functions Associated with Iterations of the Cho-Kwon-Srivastava Transform

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Abstract. In this paper we obtain subordination, superordination and sandwich results for multivalent meromorphic functions, involving the iterations of the Cho-Kwon-Srivastava operator and its combinations. Certain interesting particular cases are also pointed out.

1. Introduction and definitions

Let H be the class of analytic functions in the *open* unit disk

$$\mathbb{U} := \{z \in \mathbb{C} : |z| < 1\}$$

and let $H[a, n]$ $(a \in \mathbb{C},\ n \in \mathbb{N} := \{1, 2, 3, \dots\})$ be the subclass of H consisting of functions of the form:

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots \quad (z \in \mathbb{U}). \tag{1}$$

Also, let Σ_p denote the class of functions of the form:

$$f(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} a_{k-p} z^{k-p} \quad (p \in \mathbb{N} := \{1, 2, 3, \dots\}) \tag{2}$$

which are analytic in the *punctured unit disc*

$$\mathbb{U}^\star := \mathbb{U} \setminus \{0\}.$$

Suppose that f and F are analytic in H. We say that f is *subordinate* to F, (or F is *superordinate* to f), write as

$$f \prec F \text{ in } \mathbb{U} \text{ or } f(z) \prec F(z) \qquad (z \in \mathbb{U}),$$

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if there exists a function $\omega \in H$, satisfying the conditions of the Schwarz lemma (i.e. $\omega(0) = 0$ and $|\omega(z)| < 1$) such that

$$f(z) = F(\omega(z)) \qquad (z \in \mathbb{U}).$$

It follows that

$$f(z) \prec F(z)\ (z \in \mathbb{U}) \Longrightarrow f(0) = F(0) \quad \text{and} \quad f(\mathbb{U}) \subset F(\mathbb{U}).$$

In particular, if F is *univalent* in $\mathbb{U}$, then the reverse implication also holds (cf.[24]).

Let $\phi : \mathbb{C}^2 \times \mathbb{U} \to \mathbb{C}$ and h be univalent in $\mathbb{U}$. If p is analytic in $\mathbb{U}$ and satisfies the following differential subordination

$$\phi(p(z), zp'(z); z) \prec h(z) \qquad (z \in \mathbb{U}), \tag{3}$$

then p is called a *solution* of the first order differential subordination (3). A univalent function q is called a dominant of the solutions of the differential subordination, or more precisely a *dominant* if $p \prec q$, for all p satisfying (3). A dominant $\widetilde{q}$ that satisfies $\widetilde{q} \prec q$, for all dominant q of (3) is called the *best dominant* of (3).

Similarly, let $\phi : \mathbb{C}^2 \times \mathbb{U} \to \mathbb{C}$ and $h \in H$. Let $p \in H$ be such that $p(z)$ and $\phi(p(z), zp'(z); z)$ are univalent in $\mathbb{U}$. If $p(z)$ satisfies the following differential superordination

$$h(z) \prec \phi(p(z), zp'(z); z) \qquad (z \in \mathbb{U}) \tag{4}$$

then $p(z)$ is called a *solution* of the first order *differential superordination* (4).

An analytic function q is called a subordinant of the solutions of the differential superordination, or more precisely a *subordinant* if $q \prec p$, for all p satisfying (4). A univalent subordinant $\widetilde{q}$ that satisfies $q \prec \widetilde{q}$, for all subordinants q of (4) is said to be the *best subordinant* (see [24, 25]).

Recently, Mishra *et al.* [28] introduced and obtained subordination results of multivalent meromorphic functions defined by using the Carlson-Shaffer operator [13] and iterations of a meromorphic analogue of the Cho-Kwon-Srivastava operator [15] (see also [21, 22, 37]) and its combinations.

They defined the operator $\ell^{n,m}_{\lambda,p}(a, c) : \Sigma_p \longrightarrow \Sigma_p$ by

$$\begin{aligned} \ell^{n,m}_{\lambda,p}(a, c)f(z) &= \mathcal{L}^{\lambda,n}_p(a, c)C^{(t,m)} f(z) \\ &= \frac{1}{z^p} + \sum_{k=1}^{\infty} \left(\frac{(\lambda + p)_k (c)_k}{(a)_k (1)_k}\right)^n \left(\frac{p - kt}{p}\right)^m a_{k-p} z^{k-p} \\ &(a, c \in \mathbb{Z} \setminus \mathbb{Z}^-_0, \mathbb{Z}^-_0 := \{0, -1, -2, \cdots\}, \lambda > -p, z \in \mathbb{U}^\star) \end{aligned} \tag{5}$$

which also generalizes several previously studied familiar operators as well as provides meromorphic analogue for certain well known operators for analytic functions. The readers may refer to details in the paper by Mishra *et al.* [28].

Miller and Mocanu [24–26] and Bulboacă [10, 11] provide detailed account on the theory of differential subordination and differential superordination. Ali *et al.* [2], Bulboacă [12], Shanmugam *et al.* [31, 34] have obtained sufficient conditions on the normalized analytic function f such that *sandwich subordinations* of the following form hold true:

$$q_1(z) \prec \mathcal{I}(f) \prec q_2(z) \qquad (z \in \mathbb{U}),$$

where q_1, q_2 are univalent in $\mathbb{U}$ with $q_1(0) = q_2(0) = 1$ and $\mathcal{I}$ is a suitable functional or operator. Recently, several authors have been studied the sandwich results for analytic functions [3–9, 14, 19, 23, 27, 30, 32, 33].

For earlier investigation related to meromorphic functions and subordination see, for example, [1, 16–18, 20, 29, 36]. In the present investigation we obtain several subordination, superordination and sandwich results for multivalent meromorphic functions involving the operator $\ell^{n,m}_{\lambda,p}(a, c)$. In order to prove our main results, we need the following definitions and lemmas.

Definition 1.1. *(*[25], Definition 2,p.817; also see [24], Defintion 2.2b, p.21*) Let Q be the set of functions f that are analytic and injective on $\overline{\mathbb{U}} \setminus \mathbb{E}(f)$, where*

$$\mathbb{E}(f) := \left\{ \zeta : \zeta \in \partial\mathbb{U} \text{ and } \lim_{z\to\zeta} f(z) = \infty \right\}$$

and such that $f'(\zeta) \neq 0$ for $\zeta \in \partial\mathbb{U} \setminus \mathbb{E}(f)$.

Lemma 1.2. ([24], Theorem 3.4h, p.132) *Let q be univalent in the open unit disk $\mathbb{U}$ and θ and ϕ be analytic in a domain $\mathbb{D}$ containing $q(\mathbb{U})$ with $\phi(w) \neq 0$ when $w \in q(\mathbb{U})$. Set $\Phi(z) = zq'(z)\phi(q(z))$, and $h(z) = \theta(q(z)) + \Phi(z)$. Suppose that*

1. *Φ is starlike in $\mathbb{U}$*
   and
2. $\Re\left(\frac{zh'(z)}{\Phi(z)}\right) > 0 \qquad (z \in \mathbb{U}).$

If $p \in H[q(0), n]$ for some $n \in \mathbb{N}$ with $p(\mathbb{U}) \subset \mathbb{D}$ and

$$\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z))$$

then $p \prec q$ and q is the best dominant.

Lemma 1.3. [31] *Let q be univalent convex in the open unit disk $\mathbb{U}$ and $\psi, \gamma \in \mathbb{C}$ with $\Re\left(1 + \frac{zq''(z)}{q'(z)}\right) > \max\{0, -\Re(\psi/\gamma)\}$. If $p(z)$ is analytic and*

$$\psi p(z) + \gamma zp'(z) \prec \psi q(z) + \gamma zq'(z),$$

then $p \prec q$ and q is the best dominant.

Lemma 1.4. ([26]) *Let q be univalent in the open unit disk $\mathbb{U}$ and θ and ϕ be analytic in a domain $\mathbb{D}$ containing $q(\mathbb{U})$. Set $\Phi(z) = zq'(z)\phi(q(z))$. Suppose that*

1. *$\Phi(z)$ is univalent starlike in $\mathbb{U}$*
   and
2. $\Re\left(\frac{\theta'(q(z))}{\phi(q(z))}\right) > 0 \qquad (z \in \mathbb{U}).$

If $p \in H[q(0), 1] \cap Q$, with $p(\mathbb{U}) \subseteq \mathbb{D}$; $\theta(p(z)) + zp'(z)\phi(p(z))$ is univalent in $\mathbb{U}$ and

$$\theta(q(z)) + zq'(z)\phi(q(z)) \prec \theta(p(z)) + zp'(z)\phi(p(z)), \qquad (z \in \mathbb{U})$$

then $q \prec p$ and q is the best subordinant.

Lemma 1.5. ([25], Theorem 8, p.822) *Let q be univalent convex in the open unit disk $\mathbb{U}$ and $\gamma \in \mathbb{C}$, with $\Re(\gamma) > 0$. If $p \in H[q(0), 1] \cap Q$, $p(z) + \gamma zp'(z)$ is univalent in $\mathbb{U}$ and*

$$q(z) + \gamma zq'(z) \prec p(z) + \gamma zp'(z) \qquad (z \in \mathbb{U}),$$

then $q \prec p$ and q is the best subordinant.

Lemma 1.6. ([28]) *Let a and c be complex numbers $(a, c \notin \mathbb{Z}_0^-)$, $n, m \in \mathbb{N}$, $t > 0$, $\lambda \in \mathbb{R}$ and $\lambda > -p$. Let $f \in \Sigma_p$. Then the following identities hold.*

$$z(\ell_{\lambda,p}^{n,m}(a,c)f(z))' = \frac{p(1-t)}{t}\ell_{\lambda,p}^{n,m}(a,c)f(z) - \frac{p}{t}\ell_{\lambda,p}^{n,m+1}(a,c)f(z) \tag{6}$$

$$z(\ell_{\lambda,p}^{m}(a+1,c)f(z))' = a\ell_{\lambda,p}^{m}(a,c)f(z) - (a+p)\ell_{\lambda,p}^{m}(a+1,c)f(z) \tag{7}$$

$$z(\ell_{\lambda,p}^{m}(a,c)f(z))' = (\lambda+p)\ell_{\lambda+1,p}^{m}(a,c)f(z) - (\lambda+2p)\ell_{\lambda,p}^{m}(a,c)f(z) \tag{8}$$

$$z(\ell_{\lambda,p}^{m}(a,c)f(z))' = c\,\ell_{\lambda,p}^{m}(a,c+1)f(z) - (c+p)\ell_{\lambda,p}^{m}(a,c)f(z). \tag{9}$$

2. Subordination results

We state and prove the following results.

Theorem 2.1. *Let $\rho \in \mathbb{C}^\star$. Let the function $f \in \Sigma_p$ and let q be a univalent convex function in $\mathbb{U}$ with $q(0) = 1$. Suppose f and q satisfy the following conditions $\Re\left(1 + \frac{zq''(z)}{q'(z)}\right) > \max\left\{0, \frac{p^2}{t}\Re\left(\frac{1}{\rho}\right)\right\}$ $(z \in \mathbb{U}, t > 0, p \in \mathbb{N})$ and*

$$\frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z)) \prec q(z) - \frac{\rho t}{p^2} zq'(z) \qquad (z \in \mathbb{U}) \tag{10}$$

where $\ell_{\lambda,p}^{n,m}(a,c)$ is defined by (5). *Then*

$$z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \prec q(z) \qquad (z \in \mathbb{U}) \tag{11}$$

and q is the best dominant of (11).

Proof. Let the function g be defined by

$$g(z) := z^p \ell_{\lambda,p}^{n,m}(a,c)f(z). \tag{12}$$

Then the function $g(z)$ is analytic in $\mathbb{U}$ with $g(0) = 1$. Differentiation of (12) with respect to z followed by application of the identity (6), yield

$$z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z) = g(z) - \frac{t}{p} zg'(z) \qquad (z \in \mathbb{U}). \tag{13}$$

By using (12) and (13) in the subordination condition (10) becomes

$$g(z) - \frac{\rho t}{p^2} zg'(z) \prec q(z) - \frac{\rho t}{p^2} zq'(z) \qquad (z \in \mathbb{U}).$$

Now, an application of Lemma 1.3 with $\gamma = -\frac{\rho t}{p^2}$ and $\psi = 1$ gives the assertion in (11). This completes the proof of Theorem 2.1. $\square$

Corollary 2.2. *Let $\rho \in \mathbb{C}^\star$, $-1 \le B < A \le 1$ and $f \in \Sigma_p$. Suppose any one of the following pair of condition is satisfied $\frac{|B|-1}{|B|+1} < -\frac{p^2}{t}\Re\left(\frac{1}{\rho}\right)$ and*

$$\frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z)) \prec \frac{1+Az}{1+Bz} - \frac{\rho t}{p^2}\frac{(A-B)z}{(1+Bz)^2} \qquad (z \in \mathbb{U}).$$

Then

$$z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \prec \frac{1+Az}{1+Bz} \qquad (z \in \mathbb{U})$$

and $\frac{1+Az}{1+Bz}$ is the best dominant.

Taking $p = A = 1$ and $B = -1$ in Corollary 2.2, we get the following.

Corollary 2.3. *Let $\rho \in \mathbb{C}^\star$ and $f \in \Sigma_1$. Suppose any one of the following of condition is satisfied $\Re\left(\frac{1}{\rho}\right) < 0$ and*

$$\rho(z\ell_{\lambda,1}^{n,m+1}(a,c)f(z)) + (1-\rho)(z\ell_{\lambda,1}^{n,m}(a,c)f(z)) \prec \frac{1+z}{1-z} - \rho t\frac{2z}{(1-z)^2} \qquad (z \in \mathbb{U}).$$

Then

$$z\ell_{\lambda,1}^{m}(a,c)f(z) \prec \frac{1+z}{1-z} \qquad (z \in \mathbb{U})$$

and $\frac{1+z}{1-z}$ is the best dominant.

For $n = 1$, we state and prove the following results.

Theorem 2.4. *Let the function $q \in H$ be non zero univalent in $\mathbb{U}$ with $q(0) = 1$ and*

$$\Re\left\{1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right\} > 0 \qquad (z \in \mathbb{U}). \tag{14}$$

Let $\mu \in \mathbb{C}^\star, \nu, \eta \in \mathbb{C}$ and $\nu + \eta \neq 0$. Let $f \in \Sigma_p$ satisfy the condition $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta} \neq 0 \quad (z \in \mathbb{U})$. If

$$\mu\left[p + \frac{\nu z(\ell^m_{\lambda+1,p}(a,c)f(z))' + \eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\nu\ell^m_{\lambda+1,p}(a,c)f(z) + \eta\ell^m_{\lambda,p}(a,c)f(z)}\right] \prec \frac{zq'(z)}{q(z)}, \tag{15}$$

then

$$\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu + \eta}\right]^\mu \prec q(z) \tag{16}$$

and q is the best dominant in (16). *(The power is the principal one.)*

Proof. Let the function $g(z)$ be defined by

$$g(z) := \left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu + \eta}\right]^\mu. \tag{17}$$

Then g is analytic in $\mathbb{U}$. Logarithmic differentiation of (17) yields:

$$\frac{zg'(z)}{g(z)} = \mu\left[p + \frac{\nu z(\ell^m_{\lambda+1,p}(a,c)f(z))' + \eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\nu\ell^m_{\lambda+1,p}(a,c)f(z) + \eta\ell^m_{\lambda,p}(a,c)f(z)}\right].$$

With a view to apply Lemma 1.2, we set

$$\theta(w) := 1,\ \phi(w) := 1/w \qquad (w \in \mathbb{C} \setminus \{0\}),$$

$$\Phi(z) = zq'(z)\phi(q(z)) = \frac{zq'(z)}{q(z)} \qquad (z \in \mathbb{U})$$

and

$$h(z) = \theta(q(z)) + \Phi(z) = 1 + \frac{zq'(z)}{q(z)}.$$

By making use of the hypothesis (14), we see that $\Phi(z)$ is univalent starlike in $\mathbb{U}$. Since $h(z) = 1 + \Phi(z)$, we further more get that $\Re\left(\frac{zh'(z)}{\Phi(z)}\right) > 0$. By a routine calculation using (17) we have

$$\theta(g(z)) + zg'(z)\phi(g(z)) = 1 + \mu\left[p + \frac{\nu z(\ell^m_{\lambda+1,p}(a,c)f(z))' + \eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\nu\ell^m_{\lambda+1,p}(a,c)f(z) + \eta\ell^m_{\lambda,p}(a,c)f(z)}\right].$$

Therefore, the hypothesis (15) is equivalently written as the following:

$$\theta(g(z)) + zg'(z)\phi(g(z)) \prec 1 + \frac{zq'(z)}{q(z)} = \theta(q(z)) + zq'(z)\phi(q(z)).$$

Now, by an application of Lemma 1.2 we have $g(z) \prec q(z)$. We, thus, get the assertions in (16). This completes the proof of Theorem 2.4. $\square$

Taking $\nu = 0$, $\eta = 1$ and $q(z) = \frac{1+Az}{1+Bz}$ in Theorem 2.4, it is easy to check that the assumption (14) holds whenever $-1 \leq B < A \leq 1$; hence we obtain the next result.

Corollary 2.5. *Let $-1 \leq B < A \leq 1$ and $\mu \in \mathbb{C}^\star$. Let $f \in \Sigma_p$ and suppose that $z^p \ell^m_{\lambda,p}(a,c)f(z) \neq 0 \quad (z \in \mathbb{U}; m \in \mathbb{N}_0; \lambda > -p; p \in \mathbb{N})$. If*

$$\mu\left[p + \frac{\eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\ell^m_{\lambda,p}(a,c)f(z)}\right] \prec \frac{(A-B)z}{(1+Az)(1+Bz)},$$

then

$$\left[z^p \ell^m_{\lambda,p}(a,c)f(z)\right]^\mu \prec \frac{1+Az}{1+Bz}$$

and $\frac{1+Az}{1+Bz}$ is the best dominant. (The power is the principal one.)

Corollary 2.6. *If the function f is univalent meromorphic starlike of order α $(0 \leq \alpha < 1)$ in $\mathbb{U}^\star$ and if $(1-\alpha) = \frac{\beta}{\mu}$, $0 \leq \beta \leq 1$, then*

$$(zf(z))^\mu \prec (1-z)^{2\beta}.$$

The function $(1-z)^{2\beta}$ is the best dominant. In particular, $|zf(z)|$ is bounded by $2^{2(1-\alpha)}$ in $\mathbb{U}$. (The powers on both sides are principal ones.)

By adopting the method of proof of Theorem 2.4, the following Theorem 2.7 and 2.10 can be proved, where in the respective settings are suitably used. We only state these theorems without proofs.

Theorem 2.7. *Let the function $q \in H$ be non zero univalent in $\mathbb{U}$ with $q(0) = 1$ and $\Re\left\{1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right\} > 0$. Let $\mu \in \mathbb{C}^\star, \nu, \eta \in \mathbb{C}$ and $\nu + \eta \neq 0$. Let $f \in \Sigma_p$ satisfy the condition $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta} \neq 0$. Set*

$$\Delta(z) = \left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu + \mu\left[p + \frac{\nu z(\ell^m_{\lambda+1,p}(a,c)f(z))' + \eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\nu \ell^m_{\lambda+1,p}(a,c)f(z) + \eta \ell^m_{\lambda,p}(a,c)f(z)}\right] \quad (z \in \mathbb{U}). \tag{18}$$

If

$$\Delta(z) \prec q(z) + \frac{zq'(z)}{q(z)},$$

then

$$\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu \prec q(z) \tag{19}$$

and q is the best dominant in (19).

Taking $q(z) = \frac{1+Az}{1+Bz}$, $(-1 \leq B < A \leq 1)$, $\nu = 0$ and $\eta = 1$ in Theorem 2.7 we have the following.

Corollary 2.8. *Let $\mu \in \mathbb{C}^\star$, $-1 \leq B < A \leq 1$ and* (18) *hold true. Let $f \in \Sigma_p$ satisfy the condition $z^p \ell^m_{\lambda,p}(a,c)f(z) \neq 0$. If*

$$\left[z^p \ell^m_{\lambda,p}(a,c)f(z)\right]^\mu + \mu\left[p + \frac{z(\ell^m_{\lambda,p}(a,c)f(z))'}{\ell^m_{\lambda,p}(a,c)f(z)}\right] \prec \frac{1+Az}{1+Bz} + \frac{(A-B)z}{(1+Az)(1+Bz)},$$

then

$$\left[z^p \ell^m_{\lambda,p}(a,c)f(z)\right]^\mu \prec \frac{1+Az}{1+Bz}$$

and $\frac{1+Az}{1+Bz}$ *is the best dominant.*

Again taking $p = 1 = \nu,\ \eta = \lambda = m = 0,\ a = c$ and $q(z) = \frac{1+z}{1-z}$ in Corollary 2.8, we obtain the following.

Corollary 2.9. *Let* $f \in \Sigma$ *be such that* $zf(z) \neq 0$ *for all* $z \in \mathbb{U}$ *and let* $\mu \in \mathbb{C}^\star$. *If*

$$(zf(z))^\mu + \mu\left[1 + \frac{zf'(z)}{f(z)}\right] \prec \frac{1+z}{1-z} + \frac{2z}{(1+z)(1-z)},$$

then

$$(zf(z))^\mu \prec \frac{1+z}{1-z}$$

and $\frac{1+z}{1-z}$ *is the best dominant.*

Theorem 2.10. *Let the function* $q \in H$ *be univalent in* $\mathbb{U}$ *with* $q(0) = 1$ *and* $\Re\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\Re\left(\frac{\delta}{\gamma}\right)\right\}$. *Let* $\mu, \gamma \in \mathbb{C}^\star, \delta, \nu, \eta \in \mathbb{C}$ *and* $\nu + \eta \neq 0$. *Let* $f \in \Sigma_p$ *satisfy the condition* $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta} \neq 0 \quad (z \in \mathbb{U})$. *Set*

$$\Omega(z) = \left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu \times\left\{\delta + \gamma\mu\left[p + \frac{\nu z(\ell^m_{\lambda+1,p}(a,c)f(z))' + \eta z(\ell^m_{\lambda,p}(a,c)f(z))'}{\nu\ell^m_{\lambda+1,p}(a,c)f(z) + \eta\ell^m_{\lambda,p}(a,c)f(z)}\right]\right\} \quad (z \in \mathbb{U}). \tag{20}$$

If

$$\Omega(z) \prec \delta q(z) + \gamma z q'(z),$$

then

$$\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu \prec q(z) \tag{21}$$

and q *is the best dominant in* (21).

Taking $q(z) = \frac{1+Az}{1+Bz}$, $(-1 \leq B < A \leq 1)$, $\nu = 0 = \delta$ and $\eta = 1 = \gamma$ in Theorem 2.10 we have the following.

Corollary 2.11. *Let* $\mu \in \mathbb{C}^\star$, $-1 \leq B < A \leq 1$ *and* (20) *hold true. Let* $f \in \Sigma_p$ *satisfy the condition* $z^p \ell^m_{\lambda,p}(a,c)f(z) \neq 0$. *If*

$$\left[z^p \ell^m_{\lambda,p}(a,c)f(z)\right]^\mu \left[\delta + \mu\left(p + \frac{z(\ell^m_{\lambda,p}(a,c)f(z))'}{\ell^m_{\lambda,p}(a,c)f(z)}\right)\right] \prec \frac{(A-B)z}{(1+Az)(1+Bz)},$$

then

$$\left[z^p \ell^m_{\lambda,p}(a,c)f(z)\right]^\mu \prec \frac{1+Az}{1+Bz}$$

and $\frac{1+Az}{1+Bz}$ *is the best dominant.*

Again taking $p = 1 = \nu,\ \eta = \lambda = m = 0,\ a = c$ and $q(z) = \frac{1+z}{1-z}$ in Corollary 2.11, we obtain the following.

Corollary 2.12. *Let $f \in \Sigma$ be such that $zf(z) \neq 0$ for all $z \in \mathbb{U}$ and let $\mu \in \mathbb{C}^\star$. If*

$$(zf(z))^\mu \left[\mu\left(1 + \frac{zf'(z)}{f(z)}\right)\right] \prec \frac{2z}{(1-z)^2},$$

then

$$(zf(z))^\mu \ \prec\ \frac{1+z}{1-z}$$

and $\frac{1+z}{1-z}$ is the best dominant.

3. Superordination and sandwich results

Theorem 3.1. *Let $q \in H$ be a univalent convex function in $\mathbb{U}$ with $q(0) = 1$ and let $\rho \in \mathbb{C}^\star$. Also let the function $f \in \Sigma_p$, be such that $z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \in H[1,1] \cap Q$ and $\frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z))$ is univalent in $\mathbb{U}$. If*

$$q(z) - \frac{\rho t}{p^2} zq'(z) \prec \frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z)) \qquad (z \in \mathbb{U}). \tag{22}$$

Then

$$q(z) \prec z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \qquad (z \in \mathbb{U})$$

and q is the best subordinant.

Proof. As in the proof of our Theorem 2.1, let the function $g(z)$ be defined by (12). Then

$$z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z) = g(z) - \frac{t}{p} zg'(z) \qquad (z \in \mathbb{U}). \tag{23}$$

By using (12) and (23) in the subordination condition (22) becomes

$$q(z) - \frac{\rho t}{p^2} zq'(z) \prec g(z) - \frac{\rho t}{p^2} zg'(z) \qquad (z \in \mathbb{U}).$$

Now, an application of Lemma 1.5 with $\gamma = -\frac{\rho t}{p^2}$ gives

$$q(z) \prec g(z) = z^p \ell_{\lambda,p}^{n,m}(a,c)f(z)$$

and q is the best subordinant. The proof of Theorem 3.1 is completed. □

Taking $q(z) = \frac{1+Az}{1+Bz},\ -1 \leq B < A \leq 1$ in Theorem 3.1, we obtain the following result.

Corollary 3.2. *Let , $-1 \leq B < A \leq 1$ and $\rho \in \mathbb{C}^\star$ with $\frac{|B|-1}{|B|+1} < -\frac{p^2}{t}\Re\left(\frac{1}{\rho}\right)$. Let $f \in \Sigma_p$ suppose that $z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \in H[1,1] \cap Q$. If the function $\frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z))$ is univalent in $\mathbb{U}$, and*

$$\frac{1+Az}{1+Bz} - \frac{\rho t}{p^2}\frac{(A-B)z}{(1+Bz)^2} \prec \frac{\rho}{p}(z^p \ell_{\lambda,p}^{n,m+1}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell_{\lambda,p}^{n,m}(a,c)f(z)) \qquad (z \in \mathbb{U}),$$

then

$$\frac{1+Az}{1+Bz} \prec z^p \ell_{\lambda,p}^{n,m}(a,c)f(z) \qquad (z \in \mathbb{U})$$

and $\frac{1+Az}{1+Bz}$ is the best subordinant.

Taking $p = A = 1$ and $B = -1$ in Corollary 3.2, we get the following.

Corollary 3.3. *Let $\rho \in \mathbb{C}^\star$ with $\Re\left(\frac{1}{\rho}\right) < 0$. Let $f \in \Sigma_1$ suppose that $z\ell_{\lambda,1}^{n,m}(a,c)f(z) \in H[1,1] \cap Q$. If the function $\rho(z\ell_{\lambda,1}^{n,m+1}(a,c)f(z)) + (1-\rho)(z\ell_{\lambda,1}^{n,m}(a,c)f(z))$ is univalent in $\mathbb{U}$, and*

$$\frac{1+z}{1-z} - \rho t\frac{2z}{(1-z)^2} \prec \rho(z\ell_{\lambda,1}^{n,m+1}(a,c)f(z)) + (1-\rho)(z\ell_{\lambda,1}^{n,m}(a,c)f(z)) \qquad (z \in \mathbb{U}),$$

then

$$\frac{1+z}{1-z} \prec z\ell_{\lambda,1}^{n,m}(a,c)f(z) \qquad (z \in \mathbb{U})$$

and $\frac{1+z}{1-z}$ is the best subordinant.

Theorem 3.4. *Let the function q be non zero univalent in $\mathbb{U}$ with $q(0) = 1$ and $\Re\left\{1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right\} > 0$. Let $\mu \in \mathbb{C}^\star$, $\nu, \eta \in \mathbb{C}$ with $\nu + \eta \neq 0$. Let $f \in \Sigma_p$ be such that $\left[\frac{\nu z^p\ell_{\lambda+1,p}^{m}(a,c)f(z)+\eta z^p\ell_{\lambda,p}^{m}(a,c)f(z)}{\nu+\eta}\right]^\mu \in H[1,1] \cap Q$ and $\mu\left[p + \frac{\nu z(\ell_{\lambda+1,p}^{m}(a,c)f(z))'+\eta z(\ell_{\lambda,p}^{m}(a,c)f(z))'}{\nu\ell_{\lambda+1,p}^{m}(a,c)f(z)+\eta\ell_{\lambda,p}^{m}(a,c)f(z)}\right]$ is univalent in $\mathbb{U}$. If*

$$\frac{zq'(z)}{q(z)} \prec \mu\left[p + \frac{\nu z(\ell_{\lambda+1,p}^{m}(a,c)f(z))' + \eta z(\ell_{\lambda,p}^{m}(a,c)f(z))'}{\nu\ell_{\lambda+1,p}^{m}(a,c)f(z) + \eta\ell_{\lambda,p}^{m}(a,c)f(z)}\right], \tag{24}$$

then

$$q(z) \prec \left[\frac{\nu z^p\ell_{\lambda+1,p}^{m}(a,c)f(z) + \eta z^p\ell_{\lambda,p}^{m}(a,c)f(z)}{\nu+\eta}\right]^\mu \tag{25}$$

and q is the best subordinant. (The power is the principal one.)

Proof. With a view to apply Lemma 1.4 we set

$$\theta(w) = 1,\ \phi(w) = \frac{1}{w} \qquad (w \in \mathbb{C} \setminus \{0\})$$

and

$$\Phi(z) = zq'(z)\phi(q(z)) = \frac{zq'(z)}{q(z)} \qquad (z \in \mathbb{U}).$$

We first observe that Φ is starlike in $\mathbb{U}$. Furthermore,

$$\Re\left\{\frac{\theta'(q(z))}{\phi(z)}\right\} > 0 \qquad (z \in \mathbb{U}).$$

Hence, the condition (24) is equivalent to the following:

$$\theta(q(z)) + zq'(z)\phi(q(z)) \prec \theta(g(z)) + zg'(z)\phi(g(z)).$$

Therefore, by using Lemma 1.4, we have:

$$q(z) \prec g(z) \qquad (z \in \mathbb{U})$$

and q is the best subordinant. This is precisely the assertion of (25). The proof of Theorem 3.4 is completed. □

By adopting the method of proof of Theorem 3.4, the following Theorem 3.5 and 3.6 can be proved, where in the respective settings are suitably used. We only state these theorems without proofs.

Theorem 3.5. *Let $q \in H$ be a univalent convex function in $\mathbb{U}$ with $q(0) = 1$. Further more, suppose that q satisfies the following $\Re\{q(z)\} > 0$ and $\Re\left\{1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right\} > 0 \quad (z \in \mathbb{U})$. Let $\mu \in \mathbb{C}^{\star}, \nu, \eta \in \mathbb{C}$ and $\nu + \eta \neq 0$. Let $f \in \Sigma_p$ satisfy the following conditions $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta} \neq 0 \quad (z \in \mathbb{U})$ and $\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^{\mu} \in H[1,1] \cap Q$. If the function $\Delta(z)$ given by* (18) *is univalent in $\mathbb{U}$ and*

$$q(z) + \frac{zq'(z)}{q(z)} \prec \Delta(z),$$

then

$$q(z) \prec \left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu + \eta}\right]^{\mu} \tag{26}$$

and q is the best subordinant in (26).

Theorem 3.6. *Let the function $q \in H$ be a univalent convex in $\mathbb{U}$ with $q(0) = 1$ and $\Re\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\Re\left(\frac{\delta}{\gamma}\right)\right\}$. Let $\mu, \gamma \in \mathbb{C}^{\star}, \delta, \nu, \eta \in \mathbb{C}$ and $\nu + \eta \neq 0$. Let $f \in \Sigma_p$ satisfy the following $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta} \neq 0 \quad (z \in \mathbb{U})$ and $\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^{\mu} \in H[1,1] \cap Q$. If the function Ω given by* (20) *is univalent in $\mathbb{U}$ and*

$$\delta q(z) + \gamma z q'(z) \prec \Omega(z),$$

then

$$q(z) \prec \left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z) + \eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu + \eta}\right]^{\mu} \tag{27}$$

and q is the best subordinant in (27). *(The power is the principal one.)*

By combining Theorems 2.1 with 3.1, we obtain the following sandwich results

Theorem 3.7. *Let q_1 and q_2 be two univalent convex functions in $\mathbb{U}$ with $q_1(0) = q_2(0) = 1$. Also let the function $f \in \Sigma_p$, be such that $z^p \ell^{n,m}_{\lambda,p}(a,c)f(z) \in H[1,1] \cap Q$ and $\frac{\rho}{p}(z^p \ell^{n,m+1}_{\lambda,p}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell^{n,m}_{\lambda,p}(a,c)f(z))$ is univalent in $\mathbb{U}$. If*

$$q_1(z) - \frac{\rho t}{p^2} z q_1'(z) \prec \frac{\rho}{p}(z^p \ell^{n,m+1}_{\lambda,p}(a,c)f(z)) + \frac{p-\rho}{p}(z^p \ell^{n,m}_{\lambda,p}(a,c)f(z)) \prec q_2(z) - \frac{\rho t}{p^2} z q_2'(z),$$

then

$$q_1(z) \prec z^p \ell^{n,m}_{\lambda,p}(a,c)f(z) \prec q_2(z) \qquad (z \in \mathbb{U}) \tag{28}$$

where q_1 and q_2 are respectively the best subordinant and the best dominant in (28).

Combining Theorems 2.7 with 3.5 and Theorems 2.10 with 3.6, respectively, we get the following sandwich results:

Theorem 3.8. *Let q_1 and q_2 be univalent convex functions in $\mathbb{U}$ and further more satisfy the following conditions: $q_1(0) = q_2(0) = 1$, $\Re\{q_1(z)\} > 0$, $q_2 \neq 0$ and $\Re\left\{1 + \frac{zq_j''(z)}{q_j'(z)} - \frac{zq_j'(z)}{q_j(z)}\right\} > 0 \quad (j = 1,2;\ z \in \mathbb{U})$. Let $\mu \in \mathbb{C}^{\star}, \nu, \eta \in$*

$\mathbb{C}$ *and* $\nu+\eta\neq 0$. *Let* $f\in\Sigma_p$ *satisfy the following conditions* $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\neq 0$ $(z\in\mathbb{U})$ *and* $\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu\in H[1,1]\cap Q$. *Let the function* $\Delta(z)$ *be defined on* $\mathbb{U}$ *as in* (18). *If*

$$q_1(z)+\frac{zq_1'(z)}{q_1(z)}\prec\Delta(z)\prec q_2(z)+\frac{zq_2'(z)}{q_2(z)},$$

then

$$q_1(z)\prec\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu\prec q_2(z)\qquad(z\in\mathbb{U})\tag{29}$$

where q_1 *and* q_2 *are respectively the best subordinant and the best dominant in* (29).

Theorem 3.9. *Let* q_1 *and* q_2 *be univalent convex functions in* $\mathbb{U}$ *with* $q_1(0)=q_2(0)=1$. *Further more suppose that* q_2 *satisfies the following condition* $\Re\left\{1+\frac{zq_2''(z)}{q_2'(z)}\right\}>\max\left\{0,-\Re\left(\frac{\delta}{\gamma}\right)\right\}$ $(z\in\mathbb{U})$. *Let* $\mu\in\mathbb{C}^\star$ *and* $\nu,\eta\in\mathbb{C}$ *with* $\nu+\eta\neq 0$ *and* $\Re\left(\frac{\delta}{\gamma}\right)>0$. *Let* $f\in\Sigma_p$ *satisfy the following conditions* $\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\neq 0$ $(z\in\mathbb{U})$ *and* $\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu\in H[1,1]\cap Q$. *Let the function* $\Omega(z)$ *be defined on* $\mathbb{U}$ *as in* (20). *If*

$$\delta q_1(z)+\gamma zq_1'(z)\prec\Omega(z)\prec\delta q_2(z)+\gamma zq_2'(z),$$

then

$$q_1(z)\prec\left[\frac{\nu z^p \ell^m_{\lambda+1,p}(a,c)f(z)+\eta z^p \ell^m_{\lambda,p}(a,c)f(z)}{\nu+\eta}\right]^\mu\prec q_2(z)\qquad(z\in\mathbb{U})\tag{30}$$

where q_1 *and* q_2 *are respectively the best subordinant and the best dominant in* (30).

4. Observations and Concluding Remarks

In our present investigation, we have derived several differential subordination, superordination and sandwich results for subclasses of multivalent meromorphic functions in the punctured unit disk associated with iterations of the Cho-Kwon-Srivastava transform $\ell^{n,m}_{\lambda,p}(a,c)$ defined by (5). Furthermore, by using the relations (7), (8) and (9), we have also obtained the corresonding differential subordination, superordination and sandwich results for the transform $\ell^m_{\lambda,p}(a,c)$. For details one may see [35].

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