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Best Linear Approximation and Coefficients Characterization of Entire Functions in Doubly Connected Domains

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Abstract. In the present paper, we established the relations between growth parameters order and type in terms of coefficients occurring in generalized Faber series expansions of entire function and corresponding best linear approximation errors in supnorm in doubly connected domains.

1. Introduction

Let K be a continuum (not a point) on the complex plane $\mathbb{C}$ that does not separate the plane and let Ω be an arbitrary domain containing K such that its boundary consists of at least three points. Let U_R denote a disk of radius R with boundary T_R, and U denotes the unit disk with boundary T. If Ω is a canonical neighborhood G_R of K then the boundary ∂G_R coincides with the preimage of the circle T_R under the conformal mapping of the $\mathbb{C} \setminus K$ onto $\mathbb{C} \setminus \overline{U}$ in the extended complex plane. For $K = \overline{U}$ and $K = [-1, 1]$ the Faber approximation coincides, respectively, with Taylor and Chebyshev approximations. For a wide class of continua K, when K is a compact convex set, it is possible by approximating functions by the partial sums of the Faber series.

It has been noticed that in a number of cases the partial sums of the Faber series are much easier to compute than the corresponding approximation polynomials (see [1]). But polynomial expansions convergent on K become too sensitive with respect to the placing of singular points: the Faber series of a function f diverges at any point $z \in \mathbb{C} \setminus \overline{G_R}$ whenever some level line Γ_R contains at least one singularity of f. Also, the order of these convergence in the uniform metric on K depends on the position of analyticity domains of f in the remaining part of $\mathbb{C}$. So, if Ω is a simply connected domain containing K, then an optimal basis for approximation of functions analytic in Ω and continuous on $\overline{\Omega}$ in the metric of $\mathbb{C}(K)$ will be a fortiori not polynomial. These bases were constructed by V.D. Erokhin [3] as a natural generalization of the Faber series as follows:

Let $\Omega \setminus K$ is doubly connected domain E and H denote the conformal mapping of E onto the ring $\{w : 1 < |w| < R\}, R = mod(\Omega \setminus K) \leq +\infty$. The conformal mapping of a doubly connected domain into a ring

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can always be equivalent to a double conformal mapping of simply connected areas, at the same time one can start with any of the two simply connected domains, defined by a given doubly connected domain.

Given any two arbitrary numbered boundary continua E_1 and E_2 of a doubly connected domain E, the conformal mapping $H : E \to \{w : 1 < |w| < R\}$ can be represented as the composition $H = F_2 o F_1$, where F_1 is the conformal mapping of the simply connected domain with boundary E_1 and F_2 is the conformal mapping of the simply connected domain with boundary $F_1(E_2)$. In the case $E_1 = \partial\Omega$ and $E_2 = \partial K$, define $F = F_1$ and $\Phi = F_2$ so that $F_2(\infty) = \infty$. Then

$$H(z) = \Phi[F(z)], z \in E.$$

Denote $\xi = H^{-1}, \phi = F^{-1}$, and $\varphi = \Phi^{-1}$, it gives

$$\xi(w) = \phi[\varphi(w)], 1 < |w| < R.$$

Let $H(\Omega)$ be the space of functions analytic in Ω, equipped with the topology of uniform convergence on arbitrary compact subsets of Ω. Setting $C_\rho = \{z : |H(z)| = \rho, 1 < \rho < R\}$ and $\Omega_\rho = intC_\rho$. Following the V.D. Erokhin [3], the formulas

$$\chi(w) = \frac{1}{2\pi i} \int_{T_\rho} \frac{f(\xi(\tau))}{\tau - w} d\tau, w \in U_\rho, \tag{1.1}$$

and

$$f(z) = \frac{1}{2\pi i} \int_{C_\rho} \frac{\chi[H(\zeta)]}{F(\zeta) - F(z)} F'(\zeta) d\zeta, z \in \Omega_\rho \tag{1.2}$$

are mutually inverse and establish a linear topological isomorphism in the space $H(U_R)$ and $H(\Omega)$. Let $e_n(z)(n = 1, 2, \dots,)$ be the function defined by (1.2) with $\chi(w) = w^n$. We find the isomorphic image $\chi \in H(U_R)$ of an arbitrary function $f \in H(\Omega)$ by (1.1), and then expand it into a Taylor series. Taking the inverse transformation by (1.2), we get the following expansion

$$f(z) = \sum_{k=0}^{\infty} a_k e_k(z), z \in \Omega, \tag{1.3}$$

with coefficients for $k \in \mathbb{N}$ only

$$a_k = \frac{1}{2\pi i} \int_{T\rho} \frac{\chi(w)}{w^{k+1}} dw = \frac{1}{2\pi i} \int_{T\rho} \frac{f[\xi(w)]}{w^{k+1}} dw. \tag{1.4}$$

Note. In the Faber case, where $\partial\Omega$ agree with the level line Γ_R of the continuum K and the mapping H extends to a conformal mapping of the entire domain $\overline{\mathbb{C}} \setminus \Omega$, with $F(z) = z$ the formula (1.2) defines the classical Faber operator.

Consider the weaker topology of functions corresponding to uniform convergence on K i.e., to the form

$$\|f(z)\| \equiv \|f\| = \max_{z \in K} |f(z)|$$

and using the formulas (1.1)-(1.4) V.D. Erokhin [3] obtained the following relations:

1. $\limsup_{k\to\infty}(\|a_k\|)^{\frac{1}{k}} \le 1$.
2. $\limsup_{k\to\infty}(\|e_k\|)^{\frac{1}{k}} \le 1$.
3. $|a_k| \le A(\rho) \sup_{z\in\Omega_\rho} |f(z)|.\frac{1}{\rho^k}(k = 0, 1, 2, \dots), f \in H(\Omega_\rho), 1 < \rho \le R, \Omega_R = \Omega$.
4. $\sup_{z\in\Omega_\rho} |e_k(z)| \le A(\rho)\rho^k$ where $A(\rho)$ is a constant depending only on ρ, K and Ω.

The main purpose of the studied bases is to derive from (1)-(4) the following Bernstein theorem on the possibility of completely characterizing functions of the class $H(\Omega)$ by the best approximations on the continuum K by linear forms of the form (1.3).

Let $\Gamma_\alpha = H^{-1}(D(0,R^\alpha) = \{z \in \mathbb{C} : |z| < R^\alpha\}), R > 1 \forall \alpha \in]0,1[$. Erokhin shows that the sequence $(e_k)_{k\geq 0}$ is a common basis for the spaces $H(\Omega), H(\Gamma_\alpha), (0 < \alpha < 1)$ but generally $e_k \neq 0$ when $k < 0$. The new coefficients are define by analogy with the Faber for all $f \in H(\Gamma_\alpha), 0 < \rho < \alpha < 1$:

$$a_k = \frac{1}{2\pi i}\int_{|\zeta|=\rho} \frac{\chi(\zeta)}{\zeta^{k+1}} d\zeta$$

with for all $|\zeta| < R^\rho$,

$$\chi(\zeta) = \sum_{k=0}^{\infty} a_k \zeta^k = \frac{1}{2\pi i}\int_{|\tau|=R^\rho} \frac{f(\xi(\tau))}{\tau - \zeta} d\tau.$$

We have the following property [8]:
$f \in H(\Gamma_\alpha)$ has Γ_α as domain of holomorphy, if and only if, χ has the disc $D(0,R^\alpha)$ as domain of holomorphy.

We denote the partial sum of the n^{th} order of series (1.3) by

$$p_n(z) = \sum_{k=0}^{n} a_k e_k(z). \tag{1.5}$$

For $f \in H(\Omega)$, set

$$E_n(f) = \min_{(a_o,a_1,\ldots,a_n)} \|f(z) - p_n(z)\|.$$

Now we have the following theorem.
Theorem A. For the function $f(z) \in H(\Omega)$ it is necessary and sufficient that

$$\limsup_{n\to\infty} (E_n(f))^{\frac{1}{n}} \leq \frac{1}{R}. \tag{1.6}$$

Proof. Let $f \in H(\Omega)$, then we have

$$E_n(f) \leq \|f(z) - \sum_{k=0}^{n} a_k e_k(z)\| \leq \sum_{k=n+1}^{\infty} |a_k| \|e_k\|$$

in view of (2) and (3), we get (1.6) immediately.

Conversely, let $\{\tilde{p_n}(z) = \sum_{k=0}^{n} a_k^{(n)} e_k(z)\}_{n=0}^{\infty}$ be a sequence of forms satisfying (1.5), so $\|f - \tilde{p_n}\| = \varepsilon_n(f)$. Then

$$\|\tilde{p_{n+1}} - \tilde{p_n}\| \leq 2\varepsilon_n(f),$$

or

$$|a_{n+1}^{(n+1)}| \leq 2\varepsilon_n(f)\|a_{n+1}\|, |a_k^{(n+1)} - a_k^{(n)}| \leq 2\varepsilon_n(f)\|a_k\|, (k = 0,1,\ldots,n).$$

Using (1.6) with (1) and (2), we get

$$\lim_{n\to\infty} a_k^{(n)} = a_k (k = 0,1,\ldots)$$

it gives

$$f(z) = \sum_{k=0}^{\infty} a_k e_k(z), z \in \Omega$$

where

$$\limsup_{k\to\infty}(|a_k|)^{\frac{1}{k}} \le \frac{1}{R}. \tag{1.7}$$

Hence from (1.6),(1.7) and (4) we conclude that the series expansion (1.3) converges uniformly and absolutely in $\Omega_\rho(1<\rho<R)$ for $f\in H(\Omega)$. Hence the proof is completed.
Now we derive the following relations between $E_n(f)$ and a_k which will be useful in the sequel.

$$E_n(f) \le \|f-p_n\| = \|\sum_{k=n+1}^{\infty} a_k e_k\| \le K^* \sum_{k=n+1}^{\infty} |a_k| \tag{1.8}$$

where $K^* = mes(\Omega)$.

$$\begin{aligned}
|a_k| =& \frac{1}{2\pi i}|\int_{|w|=\rho} \frac{f[\xi(w)] - p_{\tilde{k-1}}(f,\xi(w))}{w^{k+1}}dw| \\
&\le \frac{1}{2\pi i}\int_0^{2\pi} |f[\xi(\rho e^{it})] - p_{\tilde{k-1}}(f,\xi(\rho e^{it})|\rho^{-k}dt \\
&\le \max_{0\le t\le 2\pi} |f[\xi(\rho e^{it})] - p_{\tilde{k-1}}(f,\xi(\rho e^{it})|\rho^{-k} \\
&= E_{k-1}(f)\rho^{-k}.
\end{aligned}$$

We can write

$$z=\xi(w)=\nu w+\nu_o+\frac{\nu_1}{w}+\cdots+\frac{\nu_k}{w^k}, 1<|w|<R,$$

$$\begin{aligned}
\max_{|w|=\rho}|f[\xi(w)]| &= \max_{|w|=\rho}|\frac{f(w(\nu+\frac{\nu_o}{w}+\cdots+\frac{\nu_k}{w^{k+1}}))}{w^{k+1}}| \\
&= \max_{|z|=R}|f(z)|, z=w.\nu, R=|w|.\nu.
\end{aligned}$$

Therefore, we have

$$|a_k| \le E_{k-1}(f)R^{-k} \quad for \;\; z\in\Omega. \tag{1.9}$$

Corollary 1.1. For the function $f(z)\in H(\Omega)$ to be entire it is necessary and sufficient that

$$\lim_{k\to\infty}|a_k|^{\frac{1}{k}}=0.$$

Let $M(R)=\max_{|z|=R}|f(z)|$ be the maximum modulus of $f(z)$. The growth of an entire function $f(z)$ is measured in terms of its order η and type σ defined as follows:

$$\limsup_{R\to\infty}\frac{\log\log M(R)}{\log R}=\eta \tag{1.10}$$

$$\limsup_{R\to\infty}\frac{\log M(R)}{R^\eta}=\sigma \tag{1.11}$$

for $0<\eta<\infty$.

Kumar [6] studied growth properties of entire functions over Jordan domains by using Faber polynomials. He characterized order and type in terms of L^p-approximation errors, $2\le p\le\infty$ and improved the various results of Seremeta [9] and Ganti and Srivastava [4]. Giroux [5] and Kumar and Vandna [7] characterized order and type of entire/analytic functions in terms of approximation errors by using Faber

polynomials in Jordan domains. To the best of our knowledge, coefficients characterization of order and type of an entire function in terms of best linear approximation errors in doubly connected domain have not been obtained so far.
In the present paper, we have made an attempt to bridge this gap. First we obtain coefficients characterization for order and type of an entire function over doubly connected domain. Finally, we obtain necessary and sufficient conditions of order and type of an entire function in terms of best linear approximation errors.

2. Main Results

Theorem 2.1. The function f is the restriction to doubly connected domain Ω of an entire function of finite order η if and only if

$$\mu = \limsup_{k\to\infty} \frac{k\log k}{-\log|a_k|} \tag{2.1}$$

is finite, and the order η of f is equal to μ.

Proof. Let $f(z) = \sum_{k=0}^{\infty} a_k e_k(z)$ be an entire function. Using (3) we have

$$|a_k| \le A(R)M(R)R^{-k}. \tag{2.2}$$

First we prove that $\eta \ge \mu$. Let $\mu, \varepsilon > 0$ be such that $\varepsilon < \mu < \infty$. Then using (2.1) we get

$$-(\mu-\varepsilon)\log|a_k| \le k\log k$$

or

$$\log|a_k| \ge -\frac{1}{(\mu-\varepsilon)}k\log k$$

for a sequence of values of $k \to \infty$. Now in view of (2.2) we have

$$\begin{aligned}\log M(R) &\ge \log|a_k| + \log(R^k) - \log A(R)\\ &\ge -\frac{1}{(\mu-\varepsilon)}(k\log k) + k\log R - \log A(R)\\ &= k[(\log R - \frac{1}{(\mu-\varepsilon)}\log k) - O(1)].\end{aligned}$$

The right hand side attains its maximum value at $R = (ek)^{\frac{1}{(\mu-\varepsilon)}}$. So by substituting this value of R in above inequality, we get

$$\log M(R) \ge \frac{k}{(\mu-\varepsilon)} - 0(1) = \frac{R^{(\mu-\varepsilon)}}{e(\mu-\varepsilon)}.$$

or

$$\eta = \limsup_{R\to\infty}\frac{\log\log M(R)}{\log R} \ge \mu-\varepsilon.$$

Since ε is arbitrary, it gives

$$\eta \ge \mu. \tag{2.3}$$

In order to prove reverse inequality in (2.3) we assume that

$$\limsup_{k\to\infty}\frac{k\log k}{-\log|a_k|} = \beta < \infty.$$

Then for every $\varepsilon > 0$, there exists $m(\varepsilon)$ such that for all $k \geq m$, we have

$$|a_k| \leq C' k^{-\frac{k}{(\beta+\varepsilon)}}.$$

Since $f(z) = \sum_{k=0}^{\infty} a_k e_k(z)$, we have

$$|f(z)| \leq C' k^{-\frac{k}{(\beta+\varepsilon)}} |e_k(z)|.$$

Using (4), we get

$$|f(z)| \leq C' k^{-\frac{k}{(\beta+\varepsilon)}} A(\rho)\rho^k, z \in \Omega_\rho.$$

Hence

$$M(R) \leq C' A(R)[\sum_{k=0}^{k_o} k^{-\frac{k}{(\beta+\varepsilon)}} R^k + \sum_{k_o+1}^{\infty} k^{-\frac{k}{(\beta+\varepsilon)}} R^k].$$

Following the proof of Bose and Sharma [2, Thm.IV] we obtain

$$M(R) \leq 0\{e^{(2R)^{\beta+2\varepsilon}}\}.$$

Proceeding to limits and using the arbitrariness of ε, we get

$$\limsup_{R\to\infty} \frac{\log\log M(R)}{\log R} \leq \beta. \tag{2.4}$$

Combining (2.3) and (2.4) we get the required result (2.1).

Example 2.1. Consider the function $f(z) = \sum_{k=0}^{\infty}(kR)^{-k} e_k(z)$. By using Theorem 2.1 with simple calculation we get $\mu = 1$, it gives the order of the function $f(z)$ is $\eta = 1$. Also, this function satisfies the Corollary 1.1.

Theorem 2.2. The function f is restriction to doubly connected domain Ω of an entire function of finite order η and type σ if, and only if

$$\alpha^* = e\sigma\eta \tag{2.5}$$

where

$$\alpha^* = \limsup_{k\to\infty}\{k(|a_k|)^{\frac{\eta}{k}}\}, 0 < \alpha^* < \infty.$$

Proof. Let f be an entire function of finite order η and type σ. Then

$$|f(z)| \leq e^{(\sigma+\varepsilon)R^\eta}, \quad z \in \Omega$$

and using (3), we have

$$|a_k| \leq A(R) e^{(\sigma+\varepsilon)R^\eta} R^{-k}$$

for all R sufficiently large. The minimum value of right hand side of above inequality is attained at

$$R = [\frac{k}{\eta(\sigma+\varepsilon)}]^{\frac{1}{\eta}}.$$

It gives

$$|a_k| \leq A'[\frac{e\eta(\sigma+\varepsilon)}{k}]^{\frac{k}{\eta}}$$

or

$$k(|a_k|)^{\frac{\eta}{k}} \leq e\eta(\sigma + \varepsilon) + O(1).$$

Proceeding to limits, since ε is arbitrary, we get

$$\limsup_{k\to\infty} k(|a_k|)^{\frac{\eta}{k}} \leq e\eta\sigma. \tag{2.6}$$

Conversely, let

$$\limsup_{k\to\infty} k(|a_k|)^{\frac{\eta}{k}} = \alpha^* < \infty.$$

Then for given $\varepsilon > 0$ there exists $N(\varepsilon)$ such that for all $k \geq N$, we have

$$|a_k| \leq k^{-\frac{k}{\eta}}[e\eta(\alpha^* + \varepsilon)]^{\frac{k}{\eta}}.$$

Since $f(z) = \sum_{k=0}^{\infty} a_k e_k(z)$, therefore

$$|f(z)| \leq \sum_{k=0}^{\infty} k^{-\frac{k}{\eta}}[e\eta(\alpha^* + \varepsilon)]^{\frac{k}{\eta}}|e_k(z)|.$$

Now applying (4) in above inequality, we obtain

$$|f(z)| \leq \sum_{k=0}^{\infty} k^{-\frac{k}{\eta}}[e\eta(\alpha^* + \varepsilon)]^{\frac{k}{\eta}} A(R)R^k, z \in \Omega.$$

we estimate the right hand side of the above inequality proceeding on the limits of proof of Bose and Sharma [2, Thm. V] and we get

$$|f(z)| \leq o\{e^{(\alpha^*+\varepsilon)R^\eta}\}.$$

Hence

$$M(R) \leq o\{e^{(\alpha^*+\varepsilon)R^\eta}\}$$

or

$$\frac{\log M(R)}{R^\eta} \leq \alpha^* + \varepsilon.$$

Proceeding the limits, we get

$$\limsup_{R\to\infty} \frac{\log M(R)}{R^\eta} \leq \alpha^*. \tag{2.7}$$

Combining (2.6) and (2.7), we get the required result.

Example 2.2. Using Theorem 2.2 for the function $f(z) = \sum_{k=0}^{\infty}(kR)^{-k\alpha}e_k(z)$ we get $\alpha^* = \frac{1}{R}, \eta = \frac{1}{\alpha}$ and the type $\sigma = \frac{\alpha}{eR}$.

Theorem 2.3. The function f is the restriction to doubly connected domain Ω of an entire function of finite order η if, and only if

$$\limsup_{k\to\infty} \frac{k\log k}{-\log E_k(f)} = \eta.$$

Proof. Let f is an entire function having finite order η. Then by Theorem 2.1, we have

$$|a_k| \leq C' k^{-\frac{k}{(\eta+\varepsilon)}}.$$

Using (1.8), we have

$$E_n(f) \leq K^* C' \sum_{k=n+1}^{\infty} k^{-\frac{k}{(\eta+\varepsilon)}} \leq K^* C' n^{-\frac{n}{(\eta+\varepsilon)}}$$

for all sufficiently large n. Therefore, we get

$$-\log E_n(f) \geq \frac{n \log n}{(\eta+\varepsilon)} - O(1).$$

Since ε is arbitrary, proceeding the limits, we get

$$\limsup_{n\to\infty} \frac{n \log n}{-\log E_n(f)} \leq \eta. \tag{2.8}$$

Conversely, let

$$\limsup_{k\to\infty} \frac{k \log k}{-\log E_k(f)} \leq \alpha'.$$

Suppose $\alpha' < \infty$. Then for every $\varepsilon > 0$ there exists $N(\varepsilon)$ such that for all $k > N$, we have

$$E_k(f) \leq k^{-\frac{k}{(\alpha'+\varepsilon)}}.$$

Using (1.9) we have

$$|a_k| \leq (k-1)^{-\frac{(k-1)}{(\alpha'+\varepsilon)}} R^{-k}.$$

Hence

$$|f(z)| \leq A \sum_{k=0}^{\infty} k^{-\frac{k}{(\alpha'+\varepsilon)}} A(R), \quad , z \in \Omega.$$

Now following the method used in Theorem 2.1 to estimate the right hand side, we obtain

$$M(R) \leq o\{e^{(2A(R))^{(\alpha'+2\varepsilon)}}\}.$$

Proceeding the limits, we get

$$\eta = \limsup_{R\to\infty} \frac{\log \log M(R)}{\log R} \leq \alpha'. \tag{2.9}$$

Combining (2.8) and (2.9) we get the required result.

Example 2.3. Using inequality (1.9) in Theorem 2.3 for the function $f(z) = \sum_{k=0}^{\infty} (kR)^{-k} e_k(z)$ we get the same order $\eta = 1$.

Theorem 2.4. The function f is the restriction to doubly connected domain Ω of an entire function of finite order η and type σ if, and only if

$$\limsup_{k\to\infty} \{k (E_k(f))^{\frac{\eta}{k}}\} = e\eta\sigma. \tag{2.10}$$

Proof. From Theorem 2.2, we have

$$|a_k| \leq A'[\frac{e\eta(\sigma+\varepsilon)}{k}]^{\frac{k}{\eta}}.$$

From (8) we get

$$E_n(f) \leq K^* \sum_{k=n+1}^{\infty} A'[\frac{e\eta(\sigma+\varepsilon)}{k}]^{\frac{k}{\eta}} \leq K^* A'[\frac{e\eta(\sigma+\varepsilon)}{k}]^{\frac{n}{\eta}}$$

or

$$\limsup_{n\to\infty} n(E_n(f))^{\frac{\eta}{n}} \leq e\eta\sigma.$$

The converse part can be proved similarly following on the lines of Theorem 2.2 by using (1.9). This completes the proof of Theorem 2.4.

Example 2.4. By analogy of (1.9), we define $|a_k| \leq E_{k-1}(f)R^{-k\alpha}$ for $z \in \Omega$. Now using Theorem 2.4 for the function $f(z) = \sum_{k=0}^{\infty}(kR)^{-k\alpha}e_k(z)$ we get type $\sigma = \frac{\alpha}{eR^2}$.

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