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Fixed Points of Interval Valued Neutrosophic Soft Mappings

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Abstract. In this paper, we introduce some new notions such as interval valued neutrosophic soft points, interval valued neutrosophic soft mappings, interval valued neutrosophic soft Hausdorff topological spaces, and interval valued neutrosophic soft compact topological spaces. Cantor's intersection theorem is proved for interval valued neutrosophic soft sets. The aim of this paper is to establish the existence of fixed points of interval valued neutrosophic soft mappings on interval valued neutrosophic soft compact topological spaces. Some examples are provided to support the concepts and the results presented herein.

1. Introduction

A fuzzy set A defined on a universe of discourse U is characterized by the mapping $\mu_A : U \rightarrow [0,1]$ (see, [21]). Due to certain ambiguities and uncertainties in the given data, assigning a single value $\mu_A(x)$ to x in U is a difficult task. To overcome this problem, the concept of interval valued fuzzy sets was proposed ([18]). An interval valued fuzzy set A is characterized by the mapping which assigns an interval $[\mu_A^L(x), \mu_A^U(x)]$ to x in U, where $0 \leq \mu_A^L(x) \leq \mu_A^U(x) \leq 1$. This interval represent the grade of membership of x in the set A.

However, interval valued fuzzy set theory does not provide a suitable framework to model several real life situations; specially those which require nonmembership grades along with the membership grades for each element in a domain set. For example, antibiotics are useful to treat some infectious diseases but they have side effects as well. In such cases, decision making processes cannot be modeled using the tools of interval valued fuzzy set theory. Atanassov [2] introduced the notion of intuitionistic fuzzy sets which provide a general framework to handle such real life problems. Later on, intuitionistic fuzzy sets were extended to the interval valued intuitionistic fuzzy sets [3]. In case of intuitionistic fuzzy sets truth-membership and falsity-membership depend on each other as the sum of truth-membership and falsity-membership is less than or equal to one. Thus hesitancy is the difference of truth-membership and falsity-membership.

In the cases when expert is consulted for his/her opinion on a certain statement and he/she is not sure about the range of the values of truth and falsity membership mappings, the notion of a neutrosophic set

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[17] is more appropriate than intuitionistic fuzzy set. In case of neutrosophic fuzzy sets, truth-membership, indeterminacy-membership and falsity-membership are independent of each other. Thus, neutrosophic set theory constitute a suitable setup to model the problems which involve imprecise, indeterminacy and inconsistent data (see [16]). A very handy generalization of neutrosophic sets has been introduced by Wang et al. [19]. They gave the notion of interval valued neutrosophic sets.

On the other hand, Molodtsov [15] initiated a soft set theory to provide enough tools to deal with uncertainty in a data and to represent it in a useful way. The significant attribute of soft set theory is that unlike probability and fuzzy set theory, it does not uphold a precise quantity. This theory has become full-fledged research area with applications in decision making, demand analysis, forecasting, information science, mathematics and other disciplines. Çağman et. al. [7] introduced soft toplogical spaces while soft lattice was introduced by Karaaslan et al. [11]. Maji [12] combined neutrosophic sets and soft sets. Karaaslan [9] used neutrosophic soft sets in dicicion making and also introduced possibility neutrosophic soft sets ([10]). Deli [8] introduced the concept of interval valued neutrosophic soft sets (see also [5]).

Wardowski ([20]) introduced a notion of soft mappings and obtained a fixed point result for a fixed point of a soft mapping in soft compact Hausdorff topological spaces. He also studied the properties of soft compact topological spaces. His main result is based on the fact that a decreasing sequence of nonempty soft closed subsets in soft compact topological spaces has a nonempty intersection.

Abbas et al. [1] introduced the concept of a fuzzy soft mappings on a fuzzy soft set and initiated the study of fixed points of such mappings. They studied some fundamental properties of fuzzy soft elements and fuzzy soft topological spaces.

The aim of this paper is to introduce the notions of interval valued neutrosophic soft Hausdorff topological spaces and interval valued neutrosophic soft compact topological spaces. The concept of interval valued neutrosophic soft mappings which facilitates the study fixed point results in such spaces is given. Some properties of interval valued neutrosophic soft points are discussed. Employing these concepts, interval valued neutrosophic soft Cantor's intersection theorem is proved. Finally, some necessary conditions for the existence of interval valued neutrosophic soft element which serves as a fixed point of interval valued neutrosophic soft mappings defined on interval valued neutrosophic soft Hausdorff topological spaces are studied. Our results extend, unify and generalize the comparable results in ([20]) and ([1]).

2. Preliminaries

We recall some basic definitions and results related to interval valued neutrosophic sets, interval valued neutrosophic soft sets and interval valued neutrosophic soft topological spaces.

Throughout this section, the letters U, E and $P(U)$ will denote an initial universe, a set of parameters, and the collection of all subsets of U, respectively.

Definition 2.1. ([17]) A neutrosophic set A in the universe of discourse U is identified by the set

$$A = \{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\},$$

where $\mu_A, \gamma_A, \delta_A : U \to]^-0, 1^+[$ are such that $^-0 \leq \mu_A(x) + \gamma_A(x) + \delta_A(x) \leq 3^+$ holds for all $x \in U$. Thus neutrosophic set A in U is characterized by the triplet $(\mu_A, \gamma_A, \delta_A)$ of functions, here, μ_A, γ_A, and δ_A denote the truth membership, indeterminacy-membership and false-membership functions, respectively. The values of mappings μ_A, γ_A, and δ_A at a point x in U give the grade of truth-membership, indeterminacy-membership and false-membership of x, respectively.

An interval $]^-0, 1^+[$ is a nonstandard unit interval whose left and right borders are nonstandard sets which are given by

$$\begin{aligned} (^-0) &= \{0 - \varepsilon : \varepsilon > 0 \text{ is an infinitely small number }\} \\ (1^+) &= \{1 + \varepsilon : \varepsilon > 0 \text{ is an infinitely small number }\}. \end{aligned}$$

Thus these borders are vague and imprecise. Obviously, $^-0 < 0$ and $1^+ > 1$.

For practical purposes, it is difficult to use neutrosophic set with values from non-standard subsets of $]^{-}0, 1^{+}[$.

Thus in the sequel, we consider the standard unit interval $[0, 1]$ instead of a nonstandard unit interval $]^{-}0, 1^{+}[$.

Definition 2.2. ([19]) An interval valued neutrosophic set A on the universe of discourse U is characterized by $\{(x, \mu_A(x), \gamma_A(x), \delta_A(x)) : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to Int[0, 1]$ are such that $0 \leq \sup \mu_A(x) + \sup \gamma_A(x) + \sup \delta_A(x) \leq 3$ holds and $Int[0, 1]$ denotes the set of all closed sub intervals of $[0, 1]$. Thus for each x in U, $\mu_A(x), \gamma_A(x)$ and $\delta_A(x)$ are closed sub intervals of $[0, 1]$.

The set of all interval valued neutrosophic sets on U is denoted by $IVNS(U)$.
Let $\bar{0} =< [0, 0], [1, 1], [1, 1] >$ and $\bar{1} =< [1, 1], [0, 0], [0, 0] >$.

Definition 2.3. ([6]) Let X be a subset of the set of parameters E. Then the soft set F_X over U is of the form

$$F_X = \{(e, f_X(e)) : e \in E\},$$

where $f_X : E \to P(U)$ such that $f_X(e) = \emptyset$ if $e \notin X$. For each e in E, $f_X(e)$ is called a set of $e-$elements of the soft set F_X and f_X is called an approximate function of the soft set F_X.

Hence, a soft set F_X is characterized by a set valued function taking values in $P(U)$. We denote the collection of all soft sets over a common universe U by $S(U, E)$.

If in the above definition, we replace $P(U)$ with $IVNS(U)$, then F_X is called an interval valued neutrosophic soft set ($IVNSS$ in short) over U. In this case, the approximate function f_X of interval valued neutrosophic soft set F_X is a set valued function which assigns to each parameter in X (descriptions of elements of U), an interval valued neutrosophic set.

Thus if u is a generic element of U and e is a generic element in the set of parameters in X which describes u, then each $e-$ element of interval valued neutrosophic soft set is of the form $\{(u, \mu_{f_X(e)}(u), \gamma_{f_X(e)}(u), \delta_{f_X(e)}(u)) : u \in U\}$ which contains an information about the truth membership, indeterminacy-membership and false-membership of u in $f_X(e)$ keeping in view every description e. If $e \in E - X$, then $f_X(e) = \bar{0}$. Thus, we identify interval valued neutrosophic soft set F_X by the set

$$\{(e, < (u, \mu_{f_X(e)}(u), \gamma_{f_X(e)}(u), \delta_{f_X(e)}(u)) : u \in U >) : e \in E\}.$$

The collection of all interval valued neutrosophic soft sets over (U, E) is denoted by $IVNSS(U, E)$. We will use $\sim$ as superscript in the operations of interval valued neutrosophic soft sets and N as subscript in the operations of interval valued neutrosophic sets.

Definition 2.4. ([8]) Let A and B be two subsets of a set of parameters E and $F_A, G_B \in IVNSS(U, E)$, that is, $F_A = \{(e, f_A(e)) : e \in E\}$ where

$$f_A(e) = \{(u, \mu_{f_A(e)}(u), \gamma_{f_A(e)}(u), \delta_{f_A(e)}(u)) : u \in U\}$$

and $G_B = \{(e, g_B(e)) : e \in E\}$ where

$$g_B(e) = \{(u, \mu_{g_B(e)}(u), \gamma_{g_B(e)}(u), \delta_{g_B(e)}(u)) : u \in U\}.$$

Then, $F_A \tilde{\subseteq} G_B$ if $A \subseteq B$ and $f_A(e) \subseteq_N g_B(e)$ for all $e \in A$, that is, for each u in U and for all $e \in E$

$$\begin{aligned} \inf \mu_{f_A(e)}(u) &\leq \inf \mu_{g_B(e)}(u), \ \sup \mu_{f_A(e)}(u) \leq \sup \mu_{g_B(e)}(u) \\ \inf \gamma_{f_A(e)}(u) &\geq \inf \gamma_{g_B(e)}(u), \ \sup \gamma_{f_A(e)}(u) \geq \sup \gamma_{g_B(e)}(u) \\ \inf \delta_{f_A(e)}(u) &\geq \inf \delta_{g_B(e)}(u), \ \sup \delta_{f_A(e)}(u) \geq \sup \delta_{g_B(e)}(u). \end{aligned}$$

Clearly, if $F_A \tilde{\subseteq} G_B$, then $\mu_{f_A(e)}(u) \subseteq \mu_{g_B(e)}(u)$, $\gamma_{g_B(e)}(u) \subseteq \gamma_{f_A(e)}(u)$, and $\delta_{g_B(e)}(u) \subseteq \delta_{f_A(e)}(u)$ for all u in U and for all $e \in E$.

Definition 2.5. ([8]) Let $F_A, G_B \in IVNSS(U,E)$, then

(i) $F_A \tilde{\cup} G_B = H_C$ is an interval valued neutrosophic soft set over U with $C = A \cup B$ and

$$H_C = \{(e, <(u, \mu_{h_C(e)}(u), \gamma_{h_C(e)}(u), \delta_{h_C(e)}(u)) : u \in U >: e \in E\},$$

where,

$$\mu_{h_C(e)}(u) = \begin{cases} \mu_{f_A(e)}(u) & \text{if } e \in A - B \\ \mu_{g_B(e)}(u) & \text{if } e \in B - A \\ [\max(\inf \mu_{f_A(e)}(u), \inf \mu_{g_B(e)}(u)), \max(\sup \mu_{f_A(e)}(u), \sup \mu_{g_B(e)}(u))] & \\ \quad \text{if } e \in A \cap B \end{cases}$$

$$\gamma_{h_C(e)}(u) = \begin{cases} \gamma_{f_A(e)}(u) & \text{if } e \in A - B \\ \gamma_{g_B(e)}(u) & \text{if } e \in B - A \\ [\min(\inf \gamma_{f_A(e)}(u), \inf \gamma_{g_B(e)}(u)), \min(\sup \gamma_{f_A(e)}(u), \sup \gamma_{g_B(e)}(u))] & \\ \quad \text{if } e \in A \cap B \end{cases}$$

$$\delta_{h_C(e)}(u) = \begin{cases} \delta_{f_A(e)}(u) & \text{if } e \in A - B \\ \delta_{g_B(e)}(u) & \text{if } e \in B - A \\ [\min(\inf \delta_{f_A(e)}(u), \inf \delta_{g_B(e)}(u)), \min(\sup \delta_{f_A(e)}(u), \sup \delta_{g_B(e)}(u))] & \\ \quad \text{if } e \in A \cap B \end{cases}$$

(ii) $F_A \tilde{\cap} G_B = H_C$ is an interval valued neutrosophic soft set over U with $C = A \cap B$ and

$$H_C = \{(e, <(u, \mu_{h_C(e)}(u), \gamma_{h_C(e)}(u), \delta_{h_C(e)}(u)) : u \in U >: e \in E\},$$

where

$$\begin{aligned} \mu_{h_C(e)}(u) &= [\min(\inf \mu_{f_A(e)}(u), \inf \mu_{g_B(e)}(u)), \min(\sup \mu_{f_A(e)}(u), \sup \mu_{g_B(e)}(u))], \\ \gamma_{h_C(e)}(u) &= [\max(\inf \gamma_{f_A(e)}(u), \inf \gamma_{g_B(e)}(u)), \max(\sup \gamma_{f_A(e)}(u), \sup \gamma_{g_B(e)}(u))], \\ \delta_{h_C}(u) &= [\max(\inf \delta_{f_A(e)}(u), \inf \delta_{g_B(e)}(u)), \max(\sup \delta_{f_A(e)}(u), \sup \delta_{g_B(e)}(u))]. \end{aligned}$$

(iii) $F_A \tilde{\setminus} G_B = H_A$ is an interval valued neutrosophic soft set over U with

$$H_A = \{(e, <(u, \mu_{h_C(e)}(u), \gamma_{h_C(e)}(u), \delta_{h_C(e)}(u)) : u \in U >: e \in E\},$$

where

$$\mu_{h_C(e)}(u) = \begin{cases} \mu_{f_A(e)}(u) \quad \text{if } e \in A - B \\ [\min(\inf \mu_{f_A(e)}(u), \inf \delta_{g_B(e)}(u)), \\ \min(\sup \mu_{f_A(e)}(u), \sup \delta_{g_B(e)}(u))] \quad \text{if } e \in A \cap B \end{cases}$$

$$\gamma_{h_C(e)}(u) = \begin{cases} \gamma_{f_A(e)}(u) \quad \text{if } e \in A - B \\ [\max(\inf \gamma_{f_A(e)}(u), 1 - \sup \gamma_{g_B(e)}(u)), \\ \max(\sup \gamma_{f_A(e)}(u), 1 - \inf \gamma_{g_B(e)}(u))] \quad \text{if } e \in A \cap B \end{cases}$$

$$\delta_{h_C(e)}(u) = \begin{cases} \delta_{f_A(e)}(u) \quad \text{if } e \in A - B \\ [\max(\inf \delta_{f_A(e)}(u), \inf \mu_{g_B(e)}(u)), \\ \max(\sup \delta_{f_A(e)}(u), \sup \mu_{g_B(e)}(u))] \quad \text{if } e \in A \cap B. \end{cases}$$

(iv) F_A^c (The complement of F_A) is an interval valued neutrosophic soft set over U which is characterized by the following set.

$$\{(e, <(u, \delta_{f_A(e)}(u), [1 - \sup \gamma_{f_A(e)}(u), 1 - \inf \gamma_{f_A(e)}(u)], \mu_{f_A(e)}(u)) : u \in U >) : e \in E\}.$$

Definition 2.6. ([8]) $F_A \in IVNSS(U,E)$ is said to be (i) universal interval valued neutrosophic soft set if

$f_A(e) = \bar{1}$ for each $e \in E$, and

(ii) null interval valued neutrosophic soft set if

$f_A(e) = \bar{0}$ for each $e \in E$.

The symbols $\tilde{I}$ and Φ denote universal and null interval valued neutrosophic soft sets, respectively.

Definition 2.7. ([14]) An interval valued neutrosophic soft topology τ on
$F_A \in IVNSS(U,E)$ is a collection of interval valued neutrosophic subsets of F_A satisfying:
(NT_1) $\Phi_A, F_A \in \tau$ (where $\phi_A(e) = \bar{0}$ for each $e \in A$),
(NT_2) $F_A \tilde{\cap} G_A \in \tau$ for any $F_A, G_A \in \tau$,
(NT_3) $\tilde{\cup}_{i \in J} G^i_A \in \tau \quad \forall \{G^i_A : i \in J\} \subseteq \tau$.
The pair (F_A, τ) is called an interval valued neutrosophic soft topological space and any set in τ is known as interval valued neutrosophic soft open set.

Definition 2.8. ([14]) Let (F_A, τ) be an interval valued neutrosophic soft topological space and $G_A \in IVNSS(U,E)$ with $G_A \tilde{\subseteq} F_A$. Then G_A is known as interval valued neutrosophic soft closed set if $G^c_A \in \tau$.

Definition 2.9. ([14]) Let (F_A, τ) be an interval valued neutrosophic soft topological space and $G_A \in IVNSS(U,E)$ with $G_A \tilde{\subseteq} F_A$. Then $\tau_G = \{G_A \tilde{\cap} H_A : H_A \in \tau\}$ is called relative interval valued neutrosophic soft topology on G_A. A pair (G_A, τ_G) is called relative interval valued neutrosophic soft topological space.

3. Interval Valued Neutrosophic Soft Elements

In this section, we define interval valued neutrosophic soft elements and then study some basic properties of such elements.

Definition 3.1. Let A be a subset of a set of parameters E and $e \in A$. An interval valued neutrosophic soft set F_A over the universe U is called an interval valued neutrosophic soft element corresponding to the parameter e if $f_A(e') = \bar{0}$ for each $e' \in A - \{e\}$. We denote it by F^e_A or simply by F^e.

An interval valued neutrosophic soft element F^e_A is said to be an element of interval valued neutrosophic soft set G_B if $F^e_A \tilde{\subseteq} G_B$. We write it as $F^e_A \tilde{\in} G_B$.

It is straightforward to check that there are uncountably many IVN-soft elements corresponding to each parameter in A. It is therefore possible to find some IVN-soft elements corresponding to each parameter $e \in A$ whose union results in the original $F_A \in IVNSS(U,E)$.

That is, if $F_A \in IVNSS(U,E)$, then for each parameter e in A, we may find some corresponding IVN-soft elements F^e_A such that $F_A = \widetilde{\bigcup}_{F^e \tilde{\in} F} F^e$.

We illustrate this observation with help of following example.

Example 3.2. Suppose that $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_3\}$. The tabular representation of F_A is given as follows (table 1).

Table 1: *IVNSS* F_A

U	e_1	e_2	e_3
u_1	([.5, .8], [.3, .5], [.2, .7])	([.4, .7], [.2, .3], [.1, .3])	([.3, .9], [0, .1], [0, .2])
u_2	([.4, .7], [.3, .4], [.1, .2])	([.6, .9], [.1, .2], [.1, .2])	([.4, .8], [.1, .2], [0, .5])
u_3	([.5, 1], [0, .1], [.3, .6])	([.6, .8], [.2, .4], [.1, .3])	([.4, .9], [.1, .3], [.2, .4])

Then some of the IVN-soft elements are given below:
$F^{e_1} = \{(e_1, \{\frac{<[.2,.8],[.3,.6],[.3,.8]>}{u_1}, \frac{<[.4,.7],[.3,.5],[.2,.9]>}{u_2}, \frac{<[.2,.7],[.2,.6],[.4,.9]>}{u_3}\})\}$;
$G^{e_1} = \{(e_1, \{\frac{<[.5,.6],[.4,.5],[.2,.7]>}{u_1}, \frac{<[.4,.7],[.4,.5],[.6,.7]>}{u_2}, \frac{<[.2,.7],[.2,.6],[.4,.9]>}{u_3}\})\}$;
$H^{e_2} = \{(e_2, \{\frac{<[.4,.7],[.2,.3],[.1,.3]>}{u_1}, \frac{<[.6,.9],[.1,.2],[.1,.2]>}{u_2}, \frac{<[.6,.8],[.2,.4],[.1,.3]>}{u_3}\})\}$;
$I^{e_3} = \{(e_3, \{\frac{<[.3,.9],[0,.1],[0,.2]>}{u_1}, \frac{<[.4,.8],[.1,.2],[0,.5]>}{u_2}, \frac{<[.4,.9],[.1,.3],[.2,.4]>}{u_3}\})\}$.
Note that $F^{e_1}, G^{e_1}, H^{e_2}, I^{e_3} \tilde{\in} F_A$ and $F^{e_1} \tilde{\cup} G^{e_1} \tilde{\cup} H^{e_2} \tilde{\cup} I^{e_3} = F_A$.

Definition 3.3. Two IVN-soft elements $F^e, G^{e'}$ over U are said to be distinct if either $e \neq e'$ or $f^e(e) \cap_N g^{e'}(e') = \bar{0}$.

Proposition 3.4. *If F_A, $G_B \in IVNSS(U, E)$ and $e \in E$, then the following holds:*

1. *A null interval valued neutrosophic soft set Φ is an empty IVN-soft element.*
2. *If $F_A \in IVNSS(U, E)$ such that $F_A \neq \Phi$, then F_A contains at least one nonempty IVN-soft element.*
3. *If $F_A, G_B \in IVNSS(U, E)$ then $F_A \tilde{\subseteq} G_B$ if and only if $F^e \tilde{\in} F_A$ implies that $F^e \tilde{\in} G_B$.*
4. *If $F_A, G_B \in IVNSS(U, E)$ then $F^e \tilde{\in} F_A \tilde{\cup} G_B$ if and only if F^e is a IVN-soft element of F_A or G_B.*
5. *If $F_A, G_B \in IVNSS(U, E)$ then $F^e \tilde{\in} F_A \tilde{\cap} G_B$ if and only if F^e is a IVN-soft element of F_A and G_B.*
6. *If $F_A, G_B \in IVNSS(U, E)$ and $F^e \tilde{\in} F_A \tilde{\setminus} G_B$ then F^e is an IVN-soft element of F_A but not necessarily an IVN-soft element of G_B.*

Proof. 1. It is trivial by the definition.

2. Suppose that $F_A \in IVNSS(U, E)$ such that $F_A \neq \Phi$. Then there will be at least one $e \in E$ for which $f_A(e) \neq \bar{0}$. Thus, we can define an IVN-soft element F^e such that it can be characterized by $\{(e, f_A(e))\}$. Hence, $F^e \tilde{\in} F_A$.

3. Suppose that $F_A \tilde{\subseteq} G_B$ and $F^e \tilde{\in} F_A$. That is, for $e \in A$ we have $f^e(e) \subseteq_N f_A(e) \subseteq_N g_B(e)$. Therefore $F^e \tilde{\in} G_B$. Conversely, suppose that every IVN-soft element $F^e \in F_A$ is also an IVN-soft element of G_B. That is, for every $e \in A$, $\{e, f_A(e)\}$ is a soft element of G_B which implies that for every $e \in A$, $f_A(e) \subseteq_N g_B(e)$ and hence $F_A \tilde{\subseteq} G_B$.

4. Suppose that $F^e \tilde{\in} F_A \tilde{\cup} G_B$. This implies that $f^e(e) \subseteq_N f_A(e)$ or $f^e(e) \subseteq_N g_B(e) \Leftrightarrow F^e \tilde{\in} F_A$ or $F^e \tilde{\in} G_B$.

5. Suppose that $F^e \tilde{\in} F_A \tilde{\cap} G_B$.This implies that $f^e(e) \subseteq_N f_A(e)$ and $f^e(e) \subseteq_N g_B(e) \Leftrightarrow F^e \tilde{\in} F_A$ and $F^e \tilde{\in} G_B$.

6. Suppose that $F^e \tilde{\in} F_A \tilde{\setminus} G_B$. Then either $f^e(e) \subseteq_N f_A(e) - f_B(e)$ or $f^e(e) \subseteq_N f_A(e)$. In both cases, we have $f^e(e) \subseteq_N f_A(e)$ and therefore $F^e \tilde{\in} F_A$. On the other hand, we know that the difference $f_A(e) - g_B(e)$ may have an IVN-soft subset which is not contained in $g_B(e)$. □

4. Interval Valued Neutrosophic Soft Compact Topological Spaces

In this section, we first introduce IVN-soft Hausdorff spaces and IVN-soft compact spaces, and then we study some basic properties of IVN-soft compact spaces.

Definition 4.1. An IVN-soft topological space (F_A, τ) is said to be an IVN-soft Hausdorff space if for any two distinct $F^e, F^{e'} \tilde{\in} F_A$, there exists disjoint IVN-soft open sets F_1 and F_2 (i.e. $F_1 \tilde{\cap} F_2 = \Phi$) such that $F^e \tilde{\in} F_1$ and $F^{e'} \tilde{\in} F_2$.

Proposition 4.2. *Let (F_A, τ) be an interval valued neutrosophic soft topological space and $V_A \in IVNSS(U, E)$ with $V_A \tilde{\subseteq} F_A$. Then V_A is open if and only if for every $F^e \tilde{\in} V_A$, there exists an IVN-soft open set G_A such that $F^e \tilde{\in} G_A \tilde{\subseteq} V_A$.*

Proof. Let $G_A \in \tau$. Then for every $F^e \tilde{\in} G_A$, we have $F^e \tilde{\in} G_A \tilde{\subseteq} G_A$. Conversely, let $V_A \tilde{\subseteq} F_A$ be such that for each $F^e \tilde{\in} V_A$, there exists $W_A \in \tau$ and $F^e \tilde{\in} W_A \tilde{\subseteq} V_A$. Therefore $V_A = \tilde{\cup} F^e \tilde{\subseteq} \tilde{\cup} W_A \tilde{\subseteq} V_A$ implies that $V_A \in \tau$. □

Definition 4.3. Let (F_A, τ) be an IVN-soft topological space and $K_A \tilde{\subseteq} F_A$. An IVN-soft open cover for K_A is a collection of IVN-soft open sets $\{V_i\}_{i \in I} \subseteq \tau$ whose union contains K_A.

Definition 4.4. An IVN-soft topological space (K_A, τ) is IVN-soft compact if for each IVN-soft open cover $\{V_i\}_{i \in I} \subseteq \tau$ of K_A there exists $i_1, i_2, ..., i_k \in I, k \in \mathbb{N}$ such that $K_A \tilde{\subseteq} \tilde{\cup}_{n=1}^{k} V_{i_n}$.

Definition 4.5. Let (F_A, τ) be an IVN-soft topological space and $K_A \tilde{\subseteq} F_A$. We say that K_A is IVN-soft compact in (F_A, τ) if the IVN-soft topological space (K_A, τ_K) is IVN-soft compact.

In the following example, we show that an interval valued neutrosophic soft compact space may not be an IVN-soft Hausdorff topological space.

Example 4.6. Suppose that $U = \{u_1, u_2, u_3\}$, $A = \{e_1, e_2\}$.

The tabular representation of F_A is given as:

U	e_1	e_2
u_1	$([.5, .8], [.3, .5], [.2, .7])$	$([.4, .7], [.2, .3], [.1, .3])$
u_2	$([.4, .7], [.3, .4], [.1, .2])$	$([.6, .9], [.1, .2], [.1, .2])$
u_3	$([.5, 1], [0, .1], [.3, .6])$	$([.6, .8], [.2, .4], [.1, .3])$

Table 2: The tabular representation of Φ_A

U	e_1	e_2
u_1	$([0, 0], [1, 1], [1, 1])$	$([0, 0], [1, 1], [1, 1])$
u_2	$([0, 0], [1, 1], [1, 1])$	$([0, 0], [1, 1], [1, 1])$
u_3	$([0, 0], [1, 1], [1, 1])$	$([0, 0], [1, 1], [1, 1])$

Table 3: The tabular representation of F_1

U	e_1	e_2
u_1	$([.5, .8], [.3, .5], [.2, .7])$	$([0, 0], [1, 1], [1, 1])$
u_2	$([.4, .7], [.3, .4], [.1, .2])$	$([0, 0], [1, 1], [1, 1])$
u_3	$([.5, 1], [0, .1], [.3, .6])$	$([0, 0], [1, 1], [1, 1])$

Table 4: The tabular representation of F_2

U	e_1	e_2
u_1	$([0, 0], [1, 1], [1, 1])$	$([.4, .7], [.2, .3], [.1, .3])$
u_2	$([0, 0], [1, 1], [1, 1])$	$([.6, .9], [.1, .2], [.1, .2])$
u_3	$([0, 0], [1, 1], [1, 1])$	$([.6, .8], [.2, .4], [.1, .3])$

Note that $\tau = \{\Phi_A, F_A, F_1, F_2\}$ is an interval valued neutrosophic soft compact space, where tabular representations of Φ_A, F_1, F_2 are as follows(tables 2, 3 and 4 respectively). But it is not an IVN-soft Hausdorff topological space. Indeed, for distinct IVN-soft elements F^{e_1} and F'^{e_1} as given in the tables 5 and 6 respectively, we cannot find disjoint IVN-soft open sets.

Table 5: The tabular representation of F^{e_1}

U	e_1	e_2
u_1	$([.5,.8],[.3,.5],[.2,.7])$	$([0,0],[1,1],[1,1])$
u_2	$([0,0],[1,1],[1,1])$	$([0,0],[1,1],[1,1])$
u_3	$([.5,1],[0,.1],[.3,.6])$	$([0,0],[1,1],[1,1])$

Table 6: The tabular representation of F'^{e_1}

U	e_1	e_2
u_1	$([0,0],[1,1],[1,1])$	$([0,0],[1,1],[1,1])$
u_2	$([.4,.7],[.3,.4],[.1,.2])$	$([0,0],[1,1],[1,1])$
u_3	$([0,0],[1,1],[1,1])$	$([0,0],[1,1],[1,1])$

Example 4.7. Let $F_A \in IVNSS(U,E)$ be as given in the previous example and τ a collection of all IVN-soft subsets of F_A. Then (F_A,τ) is an IVN-soft compact Hausdorff topological space.

Proposition 4.8. *Let (F_A,τ) be an IVN-soft Hausdorff topological space. Then every IVN-soft compact set in F_A is IVN-soft closed in F_A.*

Proof. Let K_A be an IVN-soft compact set in (F_A,τ) and $F^e \tilde{\in} K_A^c$. For every $G^{e'} \tilde{\in} K_A$, we have $U_A, V_A \in \tau$ such that $U_A \tilde{\cap} V_A = \Phi$ and $F^e \tilde{\in} U_A, G^{e'} \tilde{\in} V_A$. As K_A is IVN-soft compact, there exists $G^{e_1}, G^{e_2} \cdots, G^{e_k} \tilde{\in} K_A$ and $\{V_i\}_{i=1}^k \subseteq \tau$ such that $G^{e_i} \tilde{\in} V_i$ for all i and $K_A \tilde{\subseteq} \tilde{\cup}_{i=1}^k V_i$. Similarly, we may find a family $\{U_i\}_{i=1}^k \subseteq \tau$ containing F^e such that $U_i \tilde{\cap} V_i = \Phi$ for all $i = 1, \cdots, k$. Denote $U = \tilde{\cup}_{i=1}^k U_i$ and $V = \tilde{\cup}_{i=1}^k V_i$, then $F^e \tilde{\in} U \in \tau$ and $U \tilde{\cap} V = \Phi$. Therefore $U \tilde{\cap} K_A = \Phi$ and $F^e \tilde{\in} U \tilde{\subseteq} K_A^c$. Hence K_A is closed by the proposition 4.2. □

5. Interval Valued Neutrosophic Soft Mapping

We start this section by introducing IVN-soft mapping. We also give some relevant definitions and study some properties of IVN-soft mappings.

Definition 5.1. ([4]) Suppose that $F_A, G_B \in IVNSS(U,E)$. Then the Cartesian product of F_A and G_B is denoted by $(H, A \times B)$, where $h_{A\times B}(a,b) = f_A(a) \cap_N g_B(b)$.

Definition 5.2. ([4]) Let $F_A, G_B \in IVNSS(U,E)$. Then the IVN-soft relation from F_A to G_B is an IVN-soft subset of $(H, A \times B)$.

Definition 5.3. Let $F_A, G_B \in IVNSS(U,E)$. An IVN-soft relation T from F_A to G_B is called an IVN-soft mapping, denoted by $T : F_A \to G_B$ if the following conditions are satisfied:

N1. For each IVN-soft element $F^e \tilde{\in} F_A$, there exists only one IVN-soft element $G^{e'} \tilde{\in} G_B$ such that $(F^e, G^{e'}) \tilde{\in} T$, that is, for each $e \in A$ there exists $e' \in B$ such that $t_{A\times B}(e,e') = f_A(e) \cap_N g_B(e')$.

N2. For each IVN-soft empty element $F^e \tilde{\in} F_A$, we must have $(F^e, \Phi) \tilde{\in} T$.

In this case, we write $T(F^e) = G^{e'}$.

Definition 5.4. Let $F_A, G_B \in IVNSS(U,E)$, $T : F_A \to G_B$ an IVN-soft mapping and $X_C \tilde{\subseteq} F_A$. The image $T(X_C)$ of X_C under T is an interval valued neutrosophic soft set defined by

$$T(X_C) = \{\widetilde{\cup}_{F^e \tilde{\in} X_C} T(F^e) : e \in C\}.$$

Definition 5.5. Let $F_A, G_B \in IVNSS(U,E)$, $T : F_A \to G_B$ an IVN-soft mapping and $Y_D \tilde{\subseteq} G_B$. The inverse image $T^{-1}(Y_D)$ of Y_D under T is an interval valued neutrosophic soft set defined by

$$T^{-1}(Y_D) = \{\{\widetilde{\cup}_{F^e \tilde{\in} F_A} F^e\} : T(F^e) \tilde{\in} Y_D \text{ for each } e \in D\}.$$

Example 5.6. Suppose that $U = \{u_1, u_2\}$, $A = \{e_1, e_2\}$. Let F_A and G_A be two *IVNSS*:

The tabular representation of F_A

U	e_1	e_2
u_1	$([.5,.8],[.3,.5],[.2,.7])$	$([.2,.3],[.5,.7],[.2,.5])$
u_2	$([.4,.7],[.3,.4],[.1,.2])$	$([.1,.5],[.6,.8],[.2,.4])$

The tabular representation of G_A

U	e_1	e_2
u_1	$([.5,.7],[.1,.3],[.2,.3])$	$([.4,.7],[.2,.3],[.1,.3])$
u_2	$([.4,.5],[.1,.2],[.3,.4])$	$([.6,.9],[.1,.2],[.1,.2])$

Let $e \in A$ and $\widehat{G^e} = \tilde{\cup}\{G^e : G^e \tilde{\in} G_A\}$ which is the largest *IVN*-soft element in G_A corresponding to parameter e. Define $T : F_A \to G_A$ by

$$T(F^e) = \begin{cases} \widehat{G^e} & \text{if } F^e \text{ is not an } IVN\text{-soft empty element,} \\ \Phi & \text{if } F^e \text{ is an } IVN\text{-soft empty element.} \end{cases}$$

Therefore $T(F^{e_1}) = \{(e_1, \{\frac{<[.5,.7],[.1,.3],[.2,.3]>}{u_1}, \frac{<[.4,.5],[.1,.2],[.3,.4]>}{u_2})\}$ and $T(F^{e_2}) = \{(e_2, \{\frac{<[.4,.7],[.2,.3],[.1,.3]>}{u_1}, \frac{<[.6,.9],[.1,.2],[.1,.2]>}{u_2})\}$. Note that $T(F_A) = G_A$.

Proposition 5.7. *Let $F_A, G_B \in IVNSS(U, E)$ and $T : F_A \to G_B$ an IVN-soft mapping. Let $X, X_1, X_2 \tilde{\subseteq} F_A$ and $Y, Y_1, Y_2 \tilde{\subseteq} G_B$. Then the following hold:*

1. $X_1 \tilde{\subseteq} X_2 \Rightarrow T(X_1) \tilde{\subseteq} T(X_2)$,
2. $Y_1 \tilde{\subseteq} Y_2 \Rightarrow T^{-1}(Y_1) \tilde{\subseteq} T^{-1}(Y_2)$,
3. $X \tilde{\subseteq} T^{-1}(T(X))$,
4. $T(T^{-1}(Y)) \tilde{\subseteq} Y$,
5. $T(X_1 \tilde{\cup} X_2) = T(X_1) \tilde{\cup} T(X_2)$,
6. $T(X_1 \tilde{\cap} X_2) = T(X_1) \tilde{\cap} T(X_2)$,
7. $T^{-1}(Y_1 \tilde{\cup} Y_2) = T^{-1}(Y_1) \tilde{\cup} T^{-1}(Y_2)$,
8. $T^{-1}(Y_1 \tilde{\cap} Y_2) = T^{-1}(Y_1) \tilde{\cap} T^{-1}(Y_2)$.

Proof. We will prove only (1), (5) and (8) here. Proofs of rest of the properties follow on the similar lines.

1. Let $G^{e'} \tilde{\in} T(X_1)$. Then there exists an *IVN*-soft element $F^e \tilde{\in} X_1$ such that $T(F^e) = G^{e'}$. Now $X_1 \tilde{\subseteq} X_2$, $F^e \tilde{\in} X_2$ imply that $G^{e'} \tilde{\in} T(X_2)$.

5. Suppose that $G^{e'} \tilde{\in} T(X_1 \tilde{\cup} X_2)$. Then there exists an *IVN*-soft element $F^e \tilde{\in} X_1 \tilde{\cup} X_2$ such that $T(F^e) = G^{e'}$. If $F^e \tilde{\in} X_1$ then $G^{e'} \tilde{\in} T(X_1) \tilde{\subseteq} T(X_1 \tilde{\cup} X_2)$. If $F^e \tilde{\in} X_2$, then $G^{e'} \tilde{\in} T(X_2) \tilde{\subseteq} T(X_1 \tilde{\cup} X_2)$. Therefore $T(X_1 \tilde{\cup} X_2) \tilde{\subseteq} T(X_1) \tilde{\cup} T(X_2)$. Now let $G^{e'} \tilde{\in} T(X_1) \tilde{\cup} T(X_2)$. This implies that $G^{e'} \tilde{\in} T(X_1)$ or $G^{e'} \tilde{\in} T(X_2)$. Hence there exists an *IVN*-soft element $F^e \tilde{\in} X_1 \tilde{\cup} X_2$ such that $T(F^e) = G^{e'}$ and this completes the proof of (5).

8. Note that $F^e \tilde{\in} T^{-1}(Y_1 \tilde{\cap} Y_2)$ if and only if $T(F^e) \tilde{\in} Y_1 \tilde{\cap} Y_2$ if and only if $F^e \tilde{\in} T^{-1}(Y_1)$ and $F^e \tilde{\in} T^{-1}(Y_2)$ if and only if $F^e \tilde{\in} T^{-1}(Y_1) \tilde{\cap} T^{-1}(Y_2)$. □

Definition 5.8. Let (F_A, τ) and (G_B, ν) be *IVN*-soft topological spaces and $T : F_A \to G_B$ an *IVN*-soft mapping. Then T is an *IVN*-soft continuous mapping if for each $V_B \in \nu$, $T^{-1}(V_B) \in \tau$, that is the inverse image of an *IVN*-soft open set is an *IVN*-soft open set.

Example 5.9. Let $F_A \in IVNSS(U, E)$ and τ a collection of all *IVN*-soft subsets of F_A. Then (F_A, τ) is an *IVN*-soft topological space. Let $e \in A$ and $\widehat{F^e} = \tilde{\cup}\{F^e : F^e \tilde{\in} F_A\}$ which is the largest *IVN*-soft element in F_A corresponding to parameter e. Define $T : F_A \to F_A$ as

$$T(F^e) = \begin{cases} \widehat{F^e} & \text{if } F^e \text{ is not an } IVN\text{-soft empty element,} \\ \Phi & \text{if } F^e \text{ is an } IVN\text{-soft empty element.} \end{cases}$$

Then for every $V \in \tau$, $T^{-1}(V)$ is open. Therefore T is an *IVN*-soft continuous mapping.

Proposition 5.10. *Let (K_A, τ) be an IVN-soft compact topological space and $T : K_A \to K_A$ an IVN-soft continuous mapping. Then $T(K_A)$ is an IVN-soft compact in (K_A, τ).*

Proof. Suppose that $T(K_A)\tilde{\subseteq}\tilde{\cup}_i V_i$, where $V_i \in \tau$. Then $K_A\tilde{\subseteq}T^{-1}(\tilde{\cup}_i V_i) = \tilde{\cup}_i T^{-1}(V_i)$. Since $T^{-1}(V_i)$ is IVN-soft open, there exists an IVN-soft open set such that $G_i\tilde{\subseteq}T(K_A)$ and $T^{-1}(V_i) = G_i\tilde{\cap}K_A$. So $K_A\tilde{\subseteq}\tilde{\cup}_i G_i\tilde{\cap}K_A\tilde{\subseteq}\tilde{\cup}_i G_i$. Thus, there exists $i_1, i_2, ..., i_k, k \in \mathbb{N}$ such that $K_A\tilde{\subseteq}\tilde{\cup}_{n=1}^{k} G_{i_n}$. Hence we have $K_A = \tilde{\cup}_{n=1}^{k}(G_{i_n} \cap K_A) = \tilde{\cup}_{n=1}^{k} T^{-1}(V_{i_n})$ which gives that $T(K_A)\tilde{\subseteq}\tilde{\cup}_{n=1}^{k} V_{i_n}$. Hence $T(K_A)$ is IVN-soft compact. □

6. Interval Valued Neutrosophic Soft Fixed Points of Interval Valued Neutrosophic Soft Mapping

In this section study of interval valued neutrosophic soft fixed points is initiated.

Definition 6.1. Let $F_A \in IVNSS(U, E)$ and $T : F_A \to F_A$ an IVN-soft mapping. An IVN-soft element $F^e\tilde{\in}F_A$ is called IVN-soft fixed point of T if $T(F^e) = F^e$.

The following examples show the existence of IVN-soft fixed points.

Example 6.2. If $F_A \in IVNSS(U, E)$ and $T : F_A \to F_A$ is defined as identity map, then each IVN-soft element of F_A is an IVN-soft fixed point.

Example 6.3. If T is an IVN-soft mapping as in the example 5.9, then every $\widehat{F^e}$ and Φ are IVN-soft fixed points of T.

Proposition 6.4. *Let (K_A, τ) be an IVN-soft compact topological space and $\{F_i, i \in \mathbb{N}\}$ a family of IVN-soft subsets of K_A which satisfies:*

1. *$F_i \neq \Phi$ for each $i \in \mathbb{N}$;*
2. *F_i is an IVN-soft closed for each i;*
3. *$F_{i+1}\tilde{\subseteq}F_i$ for each i.*

Then $\tilde{\cap}_{i\in\mathbb{N}}F_i \neq \Phi$.

Proof. On the contrary suppose that $\tilde{\cap}_{i\in\mathbb{N}}F_i = \Phi$. By Proposition 3.25 ([8]), we have $(\tilde{\cap}_{i\in\mathbb{N}}F_i)^c = \tilde{\cup}_{i\in\mathbb{N}}F_i^c$. Therefore $K_A\tilde{\subseteq}\tilde{I} = \Phi^c = \tilde{\cup}_{i\in\mathbb{N}}F_i^c$. As K_A is an IVN-soft compact, there exists $i_1, i_2, ..., i_k \in \mathbb{N}$, $i_1 < i_2 < ... < i_k, k \in \mathbb{N}$ such that $K_A\tilde{\subseteq}\tilde{\cup}_{n=1}^{k}F_{i_n}^c$. Hence by (3) we have $F_{i_k}\tilde{\subseteq}K_A\tilde{\subseteq}(\tilde{\cap}_{n=1}^{k}F_{i_n})^c = F_{i_k}^c$, which is impossible due to (1). □

Example 6.5. Let $U = \{u_1, u_2\}$, $A = \{e_1, e_2\}$. Let F_A be defined as: (table 7)

Table 7: The tabular representation of F_A

U	e_1	e_2
u_1	([.5, .8], [.3, .5], [.2, .7])	([.2, .3], [.5, .7], [.2, .5])
u_2	([.4, .7], [.3, .4], [.1, .2])	([.1, .5], [.6, .8], [.2, .4])

Let (F_A, τ) be an IVN-soft topological space, where τ is a collection of all IVN-soft subsets of F_A.
Let us consider the following IVN-soft subsets of F_A: (tables 8 and 9)

Table 8: The tabular representation of F_1

U	e_1	e_2
u_1	([.3, .5], [.4, .6], [.3, .8])	([.1, .2], [.5, .8], [.3, .5])
u_2	([.1, .4], [.3, .5], [.2, .3])	([.1, .4], [.6, .9], [.3, .5])

Note that all the conditions of the Proposition 6.4 are satisfied and $F_1 \cap F_2 = F_2$.

Table 9: The tabular representation of F_2

U	e_1	e_2
u_1	([.3, .4], [.5, .6], [.4, .8])	([0, .1], [.5, .8], [.3, .6])
u_2	([.1, .2], [.4, .5], [.3, .5])	([.1, .2], [.6, .9], [.3, .6])

Proposition 6.6. *Let (F_A, τ) be an IVN-soft topological spaces and $T : F_A \to F_A$ an IVN-soft mapping such that for each nonempty IVN-soft element $F^e \tilde{\in} F_A$, $T(F^e)$ is a nonempty IVN-soft element of F_A. If $\tilde{\cap}_{n\in\mathbb{N}} T^n(F_A)$ contains only one nonempty IVN-soft element $F^e \tilde{\in} F_A$, then F^e is a unique IVN-soft fixed point of T.*

Proof. It can be seen that $T^n(F_A) \tilde{\subseteq} T^{n-1}(F_A)$ for each $n \in \mathbb{N}$. Suppose that $F^e \tilde{\in} \tilde{\cap}_{n\in\mathbb{N}} T^n(F_A)$, that is, $F^e \tilde{\subseteq} \tilde{\cap}_{n\in\mathbb{N}} T^n(F_A)$. Therefore $T(F^e) \tilde{\subseteq} T(\tilde{\cap}_{n\in\mathbb{N}} T^n(F_A)) \tilde{\subseteq} \tilde{\cap}_{n\in\mathbb{N}} T^{n+1}(F_A) \tilde{\subseteq} \tilde{\cap}_{n\in\mathbb{N}} T^n(F_A) = F^e$. Since $T(F^e)$ is a nonempty IVN-soft element of F_A therefore $T(F^e) = F^e$. □

Example 6.7. Let (F_A, τ) be an IVN-soft topological spaces, where $A = \{e_1, e_2, e_3\}$. Let $e \in A$ and $\widehat{F^e} = \tilde{\cup}\{F^e : F^e \tilde{\in} F_A\}$ which is the largest IVN-soft element in F_A corresponding to parameter e. Define $T : F_A \to F_A$ by

$$T(F^e) = \begin{cases} \widehat{F^{e_1}} & \text{if } F^e \text{ is a nonempty } IVN\text{-soft element,} \\ \Phi & \text{if } F^e \text{ is an } IVN\text{-soft empty element.} \end{cases}$$

Then $\tilde{\cap}_{i\in\mathbb{N}} T^n(F_A)$ contains only one nonempty IVN-soft element $\widehat{F^{e_1}}$ which will be the unique IVN-soft fixed point of T.

Theorem 6.8. *Let (K_A, τ) be an IVN-soft compact Hausdorff topological space and $T : K_A \to K_A$ an IVN-soft continuous mapping such that*

a. for each nonempty IVN-soft element $F^e \tilde{\in} K_A$, $T(F^e)$ is a nonempty IVN-soft element of K_A.,

b. for each IVN-soft closed $X \tilde{\subseteq} K_A$ if $T(X) = X$ then X contains only one nonempty IVN-soft element.

Then there exists a unique nonempty IVN-soft element $F^e \tilde{\in} K_A$ such that $T(F^e) = F^e$.

Proof. Let us consider a family of IVN-soft subsets of K_A as follows:

$$C_1 = T(K_A), C_2 = T(C_1) = T^2(K_A), ..., C_n = T(C_{n-1}) = T^n(K_A)$$

for each $n \in \mathbb{N}$. Note that $C_n \tilde{\subseteq} C_{n-1}$ for each $n \in \mathbb{N}$. By Proposition 4.8, C_n is an IVN-soft closed subset of K_A, for each $n \in \mathbb{N}$. Now by Proposition 6.4, $\cap_{n\in\mathbb{N}} C_n$ is nonempty. Let us denote $\cap_{n\in\mathbb{N}} C_n = G$. Observe that

$$T(G) = T(\cap_{n\in\mathbb{N}} T^n(K_A)) \tilde{\subseteq} \cap_{n\in\mathbb{N}} T^{n+1}(K_A) \tilde{\subseteq} \cap_{n\in\mathbb{N}} T^n(K_A) = G.$$

Now we show that $G \tilde{\subseteq} T(G)$. Suppose that there exist $F^e \tilde{\in} G$ such that F^e is not an IVN-soft element of $T(G)$. Put $E_n = T^{-1}(F^e) \tilde{\cap} C_n$. Observe that $E_n \neq \Phi$ and $E_n \tilde{\subseteq} E_{n-1}$ for each $n \in \mathbb{N}$. By Proposition 6.4, there exists a nonempty IVN-soft element $F^{e'} \tilde{\in} T^{-1}(F^e) \tilde{\cap} G$ which implies that $F^e = T(F^{e'}) \tilde{\in} T(G)$, a contradiction. Hence we have $T(G) = G$ and the proof follows by using (b) and Proposition 6.6. □

Example 6.9. Let (F_A, τ) be an IVN-soft topological spaces as given in the example 4.6. It is IVN-soft compact but it is not an IVN-soft Hausdorff. Let $\widehat{F^{e_1}} = \tilde{\cup}\{F^{e_1} : F^{e_1} \tilde{\in} F_A\}$ and $\widehat{F^{e_2}} = \tilde{\cup}\{F^{e_2} : F^{e_2} \tilde{\in} F_A\}$ which are the largest IVN-soft elements in F_A corresponding to parameters e_1 and e_2, respectively. Define $T : F_A \to F_A$ by

$$T(F^e) = \begin{cases} \widehat{F^e} & \text{if } F^e \text{ is a nonempty } IVN\text{-soft element,} \\ \Phi & \text{if } F^e \text{ is an } IVN\text{-soft empty element.} \end{cases}$$

It is easy to see that T is an IVN-soft continuous mapping and for an IVN-soft closed subset X of F_A, $T(X) \neq X$. Moreover T has not a unique IVN-soft fixed point. Indeed, T has two IVN-soft fixed points $\widehat{F^{e_1}}$ and $\widehat{F^{e_2}}$.

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