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On the Symmetrization of Jets on Vector Bundles

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Abstract. We solve the existence problems of holonomic prolongation of general linear connections and the symmetrization of semiholonomic jets on vector bundles.

1. Introduction

The classical theory of higher order jets and connections was established by C. Ehresmann, [3], [4]. For nonholonomic and semiholonomic jets we refer to the paper [9] by P. Libermann. Other important results from the theory of jets, connections and their applications can be found e. g. in [1], [6], [7], [8], [10], [13], [14], [15]. We recall that higher order jets are a very powerful tool in differential geometry and in many areas of mathematical physics. For example, holonomic jets globalize the theory of differential systems and semiholonomic jets play an important role in the calculus of variations and in the theory of PDE's. But the most important role in differential geometry and also in mathematical physics is played by classical holonomic jets. This leads to the problem of symmetrization of semiholonomic jets on vector bundles. More precisely, we solve the following problem.

Problem 1.1. *Under which conditions there is a canonical fibered map $S_E : \overline{J}^r E \to J^r E$ with $S_{E|J^r E} = id_{J^r E}$, where $E \to M$ is a vector bundle.*

We show that this problem is very closely connected with the existence of holonomic prolongation of general linear connections. In [2] we solved a similar problem for arbitrary fibered manifolds. But vector bundles have more applications in current physical theories and the present paper also generalizes the coresponding results from [2], see Remark 2.6 below.

Let $Y \to M$ be a fibered manifold and denote by $J^r Y$, $\overline{J}^r Y$ or $\widetilde{J}^r Y$ its holonomic or semiholonomic or nonholonomic prolongation, respectively. Then $J^r Y$ is the bundle of all r-jets of local sections of Y and $\widetilde{J}^r Y$ is defined by the induction $\widetilde{J}^1 Y = J^1 Y$, $\quad \widetilde{J}^r Y = J^1(\widetilde{J}^{r-1} Y \to M)$. Then we have the canonical inclusion $J^r Y \subset \widetilde{J}^r Y$ determined by $j^r_x s \mapsto j^1_x(u \mapsto j^{r-1}_u s)$ for every local section s of Y, $u \in M$. Further, write $\overline{J}^1 Y = J^1 Y$ and assume

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we have defined $\overline{J}^{r-1}Y \subset \widetilde{J}^{r-1}Y$ such that the restriction of $\beta_{\widetilde{J}^{r-2}Y} : \widetilde{J}^{r-1}Y \to \widetilde{J}^{r-2}Y$ maps $\overline{J}^{r-1}Y$ into $\overline{J}^{r-2}Y$, where $\beta_Y : J^1Y \to Y$ means the projection. Then we can define the r-th semiholonomic prolongation of Y by

$$\overline{J}^rY = \{U \in J^1\overline{J}^{r-1}Y; \beta_{\overline{J}^{r-1}Y}(U) = J^1\beta_{\widetilde{J}^{r-2}Y}(U) \in \overline{J}^{r-1}Y\}.$$

Clearly, we have $J^rY \subset \overline{J}^rY \subset \widetilde{J}^rY$.

It is well known that the basic concepts from the theory of connections can be formulated by means of jets. In particular, a (general) connection on a fibered manifold $Y \to M$ is a smooth section $\Gamma : Y \to J^1Y$. Replacing J^1Y by $\widetilde{J}^rY$ or $\overline{J}^rY$ or J^rY, we obtain the concept of an r-th order nonholonomic or semiholonomic or holonomic connection, respectively. Clearly, a connection $\Gamma : Y \to J^1Y$ can be also interpreted as the lifting map (denoted by the same symbol) $\Gamma : Y \times_M TM \to TY$. Moreover, a connection $\Gamma : E \to J^1E$ on a vector bundle $E \to M$ is called linear, if it is a linear morphism. In particular, a linear connection on the tangent bundle $E = TM$ is exactly a classical linear connection on M.

We also recall that the classical Ehresmann prolongation of a connection $\Gamma : Y \to J^1Y$ is the r-th order semiholonomic connection $\Gamma^{(r-1)} : Y \to \overline{J}^rY$ defined by

$$\Gamma^{(1)} := J^1\Gamma \circ \Gamma, \quad \Gamma^{(r-1)} := J^1\Gamma^{(r-2)} \circ \Gamma,$$

see [5]. Clearly, if $\Gamma : E \to J^1E$ is a general linear connection, then its Ehresmann prolongation $\Gamma^{(r-1)} : E \to \overline{J}^rE$ is also linear.

Denote by (x^i, y^p) the canonical coordinates on Y and let $(x^i, y^p, y^p_i = \frac{\partial y^p}{\partial x^i}, y^p_{ij} = \frac{\partial y^p_i}{\partial x^j})$ be the induced coordinates on $\overline{J}^2Y$. One evaluates directly that if $y^p_i = \Gamma^p_i(x, y)$ is the coordinate expression of Γ, then its Ehresmann prolongation $\Gamma^{(1)} : Y \to \overline{J}^2Y$ has equations

$$y^p_i = \Gamma^p_i, \quad y^p_{ij} = \frac{\partial \Gamma^p_i}{\partial x^j} + \frac{\partial \Gamma^p_i}{\partial y^q}\Gamma^q_j.$$

In what follows we denote by $C : \overline{J}^2 \to J^2$ the well known symmetrization of second order semiholonomic jets, in coordinates $(x^i, y^p, y^p_i, y^p_{ij}) \mapsto (x^i, y^p, y^p_i, \frac{1}{2}(y^p_{ij} + y^p_{ji}))$.

All manifolds are assumed to be second countable, Hausdorff, finite dimensional, without boundaries and of class C^∞. All maps between manifolds are assumed to be C^∞.

2. The main result

Let m, n, r be positive integers and let $\mathcal{VB}_{m,n}$ be the category of vector bundles with m-dimensional bases and n-dimensional fibers and their (local with respect to bases) vector bundle isomorphisms. The general concept of natural operators can be found in [6]. In particular, a $\mathcal{VB}_{m,n}$-gauge natural operator D sending general linear connections $\Gamma : E \to J^1E$ on $\mathcal{VB}_{m,n}$-objects E into r-th order holonomic connections $D_E(\Gamma) : E \to J^rE$ on E is an $\mathcal{VB}_{m,n}$-invariant family D of regular operators (functions) $D_E : \mathrm{Con}_{\mathrm{lin}}(E) \to \mathrm{Con}^r(E)$ for all $\mathcal{VB}_{m,n}$-objects $p : E \to M$, where $\mathrm{Con}_{\mathrm{lin}}(E)$ is the space of all general linear connections on E and $\mathrm{Con}^r(E)$ is the space of r-th order holonomic connections on E. The $\mathcal{VB}_{m,n}$-invariance of D means that if $\Gamma \in \mathrm{Con}_{\mathrm{lin}}(E)$ and $\Gamma_1 \in \mathrm{Con}_{\mathrm{lin}}(E_1)$ are f-related by an $\mathcal{VB}_{m,n}$-map $f : E \to E_1$ (i.e. if $J^1f \circ \Gamma = \Gamma_1 \circ f$) then $D_E(\Gamma)$ and $D_{E_1}(\Gamma_1)$ are f-related (i.e. $J^rf \circ D_E(\Gamma) = D_{E_1}(\Gamma_1) \circ f$). The regularity of D_E means that D_E transforms smoothly parametrized families of general linear connections into smoothly parametrized families of r-th order holonomic connections.

The main results of this note are the following theorems being the solution of the problem of existence of holonomic prolongation of linear general connections and the problem of existence of symmetrization of semiholonomic jets.

Theorem 2.1. *Let m, n and r be positive integers.*

(1) If $m \geq 2$ and $r \geq 3$, then there is no $\mathcal{VB}_{m,n}$-gauge natural operator D transforming general linear connections $\Gamma : E \to J^1E$ on $\mathcal{VB}_{m,n}$-objects $E \to M$ into r-th order holonomic connections $D_E(\Gamma) : E \to J^rE$ on $E \to M$.

(2) If $r = 2$, we have such D in question defined (for example) by $D_E(\Gamma) := C_E \circ \Gamma^{(1)} : E \to J^2E$ (where $C : \overline{J}^2 \to J^2$ is the usual symmetrization of second order semiholonomic jets and $\Gamma^{(r)}$ is the Ehresmann prolongation of Γ)

(3) If $r = 1$, we have such D in question defined (for example) by $D_E(\Gamma) := \Gamma : E \to J^1E$.

(4) If $m = 1$, then $\overline{J}^rE = J^rE$, and we have such D in question defined (for example) by $D_E(\Gamma) := \Gamma^{(r-1)} : E \to J^rE$.

The proof of Theorem 2.1 will be presented in Section 3 below.

Theorem 2.2. *Let r, m and n be positive integers.*

(1) If $r \geq 3$ and $m \geq 2$, then there is no $\mathcal{VB}_{m,n}$-natural transformation $S : \overline{J}^r \to J^r$ such that $S_E : \overline{J}^rE \to J^rE$ is a base preserving fibered map with $S_{E|J^rE} = id_{J^rE}$ for any $\mathcal{VB}_{m,n}$-object $E \to M$.

(2) If $r = 2$, we have such S defined (for example) by $S_E := C_E : \overline{J}^2E \to J^2E$ (the well-known symmetrization of second order semiholonomic jets).

(3) If $r = 1$, then $\overline{J}^1E = J^1E$, and then we have such S defined (for example) by $S_E := id : J^1E \to J^1E$.

(4) If $m = 1$, then $\overline{J}^rE = J^rE$, and then we have such S defined (for example) by $S_E = id_{J^rE} : J^rE \to J^rE$.

Proof. It suffices to prove the part (1) only. If such S exists, then we can define an r-th order holonomic connection $D_E(\Gamma) := S_E \circ \Gamma^{(r-1)} : E \to J^rE$ from any linear general connection $\Gamma : E \to J^1E$ on an $\mathcal{VB}_{m,n}$-object $E \to M$. This contradicts with Theorem 2.1(1). □

Remark 2.3. *We remark that in (1) the assumption $S_{E|J^rE} = id_{J^rE}$ is essential. For, we have the 0-vector bundle map $0 : \overline{J}^rE \to J^rE$.*

We have the following interesting corollaries of Theorem 2.1.

Corollary 2.4. *If $r \geq 3$, $s \geq 1$, $m \geq 2$ and $n \geq 1$ are integers, then there is no $\mathcal{VB}_{m,n}$-gauge natural operator D transforming s-th order semiholonomic linear connections $\Theta : E \to \overline{J}^sE$ on $\mathcal{VB}_{m,n}$-objects $E \to M$ into r-th order holonomic connections $D_E(\Theta) : E \to J^rE$ on $E \to M$.*

Proof. If such D exists, then we have a $\mathcal{VB}_{m,n}$-gauge natural operator D' sending general linear connections $\Gamma : E \to J^1E$ on $\mathcal{VB}_{m,n}$-objects $E \to M$ into r-th order holonomic connections $D'_E(\Gamma) := D_E(\Gamma^{(s-1)}) : E \to J^rE$ on $E \to M$. This contradicts with Theorem 2.1 (1). □

Corollary 2.5. *If $r \geq 3$, $m \geq 2$ and $n \geq 1$ are integers, then there is no $\mathcal{VB}_{m,n}$-gauge natural operator D transforming second order linear holonomic connections $\Theta : E \to J^2E$ on $\mathcal{VB}_{m,n}$-objects $E \to M$ into r-th order holonomic connections $D_E(\Theta) : E \to J^rE$ on $E \to M$.*

Proof. If such D exists, then we have $\mathcal{VB}_{m,n}$-gauge natural operator D' sending general linear connections $\Gamma : E \to J^1E$ on $\mathcal{VB}_{m,n}$-objects $E \to M$ into r-th order holonomic connections $D'_E(\Gamma) := D_E(C_E \circ \Gamma^{(1)}) : E \to J^rE$ on $E \to M$. This contradicts with Theorem 2.1 (1). □

Remark 2.6. *Theorem 2.1 (1) and Theorem 2.2 (1) can be treated as the vector bundle version of the corresponding results from [2]. Using Theorem 2.1 (1) and Theorem 2.2 (1) we immediately reobtain the above mentioned results of [2] (but not vice-versa).*

Remark 2.7. *Theorem 2.1 (1) shows that for $r \geq 3$, $m \geq 2$ and $n \geq 1$, to construct canonically an r-th order holonomic connection $D_E(\Gamma) : E \to J^rE$ from a general linear connection $\Gamma : E \to J^1E$ the using of an additional object is unavoidable. Similarly, Theorem 2.2(1) shows that for $r \geq 3$, $m \geq 2$ and $n \geq 1$, to construct a canonical symmetrization $\overline{J}^rE \to J^rE$ being the identity map on J^rE the using of an additional object is unavoidable.*

Remark 2.8. *By [11] and [12], for any fibered manifold $Y \to M$ there exists one and only one canonical construction of symmetrization $S_Y(\nabla) : \overline{J}^r Y \to J^r Y$ by means of a classical linear connection ∇ on M. If $E \to M$ is a vector bundle, $S_E(\nabla) : \overline{J}^r E \to J^r E$ is a vector bundle map being the identity map on $J^r E$. So, for any linear general connection $\Gamma : E \to J^1 E$ on a vector bundle $E \to M$ and any classical linear connection ∇ on M, we have an r-th order holonomic linear connection $S_E(\nabla) \circ \Gamma^{(r-1)} : E \to J^r E$.*

Remark 2.9. *In [2] we have also constructed a symmetrization $\overline{J}^r Y \to J^r Y$ for any r by means of a projectable classical linear connection Δ on Y.*

Remark 2.10. *It could be interesting to replace in Corollary 2.4 s-th order semiholonomic linear connection Θ by s-th order holonomic linear connection Θ (if $s = 2$, we made it in Corollary 2.5). For $r > s \geq 3$, the problem is still open (if $r \leq s$, we have $\pi^s_r \circ \Theta : E \to J^r E$).*

3. Proof of Theorem 2.1

Remark 3.1. *We remark that part (1) of Theorem 2.1 is the vector bundle version of Proposition 3 in [2]. On the other hand, the proof of part (1) of Theorem 2.1 will be much more complicated than the one of Proposition 3 in [2] because of the following two reasons. (1) The $\mathcal{VB}_{m,n}$-orbit of $(0,0)$ is not dense in $\mathcal{VB}_{m,n}$-objects. (To dispose of this problem we will consider the (dense) $\mathcal{VB}_{m,n}$-orbit of $(0, e_1)$ and define B by (1), i.e. almost as in the proof of Proposition 3 in [2] with (x, e_1) instead of $(x, 0)$. However, then we cannot deduce that $B(\omega)$ is linear in ω because the fiber homotheties do not preserve (x, e_1). So, we cannot proceed later as in the proof of Proposition 3 in [2].) (2) The fibered diffeomorphism $(x^1, \dots, x^m, y^1 + x^1, y^2, \dots, y^n)$, which was essentially used in the proof of Proposition 3 in [2], is not a $\mathcal{VB}_{m,n}$-map.*

Proof. [Proof of Theorem 2.1] It suffices to prove the part (1) only. We will use the following non trivial modification of the proof of Proposition 3 in [2]. Let $x^1, \dots, x^m$ be the usual coordinates on $\mathbb{R}^m$ and $y^1, \dots, y^n$ be the usual coordinates on $\mathbb{R}^n$. Let $\mathcal{M}f_m$ be the category of m-dimensional manifolds and their embeddings. Suppose D is a $\mathcal{VB}_{m,n}$-gauge natural operator transforming general linear connections Γ on $\mathcal{VB}_{m,n}$-objects $p : E \to M$ into r-th order holonomic connections $D_E(\Gamma) : E \to J^r E$ on $E \to M$ and $r \geq 3$ and $m \geq 2$.

Using D, we define an $\mathcal{M}f_m$-natural operator B transforming 1-forms ω on m-manifolds M into sections $B_M(\omega) : M \to J^r(M, \mathbb{R})_1 \subset J^r(M, \mathbb{R})$ of $J^r(M, \mathbb{R}) \to M$ by

$$B_M(\omega)(x) = \mathrm{pr}_1 \circ D_{M\times\mathbb{R}^n}(\Gamma^o_M + \omega \otimes y^1 \frac{\partial}{\partial y^1})(x, e_1) \tag{1}$$

for any m-manifold M, any 1-form ω on M and any $x \in M$, where Γ^o_M denotes the trivial (linear) connection on the trivial bundle (trivial $\mathcal{VB}_{m,n}$-object) $M \times \mathbb{R}^n$ over M, $e_1 = (1, 0, \dots,) \in \mathbb{R}^n$ and $\mathrm{pr}_1 : J^r(M \times \mathbb{R}^n) \tilde{=} \times^n_M J^r(M, \mathbb{R}) \to J^r(M, \mathbb{R})$ is the projection onto the first factor.

We can write $B_{\mathbb{R}^m}(0)(0) = j^r_0 \gamma$ for some $\gamma : \mathbb{R}^m \to \mathbb{R}$ with $\gamma(0) = 1$. Using the invariance of B with respect to the homotheties $\frac{1}{t}\mathrm{id}_{\mathbb{R}^m}$ for $t > 0$ and then putting $t \to 0$, we deduce

$$B_{\mathbb{R}^m}(0)(0) = j^r_0 1.$$

Let $f : \mathbb{R} \to \mathbb{R}$ be a map such that $f(0) = 0$. Then $\Phi = (x^1, \dots, x^m, y^1 + f(x^1)y^1, y^2, \dots, y^n)$ is a $\mathcal{VB}_{m,n}$-map over some neighbourhood of $0 \in \mathbb{R}^m$ (over $1 + f(x^1) \neq 0$). It preserves $(0, e_1)$ and sends $j^r_0 1$ into $j^r_0(1 + f(x^1))$. Moreover, Φ sends (the germ at $0 \in \mathbb{R}^m$) of $\Gamma^o_{\mathbb{R}^m} = \sum^m_{i=1} dx^i \otimes \frac{\partial}{\partial x^i}$ into $\Gamma^o_{\mathbb{R}^m} + \frac{f'(x^1)}{1+f(x^1)} dx^1 \otimes y^1 \frac{\partial}{\partial y^1}$. For, the flow of $\frac{\partial}{\partial x^1}$ is $\varphi_t = (x^1 + t, x^2, \dots, x^m, y^1, \dots, y^m)$. Then the flow Ψ_t of $\Phi_* \frac{\partial}{\partial x^1}$ satisfies $\Psi_t(x, y) = \Phi \circ \varphi_t \circ \Phi^{-1}(x, y) = (x^1 + t, \dots, x^m, \frac{1+f(x^1+t)}{1+f(x^1)} y^1, y^2, \dots, y^m)$, so that $\Phi_* \frac{\partial}{\partial x^1} = \frac{\partial}{\partial x^1} + \frac{f'(x^1)}{1+f(x^1)} y^1 \frac{\partial}{\partial y^1}$. Similarly, $\Phi_* \frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i}$ for $i = 2, \dots, m$ and $\Phi_* dx^i = dx^i$ for $i = 1, \dots, m$. Then $\Phi_* \Gamma^o_{\mathbb{R}^m} = \Gamma^o_{\mathbb{R}^m} + \frac{f'(x^1)}{1+f(x^1)} dx^1 \otimes y^1 \frac{\partial}{\partial y^1}$ and we have

$$B_{\mathbb{R}^m}(\frac{f'(x^1)}{1 + f(x^1)} dx^1)(0) = j^r_0(1 + f(x^1)).$$

Putting $f(x^1) = e^{x^1} - 1$, we get $B(dx^1)(0) = j^r_0 e^{x^1}$. Next, using the $\mathcal{M}f_m$-invariance and regularity of B and the rank theorem, we get

$$B(dg)(0) = j^r_0(e^g)$$

for any map $g : \mathbb{R}^m \to \mathbb{R}$ with $g(0) = 0$. In particular, for $g = \frac{1}{2}(x^1)^2$ we have

$$B_{\mathbb{R}^m}(x^1dx^1)(0) = j^r_0(e^{\frac{1}{2}(x^1)^2}). \tag{2}$$

We can write $B_{\mathbb{R}^m}(x^2dx^1)(0) = j^r_0(\sigma(x^1, \dots, x^m))$ for some $\sigma : \mathbb{R}^m \to \mathbb{R}$ with $\sigma(0) = 1$. Using the invariance of B with respect to $(x^1, x^2, \frac{1}{t}x^3, \dots, \frac{1}{t}x^m)$ for $t > 0$ and next putting $t \to 0$, we can write

$$B_{\mathbb{R}^m}(x^2dx^1)(0) = j^r_0\Big(\sum_{0\le k+l\le r} a_{k,l}(x^1)^k(x^2)^l\Big)$$

for some uniquely determined real numbers $a_{i,j}$ for non-negative integers i, j with $i + j \le r$ and with $a_{0,0} = 1$. We see that x^2dx^1 is preserved by $b_t = (tx^1, \frac{1}{t}x^2, x^3, \dots, x^m)$ for any $t > 0$. Then, using the invariance of B with respect to b_t, we get

$$B_{\mathbb{R}^m}(x^2dx^1)(0) = j^r_0\Big(\sum_k a_{k,k}(x^1x^2)^k\Big).$$

By the $\mathcal{M}f_m$-invariance and regularity of B and the rank theorem, we get

$$B_{\mathbb{R}^m}(fdg)(0) = j^r_0\Big(\sum_k a_{k,k}(fg)^k\Big)$$

for any maps $f, g : \mathbb{R}^m \to \mathbb{R}$ with $f(0) = g(0) = 0$. In particular, for $f = g = x^1$ we have

$$B_{\mathbb{R}^m}(x^1dx^1)(0) = j^r_0\Big(\sum_k a_{k,k}(x^1)^{2k}\Big).$$

Using (2), we get $a_{k,k} = \frac{1}{2^k}\frac{1}{k!}$. Consequently,

$$B_{\mathbb{R}^m}(x^2dx^1)(0) = j^r_0(e^{\frac{1}{2}x^2x^1}). \tag{3}$$

Next, by the $\mathcal{M}f_m$-invariance and regularity of B and the rank theorem, we have

$$B_{\mathbb{R}^m}(fdg)(0) = j^r_0(e^{\frac{1}{2}fg})$$

for any maps $f, g : \mathbb{R}^m \to \mathbb{R}$ with $f(0) = g(0) = 0$. Then, for $f = (1 + x^1)^2x^2$ and $g = \frac{x^1}{1+x^1}$, since $(1 + x^1)^2x^2d(\frac{x^1}{1+x^1}) = x^2dx^1$ and $(1 + x^1)^2x^2\frac{x^1}{1+x^1} = (1 + x^1)x^2x^1$ near 0, we have

$$B_{\mathbb{R}^m}(x^2dx^1)(0) = j^r_0(e^{\frac{1}{2}(1+x^1)x^2x^1}). \tag{4}$$

Now, by (3) and (4),

$$j^r_0(x^2x^1) = j^r_0((1 + x^1)x^2x^1).$$

Consequently, $j^r_0(x^1x^2x^1) = 0$. But this is impossible if $r \ge 3$. □

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