Filomat 33:5 (2019), 1353–1359
https://doi.org/10.2298/FIL1905353S

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Solution of the Rational Difference Equation $x_{n+1} = \frac{x_{n-17}}{1+x_{n-5}.x_{n-11}}$

Dağıstan Simşek[a,c], Burak Oğul[b], Cengiz Çınar[d]

[a]*Department of Industrial Engineering, Faculty of Engineering and Natural Sciences, Konya Technical University, Konya, Turkey*
[b]*Institute of Science, Kyrgyz Turkish Manas University, Bishkek, Kyrgyzstan*
[c]*Department of Applied Mathematics and Informatics, Faculty of Science, Kyrgyz Turkish Manas University, Bishkek, Kyrgyzstan*
[d]*Department of Mathematics Education, Faculty of Education, Gazi University, Ankara, Turkey*

Abstract. In this paper, solution of the following difference equation is examined

$$x_{n+1} = \frac{x_{n-17}}{1 + x_{n-5}.x_{n-11}},$$

where the initial conditions are positive reel numbers.

1. Introduction

Difference equations appear naturally as discrete analogs and as numerical solutions of differential and delay differential equations, having applications in biology, ecology, physics.

Recently, a high attention to studying the periodic nature of nonlinear difference equations has been attracted. For some recent results concerning the periodic nature of scalar nonlinear difference equations, among other problems (see, for example, [1–35]).

Cinar ([3–5]), studied the following problems with positive initial values:

$$x_{n+1} = \frac{x_{n-1}}{1 + ax_n x_{n-1}}, \quad n = 0, 1, ...,$$

$$x_{n+1} = \frac{x_{n-1}}{-1 + ax_n x_{n-1}}, \quad n = 0, 1, ...,$$

$$x_{n+1} = \frac{ax_{n-1}}{1 + bx_n x_{n-1}}, \quad n = 0, 1, ...,$$

respectively.

Simsek et. al. ([28–30, 32]), studied the following problems with positive initial values

$$x_{n+1} = \frac{x_{n-3}}{1 + x_{n-1}}, \quad n = 0, 1, ...,$$

2010 *Mathematics Subject Classification*. Primary 39A10
Keywords. Difference equations, rational difference equations, period 18 solution
Received: 18 July 2018; Revised: 21 January 2019; Accepted: 10 February 2019
Communicated by Fahreddin Abdullayev
Email addresses: dsimsek@ktun.edu.tr, dagistan.simsek@manas.edu.kg (Dağıstan Simşek), burak_1745@hotmail.com (Burak Oğul), ccinar2525@gmail.com (Cengiz Çınar)

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-2}}, \quad n = 0, 1, ...,$$

$$x_{n+1} = \frac{x_{n-5}}{1 + x_{n-1}x_{n-3}}, \quad n = 0, 1, ...,$$

$$x_{n+1} = \frac{x_{n-3}}{1 + x_n x_{n-1} x_{n-2}}, \quad n = 0, 1, ...,$$

respectively.

Elsayed [15], studied the global result, boundedness, and periodicity of solutions of the difference equation

$$x_{n+1} = a + \frac{bx_{n-l} + cx_{n-k}}{dx_{n-l} + ex_{n-k}}, \quad n = 0, 1, ...,$$

where the parameters a, b, c, d and e are positive real numbers and the initial conditions $x_{-t}, x_{-t+1}, ..., x_0$ are positive real numbers where $t = max\{l, k\}, l \neq k$.

DeVault [8], studied the following problems

$$x_{n+1} = \frac{A}{x_n} + \frac{1}{x_{n-2}}, \quad n = 0, 1, ...,$$

and showed every positive solution of the equation where $A \in (0, \infty)$.

Ibrahim [18], studied the solutions of non-linear difference equation

$$x_{n+1} = \frac{x_n x_{n-2}}{x_{n-1} (a + bx_n x_{n-2})}, \quad n = 0, 1, ...,$$

where the initial values x_0, x_{-1} and x_{-2} non-negative real numbers with $bx_0\ x_{-2} \neq -a$ and $x_{-1} \neq 0$. He investigated some properties for this difference equation such as the local stability and the boundedness for the solutions.

In this work, the following non-linear difference equation is studied

$$x_{n+1} = \frac{x_{n-17}}{1 + x_{n-5}x_{n-11}}, \quad n = 0, 1, ..., \tag{1}$$

where $x_{-17}, x_{-16}, ..., x_{-1}, x_0 \in (0, \infty)$ is investigated.

2. Main Result

Let $\overline{x}$ be the unique positive equilibrium of the equation (1), then clearly

$$\overline{x} = \frac{\overline{x}}{1 + \overline{x}.\overline{x}} \Rightarrow \overline{x} + \overline{x}^3 = \overline{x} \Rightarrow \overline{x}^3 = 0 \Rightarrow \overline{x} = 0,$$

so, $\overline{x} = 0$ can be obtained. For any $k \geq 0$ and $m > k$, notation $i = \overline{k, m}$ means $i = k, k + 1, ..., m$.

Theorem 2.1. *Consider the difference equation* (1)*. Then the following statements are true:*

a) The sequences $(x_{18n-17}), (x_{18n-16}), ..., (x_{18n-1}), (x_{18n})$ *are decreased and* $a_1, ..., a_{18} \geq 0$ *is existed in such that:*

$$\lim_{n\to\infty} x_{18n-17+k} = a_{1+k}, \quad k = \overline{0, 17}.$$

b) $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, ...)$ *is a solution of equation* (1) *having period eighteen.*

c)

$$\prod_{k=0}^{2}\lim_{n\to\infty}x_{18n-17-j+6k}=0,\quad j=\overline{0,5};$$

or

$$\prod_{k=0}^{2}a_{6k+i}=0,\quad i=\overline{1,6}.$$

d) If there exist $n_0\in\mathbb{N}$ such that $x_{n+1}\le x_{n-11}$ for all $n\ge n_0$, then

$$\lim_{n\to\infty}x_n=0.$$

e) The following formulas can be generated:

$$x_{18n+k+1}=x_{-17+k}\left(1-\frac{x_{-5+k}x_{-11+k}}{1+x_{-5+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{3j}\frac{1}{1+x_{6i-5+k}x_{6i-11+k}}\right),\quad k=\overline{0,5},$$

$$x_{18n+k+7}=x_{-11+k}\left(1-\frac{x_{-5+k}x_{-17+k}}{1+x_{-5+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{3j+1}\frac{1}{1+x_{6i-5+k}x_{6i-11+k}}\right),\quad k=\overline{0,5},$$

$$x_{18n+k+13}=x_{-5+k}\left(1-\frac{x_{-11+k}x_{-17+k}}{1+x_{-5+k}x_{-11+k}}\sum_{j=0}^{n}\prod_{i=1}^{3j+2}\frac{1}{1+x_{6i-5+k}x_{6i-11+k}}\right),\quad k=\overline{0,5}.$$

f) If $x_{18n+k}\to a_k\neq 0$, $x_{18n+6+k}\to a_{6+k}\neq 0$, then $x_{18n+12+k}\to 0$ as $n\to\infty$, $k=\overline{1,6}$.

Proof. a) Firstly, from (1), we get

$$x_{n+1}\left(1+x_{n-5}x_{n-11}\right)=x_{n-17}.$$

If $x_{n-5}x_{n-11}\in(0,+\infty)$, then

$$\left(1+x_{n-5}x_{n-11}\right)\in(1,+\infty).$$

Since $x_{n+1}<x_{n-17}$, $n\in\mathbb{N}$, we obtain that there exist

$$\lim_{n\to\infty}x_{18n-17+k}=a_{1+k},\quad k=\overline{0,17}$$

b) $(a_1,a_2,...,a_{18},a_1,a_2,...,a_{18},...)$ is a solution of equation (1) having period eighteen.

c) In view of the equation (1),

$$x_{18n+1}=\frac{x_{18n-17}}{1+\prod_{k=0}^{1}x_{18n-11+6k}}$$

is obtained. If the limits are put on both sides of the above equality,

$$\prod_{k=0}^{2}\lim_{n\to\infty}x_{18n-17+6k}=0\quad\text{or}\quad\prod_{k=0}^{2}a_{6k+1}=0$$

is obtained. Similarly, we can find $x_{18n+2},x_{18n+3},x_{18n+4},x_{18n+5},x_{18n+6}$.

d) If there exist $n_0\in\mathbb{N}$ such that $x_{n+1}\le x_{n-11}$ for all $n\ge n_0$, then, $a_1\le a_7\le a_{13}\le a_1,...,$ $a_6\le a_{12}\le a_{18}\le a_6$. Using (c), we get

$$\prod_{k=0}^{2}a_{6k+i}=0,\quad i=\overline{1,6}.$$

Then, we see that

$$\lim_{n\to\infty}x_n=0.$$

Hence the proof of (d) completed.

e) Subtracting x_{n-17} from the left and right-hand sides of equation (1), we obtain:

$$x_{n+1} - x_{n-17} = \frac{1}{1 + x_{n-5}x_{n-11}} \left(x_{n-5} - x_{n-23}\right). \tag{2}$$

From (2), for $n \geq 6$ following formula is produced.

$$\begin{aligned}
x_{6n-35} - x_{6n-53} &= (x_1 - x_{-17}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i-5}x_{6i-11}}, \\
x_{6n-34} - x_{6n-52} &= (x_2 - x_{-16}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i-4}x_{6i-10}}, \\
x_{6n-33} - x_{6n-51} &= (x_3 - x_{-15}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i-3}x_{6i-9}}, \\
x_{6n-32} - x_{6n-50} &= (x_4 - x_{-14}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i-2}x_{6i-8}}, \\
x_{6n-31} - x_{6n-49} &= (x_5 - x_{-13}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i-1}x_{6i-7}}, \\
x_{6n-30} - x_{6n-48} &= (x_6 - x_{-12}) \prod_{i=1}^{n-6} \tfrac{1}{1+x_{6i}x_{6i-6}},
\end{aligned} \tag{3}$$

Replacing n by $3j$ in (3) and summing from $j = 0$ to $j = n$, we obtain:

$$x_{18n+1+k} - x_{-17+k} = (x_{1+k} - x_{-17+k}) \sum_{j=0}^{n} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}, \quad k = \overline{0,5}.$$

Also, replacing n by $3j + 1$ in (3) and summing from $j = 0$ to $j = n$, we obtain:

$$x_{18n+7+k} - x_{-11+k} = (x_{7+k} - x_{-11+k}) \sum_{j=0}^{n} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}, \quad k = \overline{0,5}.$$

Also, replacing n by $3j + 2$ in (3) and summing from $j = 0$ to $j = n$, we obtain:

$$x_{18n+13+k} - x_{-5+k} = (x_{13+k} - x_{-5+k}) \sum_{j=0}^{n} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}, \quad k = \overline{0,5}.$$

Now, we obtained of the above formulas:

$$x_{18n+k+1} = x_{-17+k} \left(1 - \frac{x_{-5+k}x_{-11+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right), \quad k = \overline{0,5},$$

$$x_{18n+k+7} = x_{-11+k} \left(1 - \frac{x_{-5+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right), \quad k = \overline{0,5},$$

$$x_{18n+k+13} = x_{-5+k} \left(1 - \frac{x_{-11+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right), \quad k = \overline{0,5}.$$

f) Suppose that $a_{1+k} = a_{7+k} = a_{13+k} = 0$ for $k = \overline{0,5}$. By (e), the following formulas are produced below

$$\lim_{n\to\infty} x_{18n+1+k} = \lim_{n\to\infty} x_{-17+k} \left(1 - \frac{x_{-5+k}x_{-11+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{1+k} = x_{-17+k}\left(1 - \frac{x_{-5+k}x_{-11+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{\infty} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{1+k} = 0 \Rightarrow \frac{1 + x_{-5+k}x_{-11+k}}{x_{-5+k}x_{-11+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}. \tag{4}$$

Similarly,

$$\lim_{n\to\infty} x_{18n+7+k} = \lim_{n\to\infty} x_{-11}\left(1 - \frac{x_{-5+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{7+k} = x_{-11+k}\left(1 - \frac{x_{-5+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{\infty} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{7+k} = 0 \Rightarrow \frac{1 + x_{-5+k}x_{-11+k}}{x_{-5+k}x_{-17+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}. \tag{5}$$

From (4) and (5);

$$\frac{1 + x_{-5+k}x_{-11+k}}{x_{-5+k}x_{-11+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}} > \frac{1 + x_{-5+k}x_{-11+k}}{x_{-5+k}x_{-17+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}};$$

$$\frac{1 + x_{-5+k}.x_{-11+k}}{x_{-5+k}.x_{-11+k}} > \frac{1 + x_{-5+k}.x_{-11+k}}{x_{-5+k}.x_{-17+k}},$$

$$\frac{1}{x_{-5+k}.x_{-11+k}} > \frac{1}{x_{-5+k}.x_{-17+k}},$$

$$x_{-5+k}.x_{-17+k} > x_{-5+k}.x_{-11+k} \Rightarrow x_{-17+k} > x_{-11+k}.$$

thus $x_{-17+k} > x_{-11+k}$. Similarly,

$$\lim_{n\to\infty} x_{18n+13+k} = \lim_{n\to\infty} x_{-5+k}\left(1 - \frac{x_{-11+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{n} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{13+k} = x_{-5+k}\left(1 - \frac{x_{-11+k}x_{-17+k}}{1 + x_{-5+k}x_{-11+k}} \sum_{j=0}^{\infty} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}\right),$$

$$a_{13+k} = 0 \Rightarrow \frac{1 + x_{-5+k}x_{-11+k}}{x_{-11+k}x_{-17+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}}. \tag{6}$$

From (5) and (6);

$$\frac{1 + x_{-5+k}x_{-11+k}}{x_{-5+k}x_{-17+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j+1} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}} > \frac{1 + x_{-5+k}x_{-11+k}}{x_{-11+k}x_{-17+k}} = \sum_{j=0}^{\infty} \prod_{i=1}^{3j+2} \frac{1}{1 + x_{6i-5+k}x_{6i-11+k}};$$

$$\frac{1 + x_{-5+k}.x_{-11+k}}{x_{-5+k}.x_{-17+k}} > \frac{1 + x_{-5+k}.x_{-11+k}}{x_{-11+k}.x_{-17+k}},$$

$$\frac{1}{x_{-5+k}.x_{-17+k}} > \frac{1}{x_{-11+k}.x_{-17+k}},$$

$$x_{-11+k}.x_{-17+k} > x_{-5+k}.x_{-17+k} \Rightarrow x_{-11+k} > x_{-5+k}.$$

Thus $x_{-11+k} > x_{-5+k}$. We obtain $x_{-17+k} > x_{-11+k} > x_{-5+k}$. We arrive at a contradiction, which completes the proof of the theorem. □

Example 2.2. Consider the following equation $x_{n+1} = \frac{x_{n-17}}{1+x_{n-5}.x_{n-11}}$. If the initial conditions are selected follows: $x_{-17} = 0.\underbrace{9...9}_{20}$, $x_{-16} = 0.\underbrace{9...9}_{19}$, $x_{-15} = 0.\underbrace{9...9}_{18}$, ..., $x_0 = 0.\underbrace{9...9}_{3}$

The graph of the solution is given below.

$x_n = \{0.5, 0.5, 0.5, 0.500003, 0.500025, 0.50025, 0.666667, 0.666667, 0.666667, 0.666668, 0.666678, 0.666778, 0.75, 0.75, 0.749999, 0.749991, 0.749913, 0.749125, 0.333333, 0.333333, 0.333334, 0.333336, 0.333361, 0.333611, 0.533$ $0.533, 0.533, 0.533335, 0.533346, 0.533458, 0.636792, 0.636792, 0.636792, 0.636784, 0.636708, 0.635947, 0.248826, 0.248826, 0.248827, 0.248829, 0.248854, 0.249103, 0.460385, 0.460385, 0.460385, 0.460386, 0.460397, 0.460506, 0.571342, 0.571342, 0.571341, 0.571333, 0.571258, 0.570503, 0.197006, 0.197006, 0.197007, 0.197009, 0.197033, 0.197275, 0.413808, 0.413808, ...\}$.

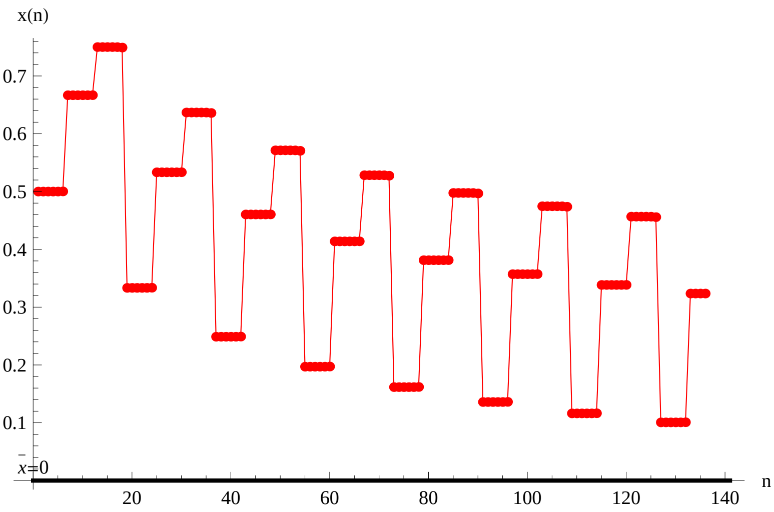

Figure 1: x_n graph of the solution

References

[1] R.P. Agarwal, E.M. Elsayed, On the solution of fourth-order rational recursive sequence, Adv. Studies Cont. Math. 20 (2010) 525–545.

[2] A.M. Amleh, E.A. Grove, G. Ladas,D.A. Georgiou, On the recursive sequence $y_{n+1} = \alpha + \frac{y_{n-1}}{y_n}$, J. Math. Anal. Appl. 233 (1999) 790–798.

[3] C. Cinar, On the positive solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{1+\alpha x_n x_{n-1}}$, J. Appl. Math. Comput. 158 (2004) 890–812.

[4] C. Cinar, On the positive solutions of the difference equation $x_{n+1} = \frac{x_{n-1}}{-1+\alpha x_n x_{n-1}}$, Appl. Math. Comput. 158 (2004) 793–797.

[5] C. Cinar, On the positive solutions of the difference equation $x_{n+1} = \frac{\alpha x_{n-1}}{1+b x_n x_{n-1}}$, Appl. Math. Comput. 156 (2004) 587–590.

[6] C. Cinar, T. Mansour, I. Yalcinkaya, On the difference equation of higher order, Utilitas Math. 92 (2013) 161–166.

[7] S.E. Das, M. Bayram, On a system of rational difference equations, World Appl. Sci. J. 10 (2010) 1306–1312.

[8] R. DeVault, G. Ladas, S.W. Schultz, On the recursive sequence $x_{n+1} = \frac{A}{x_n} + \frac{1}{x_{n-2}}$, Proc. Amer. Math. Soc. 126 (1998) 3257–3261.

[9] E.M. Elabbasy, H. El-Metwally, E.M. Elsayed, On the difference equation $x_{n+1} = \frac{\alpha_x n_k}{\beta} + \gamma\pi = 0$, J. Concrete Appl. Math. 5 (2007) 101–103.

[10] E.M. Elsayed, Solution and attractivity for a rational recursive sequence, Discrete Dynamics Nature Soc. 2011 (2011) 1–17.

[11] E.M. Elsayed, On the solution of some difference equation, European J. Pure Appl. Math. 4 (2011) 287–303.

[12] E.M. Elsayed, On the dynamics of a higher order rational recursive sequence, Commun. Math. Anal. 12 (2012) 117–133.

[13] E.M. Elsayed, Behavior and expression of the solutions of some rational difference equations, J. Comput. Anal. Appl. 15 (2013) 73–81.

[14] E.M. Elsayed, Solution of rational difference system of order two, J. Comput. Anal. Appl. 33 (2014) 751–765.

[15] E.M. Elsayed, Dynamics and behavior of a higher order rational difference equation, J. Nonlinear Sci. Appl. 5 (2016) 1463–1474.

[16] M.E. Erdogan, C. Cinar, I. Yalcinkaya, On the dynamics of the recursive sequence, Mathematical and Computer Modelling 54(5) (2011) 1481–1485.

[17] C.H. Gibbons, M.R.S. Kulenovic, G. Ladas, On the recursive sequence $\frac{\alpha+\beta xn-1}{\chi+\beta xn-1}$, Math. Sci. Res. Hot-Line 4(2) (2000) 1–11.

[18] T.F. Ibrahim, On the third order rational difference equation $x_{n+1} = \frac{x_n x_{n-2}}{x_{n-1}(a+bx_n x_{n-2})}$, Internat. J. Cont. Math. Sci. 4 (2009) 1321–1334.

[19] T.F. Ibrahim, Periodicity and global attractivity of difference equation of higher order, J. Comput. Anal. Appl. 16 (2014) 552–564.

[20] T.F. Ibrahim, N. Touafek, On a third order rational difference equation with variable coefficients, DCDIS Ser. B: Appl. Algorithms 20 (2013) 251–264.

[21] T.F. Ibrahim, Q. Zhang, Stability of an anti-competitive system of rational difference equations, Arch. Sci. 66:5 (2013) 44–58.

[22] R. Karatas, C. Cinar, D. Simsek, On positive solutions of the difference equation $x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}x_{n-5}}$, Internat. J. Cont. Math. Sci 10 (2006) 495–500.

[23] M.R.S. Kulenovic, G. Ladas, W.S. Sizer, On the recursive sequence $\frac{\alpha x_n+\beta x_{n-1}}{\chi x_n+\beta x_{n-1}}$, Math. Sci. Res. Hot-Line 2(5) (1998) 1–16.

[24] M. Lui, Z. Guo, Solvability of a higher-order nonlinear neutral delay difference equation, Adv. Difference Eq. 2010 (2010) 620–627.

[25] D. Simsek, On the recursive sequence $x_{n+1} = \frac{x_{n-11}}{1+x_{n-1}x_{n-3}x_{n-5}x_{n-7}x_{n-9}}$, Selcuk Univ. Res. Center Appl. Math. 28 (2007) 11–23.

[26] D. Şimşek, F.G. Abdullayev, On the recursive sequence $x_{n+1} = \frac{x_{n-(4k+3)}}{1+\prod_{t=0}^{2} x_{n-(k+1)t-k}}$, J. Math. Sci. 222 (2017) 762–771.

[27] D. Şimşek, F.G. Abdullayev, On the recursive sequence $x_{n+1} = \frac{x_{n-(k+1)}}{1+x_n x_{n-1}\ldots x_{n-k}}$, J. Math. Sci. 234 (2018) 73–81.

[28] D. Simsek, C. Cinar, I. Yalcinkaya, On the recursive sequence $x_{n+1} = \frac{x_{n-3}}{1+x_{n-1}}$, Internat. J. Contemp. 9(12) (2006) 475–480.

[29] D. Simsek, C. Cinar, I. Yalcinkaya, On the recursive sequence $x_{n+1} = \frac{x_{n-5}}{1+x_{n-2}}$, Internat. J. Pure Appl. Math. 27 (2006) 501–507.

[30] D. Simsek, C. Cinar, I. Yalcinkaya, On the recursive sequence $x_{n+1} = \frac{x_{n-5}}{1+x_{n-1}x_{n-3}}$, Internat. J. Pure Appl. Math. 28 (2006) 117–124.

[31] D. Şimşek, A. Doğan, On a class of recursive sequence, Manas J. Engin. 2 (2014) 16–22.

[32] D. Şimşek, M. Eroz, Solutions of the rational difference equations $x_{n+1} = \frac{x_{n-3}}{1+x_n x_{n-1}x_{n-2}}$, Manas J. Engin. 4 (2016) 12–20.

[33] D. Şimşek, B. Ogul, Solutions of the rational difference equations $x_{n+1} = \frac{x_{n-(2k+1)}}{1+x_{n-k}}$, Manas J. Engin. 5:3 (2017) 57–68.

[34] D. Simsek, B. Ogul, F. Abdullayev, Solutions of the rational difference equations $x_{n+1} = \frac{x_{n-11}}{1+x_{n-2}x_{n-5}x_{n-8}}$, AIP Conf. Proc. 1880:1 (2017) 040003.

[35] H.D. Voulov, Periodic solutions to a difference equation with maximum, Proc. Amer. Math. Soc. 131 (2002) 2155–2160.