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Some New Results on (s, q) −Dass-Gupta-Jaggi Type Contractive Mappings in b-Metric-Like Spaces

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Abstract. In this paper we consider cyclic $(s - q)$-Dass-Gupta-Jaggi type contractive mapping in b-metric like spaces. By using our new approach for the proof that one Picard's sequence is Cauchy in the context of b-metric-like space, our results generalize, improve and complement several results in the existing literature. Moreover, we showed that the cyclic type results of Kirk et al. are equivalent with the corresponding usual fixed point ones for Dass-Gupta-Jaggi type contractive mappings. Finally, some examples are presented here to illustrate the usability of the obtained theoretical results.

1. Introduction and preliminaries

Let X be a nonempty set and $f : X \to X$ a self-mapping of it. A solution to an equation $fx = x$ is called a fixed point of f. Results dealing with the existence and construction of a solution to an operator equation $fx = x$ form the part of so-called Fixed Point Theory. It is well known that the Banach contraction principle [9] is one of the most important and attractive results in nonlinear analysis and in mathematical analysis in general. Also, the whole fixed point theory is a significant subject in different fields like geometry, differential equations, informatics, physics, economics, engineering, etc. After the existence of the solutions is guaranteed, the numerical methodology will be established in order to obtain an approximated solution to the fixed point problem.

Fixed point of functions depend heavily on the considered spaces that are defined using intuitive axioms. In particular, generalized variants of standard metric spaces are proposed.

This paper is organized as follows. First we present definitions and basic notions of some known generalized metric spaces: partial-metric, metric-like, b-metric, partial b-metric and b-metric-like spaces. Afterwards we give a process diagram, where arrows stand for inclusions, while inverse inclusions do not hold. Eventually we show the proof to the theorem which consists of our main result obtained in this paper.

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We shall present now some definitions and basic notions of generalized metric spaces.

Definition 1.1. *[27] Let X be a nonempty set. A mapping $p : X \times X \to [0, +\infty)$ is said to be a p-metric if the following conditions hold for all $u, v, w \in X$:*

($p1$) $u = v$ *if and only if* $p(u,u) = p(u,v) = p(v,v)$;

($p2$) $p(u,u) \leq p(u,v)$;

($p3$) $p(u,v) = p(v,u)$;

($p4$) $p(u,v) \leq p(u,w) + p(w,v) - p(w,w)$.

Then, the pair (X, p) is called a partial metric space.

Definition 1.2. *[19] Let X be a nonempty set. A mapping $\sigma : X \times X \to [0, +\infty)$ is said to metric-like if the following conditions hold for all $u, v, w \in X$:*

($\sigma1$) $\sigma(u,v) = 0$ *implies* $u = v$;

($\sigma2$) $\sigma(u,v) = \sigma(v,u)$;

($\sigma3$) $\sigma(u,w) \leq \sigma(u,v) + \sigma(v,w)$.

In this case, the pair (X, σ) is called a metric-like space.

Definition 1.3. *[8, 12] Let X be a nonempty set and $s \geq 1$ a given real number. A mapping $b : X \times X \to [0, +\infty)$ is called a b-metric on the set X if the following conditions hold for all $u, v, w \in X$:*

($b1$) $b(u,v) = 0$ *if and only if* $u = v$;

($b2$) $b(u,v) = b(v,u)$;

($b3$) $b(u,w) \leq s[b(u,v) + b(v,w)]$.

In this case, the pair (X, b) is called a b-metric space (with the constant $s \geq 1$).

Definition 1.4. *[40, 49] Let X be a nonempty set and $s \geq 1$. A mapping $p_b : X \times X \to [0, +\infty)$ is called a partial b-metric on the set X if the following conditions hold for all $u, v, w \in X$:*

(p_b1) $u = v$ *if and only if* $p_b(u,u) = p_b(u,v) = p_b(v,v)$;

(p_b2) $p_b(u,u) \leq p_b(u,v)$;

(p_b3) $p_b(u,v) = p_b(v,u)$;

(p_b4) $p_b(u,v) \leq s[p_b(u,w) + p_b(w,v)] - p_b(w,w)$.

Then, the pair (X, p_b) is called a partial b-metric space.

Definition 1.5. *[4] Let X be a nonempty set and $s \geq 1$. A mapping $b_\sigma : X \times X \to [0, +\infty)$ is called a b-metric-like on the set X if the following conditions hold for all $u, v, w \in X$:*

($b_\sigma1$) $b_\sigma(u,v) = 0$ *implies* $u = v$;

($b_\sigma2$) $b_\sigma(u,v) = b_\sigma(v,u)$;

($b_\sigma3$) $b_\sigma(u,w) \leq s[b_\sigma(u,v) + b_\sigma(v,w)]$.

In this case, the pair (X, b_σ) is called a b-metric-like space with the coefficient $s \geq 1$.

Now, we give the process diagram of the classes of generalized metric spaces which were introduced earlier:

$$\begin{array}{ccccc} \text{Metric space} & \to & \text{Partial metric space} & \to & \text{Metric-like space} \\ \downarrow & & \downarrow & & \downarrow \\ b\text{-Metric space} & \to & \text{Partial } b\text{-metric space} & \to & b\text{-Metric-like space} \end{array}$$

For more details on other generalized metric spaces see [2]-[38].

The next proposition helps us to construct some more examples of b-metric (resp. partial b-metric, b-metric-like) spaces.

Proposition 1.6. *([14], Proposition 1.) Let (X, d) (resp. (X, p), (X, σ)) be a metric (resp. partial metric, metric-like) space and $b(u,v) = (d(u,v))^k$ (resp. $p_b(u,v) = (p(u,v))^k$, $b_\sigma(u,v) = (\sigma(u,v))^k$), where $k > 1$ is a real number. Then b (resp. p_b, b_σ) is b-metric (resp. partial b-metric, b-metric-like) with the coefficient $s = 2^{k-1}$.*

Proof. The proof follows from the fact that

$$\alpha^k + \beta^k \leq (\alpha + \beta)^k \leq (\gamma + \delta)^k \leq 2^{k-1} \left(\gamma^k + \delta^k\right),$$

for all nonnegative $\alpha, \beta, \gamma, \delta$ with $\alpha + \beta \leq \gamma + \delta$. □

It is clear that each metric-like space, i.e. each partial b-metric space, is a b-metric-like space, while the converse is not true. For more such examples and details see [4]-[7], [15], [17]-[20], [27], [29]-[37], [40]-[42], [49], [50] and [52].

The definitions of convergent and Cauchy sequences are formally the same in partial metric, metric-like, partial b-metric and b-metric like spaces. Therefore, we give only the definition of convergence and Cauchyness of the sequences in b-metric-like space. Also, these two notions are formally the same in metric and b-metric spaces.

Definition 1.7. *[4] Let $\{r_n\}$ be a sequence in a b-metric-like space (X, b_σ) with the coefficient s. Then*

(i) The sequence $\{r_n\}$ is said to be convergent to r if $\lim_{n\to\infty} b_\sigma (r_n, r) = b_\sigma (r, r)$;

(ii) The sequence $\{r_n\}$ is said to be b_σ-Cauchy in (X, b_σ) if $\lim_{n,m\to\infty} b_\sigma (r_n, r_m)$ exists and is finite;

(iii) One says that a b-metric-like space (X, b_σ) is b_σ-complete if for every b_σ-Cauchy sequence $\{r_n\}$ in X there exists an $r \in X$, such that $\lim_{n,m\to\infty} b_\sigma (r_n, r_m) = b_\sigma (r, r) = \lim_{n\to\infty} b_\sigma (r_n, r)$.

Remark 1.8. *([14], Remark 1.) In a b−metric-like space the limit of a sequence need not be unique and a convergent sequence need not be a b_σ-Cauchy sequence (see Example 7. in [17]). However, if the sequence $\{r_n\}$ is b_σ-Cauchy with $\lim_{n,m\to\infty} b_\sigma (r_n, r_m) = 0$ in the b_σ-complete b-metric-like space (X, b_σ) with the coefficient $s \geq 1$, then the limit of such a sequence is unique. Indeed, in such a case if $r_n \to r$ $(b_\sigma (r_n, r) \to b_\sigma (r, r))$ as $n \to \infty$ we get that $b_\sigma (r, r) = 0$. Now, if $r_n \to r_1$ and $r_n \to r_2$ where $r_1 \neq r_2$, we obtain that:*

$$\frac{1}{s} b_\sigma (r_1, r_2) \leq b_\sigma (r_1, r_n) + b_\sigma (r_n, r_2) \to b_\sigma (r_1, r_1) + b_\sigma (r_2, r_2) = 0 + 0 = 0. \quad (1)$$

From $(b_\sigma 1)$ it follows that $r_1 = r_2$, which is a contradiction. The same is true as well for partial metric, metric like and partial b-metric spaces.

The next definition and the corresponding proposition are important in the context of fixed point theory.

Definition 1.9. *[1] The self-mappings $f, g : X \to X$ are weakly compatible if $f (g (x)) = g (f (x))$, whenever $f (x) = g (x)$.*

Proposition 1.10. *[1] Let f and g be weakly compatible self-maps of a nonempty set X. If they have a unique point of coincidence $v = f (u) = g (u)$, then v is the unique common fixed point of f and g.*

In this paper we shall use the following result to prove that certain Picard sequences are Cauchy. The proof is completely identical with the corresponding one in [23] (see also [42]).

Lemma 1.11. *Let $\{r_n\}$ be a sequence in a b-metric-like space (X, b_σ) with the coefficient $s \geq 1$ such that*

$$b_\sigma (r_n, r_{n+1}) \leq \mu b_\sigma (r_{n-1}, r_n) \quad (2)$$

for some μ, $0 \leq \mu < \frac{1}{s}$, and each $n = 1, 2, ...$ Then $\{r_n\}$ is a b_σ-Cauchy sequence in (X, b_σ) such that $\lim_{n,m\to\infty} b_\sigma (r_n, r_m) = 0$.

Remark 1.12. *It is worth noticing that the previous lemma holds in the context of b-metric-like spaces for each $\mu \in [0, 1)$. For more details see [3, 49].*

Otherwise, in some papers many authors for the proof that certain Picard sequence is a Cauchy often use one of the next lemmas:

Lemma 1.13. *[50] Let $\{r_n\}$ be a sequence on a complete b-metric-like space (X, b_σ) with the coefficient $s \geq 1$ such that*

$$\lim_{n\to\infty} b_\sigma (r_n, r_{n+1}) = 0. \tag{3}$$

If $\lim_{n,m\to\infty} b_\sigma (r_n, r_m) \neq 0$, there exists $\varepsilon > 0$ and two sequences $\{m_k\}_{k=1}^{+\infty}, \{n_k\}_{k=1}^{+\infty}$ of positive integers with $n_k > m_k > k$ such that

$$b_\sigma (r_{n_k}, r_{m_k}) \geq \varepsilon, b_\sigma (r_{m_k}, r_{n_k-1}) < \varepsilon, \frac{\varepsilon}{s^2} \leq \limsup_{k\to\infty} b_\sigma (r_{n_k-1}, r_{m_k-1}) \leq \varepsilon s, \tag{4}$$

$$\frac{\varepsilon}{s} \leq \limsup_{k\to\infty} b_\sigma (r_{n_k-1}, r_{m_k}) \leq \varepsilon, \ \frac{\varepsilon}{s} \leq \limsup_{k\to\infty} b_\sigma (r_{n_k}, r_{m_k-1}) \leq \varepsilon s^2. \tag{5}$$

Lemma 1.14. *[34] Let (X, σ) be a metric-like space and let $\{r_n\}, n \in \mathbb{N}$ be a sequence in X such that $\lim_{n\to\infty} \sigma (r_n, r_{n+1}) = 0$. If $\lim_{n,m\to\infty} \sigma (r_n, r_m) \neq 0$, then there exists $\varepsilon > 0$ and two sequences $\{n_k\}$ and $\{m_k\}$ of positive integer numbers with $n_k > m_k > k$ such that the limit of all of the sequences is $\lim_{\substack{k\to\infty \\ a,b\in\{-1,0,1\}}} \sigma (r_{n_k+a}, r_{m_k+b}) = \varepsilon^+$.*

In the sequel we give the definitions of known notions in existing literature as well as some known results.

• First, let $X \neq \emptyset$. A mapping $f : X \to X$ is said to be an $\eta-$admissible mapping if $\eta (u, v) \geq 1$ implies $\eta (fu, fv) \geq 1$, for all $u, v \in X$ and $\eta : X \times X \to [0, \infty)$.

• Further, let Φ refers to the class of all functions $\phi : [0, +\infty) \to [0, +\infty)$, satisfying the following conditions:

(**1**) ϕ is non-decreasing and continuous;

(**2**) $\lim_{n\to\infty} \phi^n (t) = 0$ for all $t > 0$.

It is clear that (**2**) implies: $\phi (t) < t$ for each $t > 0$.

• Let (X, b_σ) be a b-metric-like space, $l \in \mathbb{N}$, $A_1, A_2, ..., A_l$ are $b_\sigma-$closed subsets of $X, Y = \cup_{i=1}^{l} A_i$ and $\eta : Y \times Y \to [0, +\infty)$ be a given mapping. The mapping $f : Y \to Y$ is called a cyclic η_s^q-rational contractive if

(a)

$$f\left(A_j\right) \subseteq A_{j+1}, j = 1, 2, ..., l, \text{ where } A_{l+1} = A_1, \tag{6}$$

(b) for any $u \in A_i$ and $v \in A_{i+1}, i = 1, 2, ., .., l$, such that $\eta (u, fu) \eta (v, fv) \geq 1$, we get

$$2s^q b_\sigma (fu, fv) \leq \phi (N (u, v)), \tag{7}$$

for all $q > 1$ where

$$N (u, v) = \max \left\{ b_\sigma (u, v), b_\sigma (v, fu), \frac{b_\sigma (u, v) b_\sigma (v, fv)}{1 + b_\sigma (u, fu)}, \frac{b_\sigma (v, fv) [1 + b_\sigma (u, fu)]}{1 + b_\sigma (u, v)}, \frac{b_\sigma (u, fv) + b_\sigma (v, fu)}{4s} \right\}. \tag{8}$$

If we take $X = A_i, i = 1, 2, ..., l$, in the above case, then the mapping f reduces to η_s^q-rational contraction mapping of Dass-Gupta-Jaggi type (see [10], [13], [16], [21], [51]).

The next notion is significant enough in the subject of admissible mappings.

Definition 1.15. *([18], Definition 7.) Let (X, b_σ) be a b-metric-like space and $\eta : X \times X \to [0, \infty)$ be an admissible mapping. It is said that $f : X \to X$ is $\eta-$continuous on (X, b_σ), if $\lim_{n\to\infty} r_n = r$, $\eta (r_n, r_{n+1}) \geq 1$ implies $\lim_{n\to\infty} fr_n = fr$.*

In ([18], Theorem 2) authors proved the next result:

Theorem 1.16. *Let (X, b_σ) be a $b_\sigma-$complete b-metric-like space, l be a positive integer, $A_1, A_2, ..., A_l$ be non-empty $b_\sigma-$closed subsets of $X, Y = \cup_{i=1}^{l} A_i$ and $\eta : Y \times Y \to [0, +\infty)$ be a mapping. Assume that $f : Y \to Y$ is a cyclic η_s^q-rational contractive mapping satisfying the following conditions:*

(i) a mapping f is $\eta-$admissible;

(ii) $\eta(r_0, fr_0) \geq 1$ for some $r_0 \in X$;

(iii) Either mapping f is $\eta-$continuous, or for any sequence $\{r_n\}$ in X and for all $n \geq 0$, if $\eta(r_n, r_{n+1}) \geq 1$ and $\lim_{n\to\infty} r_n = r$, then, $\eta(r, fr) \geq 1$. Therefore, f has a fixed point $r \in \cap_{i=1}^{l} A_i$. Furthermore, if

(iv) for all $r \in \nabla = \{a \in X : fa = a\}$, we get $\eta(r, r) \geq 1$,

then, the uniqueness of the fixed point is realized.

The usefulness of b-metric space in solving certain problems have been shown in [11] where the existence of a solution for the integral inclusion of Fredholm type for multivalued operators is proved, and in [39] for the same problem of single valued operators. [22] again investigates the solution for integral equations while [48] considers solutions to a periodic problem.

For the partial metric spaces [43] gives us homotopy results for multivalued mappings, and [47] deals with common fixed points and coincidence points, both in a metric space and in a partial metric space. Generalized contractions like the one discussed in this paper are also considered in [45] in an ordered framework space, in [46] with an application to a periodic ordinary differential equation and in [28].

For a recent application of fixed point methodologies to the solution of a first–order periodic differential problem, for example, one can refer to [44].

2. Main results

In this section we complement, generalize, extend, unify, enrich and improve recent results announced in [20]-[25], [30], [32]. In our first new result we consider η_s^q-rational contractive mapping:

Theorem 2.1. *Let (X, b_σ) be a $b_\sigma-$complete b-metric-like space, $\eta : X \times X \to [0, \infty)$, $f : X \to X$ given mappings. If for all $u, v \in X$ with $\eta(u, fu)\eta(v, fv) \geq 1$ implies $2s^q b_\sigma(fu, fv) \leq \phi(N(u, v))$ where $q > 1, \phi \in \Phi$ and*

$$N(u,v) = \max\left\{b_\sigma(u,v), b_\sigma(v,fu), \frac{b_\sigma(u,v)\, b_\sigma(v,fv)}{1+b_\sigma(u,fu)}, \frac{b_\sigma(v,fv)[1+b_\sigma(u,fu)]}{1+b_\sigma(u,v)}, \frac{b_\sigma(u,fv)+b_\sigma(v,fu)}{4s}\right\}. \tag{9}$$

Assume that $f : X \to X$ satisfies the following conditions:

(i) The mapping f is $\eta-$admissible;

(ii) There exists $r_0 \in X$ such that $\eta(r_0, fr_0) \geq 1$;

(iii) Either mapping f is $\eta-$continuous, or for any sequence $\{r_n\}$ in X and for all $n \geq 0$, if $\eta(r_n, r_{n+1}) \geq 1$ and $\lim_{n\to\infty} r_n = r$, then, $\eta(r, fr) \geq 1$.

Then f has a fixed point $r \in X$.

Furthermore, if

(iv) for all $r \in \nabla = \{a \in X : fa = a\}$, we have that $\eta(r, r) \geq 1$,

then f has a unique fixed point in X.

Proof. Define the sequence $r_n = f^n r_0$. From (i) and (ii) follows by induction that $\eta(r_n, r_{n+1}) \geq 1$ for all $n \in \mathbb{N}$. Now, we can estimate $b_\sigma(r_n, r_{n+1})$. Indeed, since $\eta(r_n, r_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ we get

$$b_\sigma(r_n, r_{n+1}) \leq 2s^q b_\sigma(r_n, r_{n+1}) = 2s^q b_\sigma(fr_{n-1}, fr_n) \leq \phi(N(r_{n-1}, r_n)), \tag{10}$$

where

$$N(r_{n-1}, r_n) = \max\left\{b_\sigma(r_{n-1}, r_n), b_\sigma(r_n, r_n), \frac{b_\sigma(r_{n-1}, r_n)\, b_\sigma(r_n, r_{n+1})}{1+b_\sigma(r_{n-1}, r_n)},\right.$$

$$\left.\frac{b_\sigma(r_n, r_{n+1})[1+b_\sigma(r_{n-1}, r_n)]}{1+b_\sigma(r_{n-1}, r_n)}, \frac{b_\sigma(r_{n-1}, r_{n+1})+b_\sigma(r_n, r_n)}{4s}.\right\}. \tag{11}$$

If $r_n = r_{n+1}$ for some n then r_n is a fixed point of f. Therefore, suppose that $r_n \neq r_{n+1}$, that is, $b_\sigma(r_n, r_{n+1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. In this case we have

$$N(r_{n-1}, r_n) \leq \max\left\{b_\sigma(r_{n-1}, r_n), 2sb_\sigma(r_{n-1}, r_n), b_\sigma(r_n, r_{n+1}), \frac{sb_\sigma(r_{n-1}, r_n) + sb_\sigma(r_n, r_{n+1}) + 2sb_\sigma(r_{n-1}, r_n)}{4s}\right\}$$

$$= \max\left\{2sb_\sigma(r_{n-1}, r_n), b_\sigma(r_n, r_{n+1}), \frac{3sb_\sigma(r_{n-1}, r_n) + sb_\sigma(r_n, r_{n+1})}{4s}\right\}$$

$$= \max\left\{2sb_\sigma(r_{n-1}, r_n), b_\sigma(r_n, r_{n+1}), \frac{3}{4}b_\sigma(r_{n-1}, r_n) + \frac{1}{4}b_\sigma(r_n, r_{n+1})\right\}. \quad (12)$$

From the known estimation $\frac{a+b}{2} \leq \max\{a, b\}$ for $a = \frac{3}{2}b_\sigma(r_{n-1}, r_n)$ and $b = \frac{1}{2}b_\sigma(r_n, r_{n+1})$ it follows that $\frac{3}{4}b_\sigma(r_{n-1}, r_n) + \frac{1}{4}b_\sigma(r_n, r_{n+1}) \leq \max\left\{\frac{3}{2}b_\sigma(r_{n-1}, r_n), \frac{1}{2}b_\sigma(r_n, r_{n+1})\right\}$. Further, (12) becomes

$$N(r_{n-1}, r_n) \leq \max\{b_\sigma(r_n, r_{n+1}), 2sb_\sigma(r_{n-1}, r_n)\} \quad (13)$$

and $\phi(N(r_{n-1}, r_n)) < N(r_{n-1}, r_n)$. If $\max\{b_\sigma(r_n, r_{n+1}), 2sb_\sigma(r_{n-1}, r_n)\} = b_\sigma(r_n, r_{n+1})$ then (10) implies a contradiction. Hence, we obtain the next estimation:

$$2s^q b_\sigma(r_n, r_{n+1}) < 2sb_\sigma(r_{n-1}, r_n), \quad (14)$$

where $s \geq 1$ and $q > 1$.

The case $s > 1$ implies

$$b_\sigma(r_n, r_{n+1}) \leq \mu b_\sigma(r_{n-1}, r_n), \mu = \frac{1}{s^{q-1}} \in (0, 1). \quad (15)$$

According to Lemma 1.10. it follows that the sequence $\{r_n\}$ is a b_σ–Cauchy and $\lim_{n,m\to\infty} b_\sigma(r_n, r_m) = 0$. This means that there exists a unique point $r \in X$ such that:

$$b_\sigma(r, r) = \lim_{n\to\infty} b_\sigma(r_n, r) = \lim_{n,m\to\infty} b_\sigma(r_n, r_m) = 0. \quad (16)$$

Now, we will show that r is a fixed point of f i.e., $fr = r$.This is evident in the case that the mapping f is a η–continuous. Further, suppose that for any sequence $\{r_n\}$ in X and for all $n \geq 0$, if $\eta(r_n, r_{n+1}) \geq 1$ and $\lim_{n\to\infty} r_n = r$, then, $\eta(r, fr) \geq 1$.

Let, $b_\sigma(r, fr) > 0$. Since $\eta(r_n, fr_n)\eta(r, fr) \geq 1$ then according to $(b_\sigma 3)$ and the given contractive condition, we have

$$b_\sigma(r, fr) \leq sb_\sigma(r, r_{n+1}) + sb_\sigma(fr_n, fr) \leq sb_\sigma(r, r_{n+1}) + \frac{1}{2s^{q-1}}\phi(N(r_n, r)) \leq sb_\sigma(r, r_{n+1}) + \frac{1}{2s^{q-1}}N(r_n, r), \quad (17)$$

where

$$N(r_n, r) = \max\left\{b_\sigma(r_n, r), b_\sigma(r, r_{n+1}), \frac{b_\sigma(r_n, r)b_\sigma(r, fr)}{1 + b_\sigma(r_n, r_{n+1})}, \frac{b_\sigma(r, fr)[1 + b_\sigma(r_n, r_{n+1})]}{1 + b_\sigma(r_n, r)}, \frac{b_\sigma(r_n, fr) + b_\sigma(r, r_{n+1})}{4s}\right\}$$

$$\leq \max\left\{b_\sigma(r_n, r), b_\sigma(r, r_{n+1}), \frac{b_\sigma(r_n, r)b_\sigma(r, fr)}{1 + b_\sigma(r_n, r_{n+1})}, \frac{b_\sigma(r, fr)[1 + b_\sigma(r_n, r_{n+1})]}{1 + b_\sigma(r_n, r)}, \frac{sb_\sigma(r_n, r) + sb_\sigma(r, fr) + b_\sigma(r, r_{n+1})}{4s}\right\}$$

$$\to \max\left\{0, 0, 0, b_\sigma(r, fr), \frac{1}{4}b_\sigma(r, fr)\right\} \text{ as } n \to \infty,$$

$$= b_\sigma(r, fr). \tag{18}$$

Now, letting in (17) the limit as $n \to \infty$ we get $b_\sigma(r, fr) \le \frac{1}{2s^{q-1}} b_\sigma(r, fr)$. The obtained relation is possible only if $b_\sigma(r, fr) = 0$. That is, we proved that the point r is the fixed point of f.

Finally, we will prove the uniqueness of the fixed point f satisfying the condition (iv). Let $r, y \in X$ be two difference fixed points of f. Then, we obtain that $b_\sigma(r, y) > 0, \eta(r, r) \ge 1, \eta(y, y) \ge 1$. Further, since $\eta(r, r)\eta(y, y) \ge 1$ we get that

$$b_\sigma(r, y) \le 2s^q b_\sigma(r, y) \le \phi(N(r, y)), \tag{19}$$

where

$$N(r, y) = \max\left\{b_\sigma(r, y), b_\sigma(y, r), \frac{b_\sigma(r, y) b_\sigma(y, y)}{1 + b_\sigma(r, r)}, \frac{b_\sigma(y, y)[1 + b_\sigma(r, r)]}{1 + b_\sigma(r, y)}, \frac{b_\sigma(r, y) + b_\sigma(y, r)}{4s}\right\}$$

$$= \max\left\{b_\sigma(r, y), \frac{b_\sigma(r, y) \cdot 0}{1 + 0}, \frac{0[1 + 0]}{1 + b_\sigma(r, y)}, \frac{b_\sigma(r, y)}{2s}\right\} = b_\sigma(r, y). \tag{20}$$

Now, (19) becomes

$$b_\sigma(r, y) \le 2s^q b_\sigma(r, y) \le \phi(N(r, y)) < b_\sigma(r, y), \tag{21}$$

which is a contradiction. Hence, uniqueness is proved.

The case $s = 1$ means that (X, b_σ) is actually a complete metric-like space (b_σ become σ). First we show that $\sigma(r_n, r_{n+1}) \to 0$ as $n \to \infty$. From (14) it follows that $\sigma(r_n, r_{n+1}) < \sigma(r_{n-1}, r_n) \to \sigma \ge 0$ as $n \to \infty$. Suppose that $\sigma > 0$. Then, we have that $2\sigma(r_n, r_{n+1}) \le \phi(2\sigma(r_{n-1}, r_n))$, obtaining in the limit $n \to \infty : 2\sigma \le \phi(2\sigma) < 2\sigma$, that is a contradiction, thus $\sigma = 0$. Now, we can use Lemma 1.12 in order to prove that the sequences $r_n = f^n r_0$ is a $\sigma-$Cauchy. Indeed, putting $u = r_{n_k}, v = r_{m_k}$ in the given contractive condition, we obtain

$$2\sigma(r_{n_k+1}, r_{m_k+1}) \le \phi\left(\max\left\{\sigma(r_{n_k}, r_{m_k}), \sigma(r_{m_k}, r_{n_k+1}), \frac{\sigma(r_{n_k}, r_{m_k})\sigma(r_{m_k}, r_{m_k+1})}{1 + \sigma(r_{n_k}, r_{n_k+1})},\right.\right.$$

$$\left.\left.\frac{\sigma(r_{m_k}, r_{m_k+1})[1 + \sigma(r_{n_k}, r_{n_k+1})}{1 + \sigma(r_{n_k}, r_{m_k})}, \frac{\sigma(r_{n_k}, r_{m_k+1}) + \sigma(r_{m_k}, r_{m_k+1})}{4}\right\}\right). \tag{22}$$

Now, letting the limit in (22) as $k \to \infty$ we get:

$$2\varepsilon \le \phi\left(\max\left\{\varepsilon, \varepsilon, 0, 0, \frac{\varepsilon}{2}\right\}\right) = \phi(\varepsilon) < \varepsilon. \tag{23}$$

A contradiction as $\varepsilon > 0$. Hence, the sequence $\{r_n\}$ is a $\sigma-$Cauchy. The rest of the proof is the same as the corresponding one in the case $s > 1$. □

Remark 2.2. *It is useful to notice that the third member* $\frac{\omega(k,fk)\omega(\mu,f\mu)}{\omega(k,\mu)}$ *in [18] raises some doubts. Indeed, it follows from the proof of Theorem 2 from [18] as well as from Examples 5 and 6 in the same paper. For the first case see page 9, lines 3- and 4- where we obtain the form* $\frac{0}{0}$*, a division by zero.*

Furthermore, in both examples 0 is the unique fixed point of the defined mapping f (Γ in notation of [18]). However, for the pair $(k, \mu) = (0, 0)$ we get, in example 5:

$$N(0, 0) = \max\left\{\omega(0, 0), \omega(0, 0), \frac{\omega(0, 0)\omega(0, 0)}{\omega(0, 0)}, \frac{\omega(0, 0)[1 + \omega(0, 0)]}{1 + \omega(0, 0)}, \frac{\omega(0, 0) + \omega(0, 0)}{4 \cdot 1}\right\}$$

$$= \max\left\{0, 0, \frac{0}{0}, 0, 0\right\} = \max\left\{0, \frac{0}{0}\right\} = ?. \tag{24}$$

Similarly, in Example 6 we get again the same form $N(0,0) = \max\left\{\frac{0}{0}, 0\right\} = ?$.

The main motivation for our second new result are recent announced papers [33]-[37]. Now, we can prove this significant and important new result which complements and give complete new observations for cyclic type mappings (for more details see [26]).

In the following theorem, we prove the equivalence of the two theorems. We assume that the contractive condition in Theorem 2 in [18] is the same as in the Theorem 2.1 proved above. Otherwise, Theorem 2 in [18] is not correct.

Theorem 2.3. *Theorem 2.1 and Theorem 2 from [18] are equivalent.*

Proof. If we take $X = A_i, i = 1,2,...,l$, in Theorem 2 from [18], then the mapping f reduces to η_s^q-rational contraction mapping, that is, Theorem 2 from [18] implies our Theorem 2.1. Suppose firstly that $s > 1$. If we take $X = A_i, i = 1,2,...,l$, in Theorem 2 from [18], then the mapping f reduces to η_s^q-rational contraction mapping, that is, Theorem 2 from [18] implies our Theorem 2.1. Conversely, we shall prove that Theorem 2.1 implies Theorem 2 from [18]. Indeed, following the lines of the proof of Theorem 2 from [18] we obtain that the Picard's sequence $r_n = fr_{n-1}, n \in \mathbb{N}$ is $b_\sigma-$Cauchy in b-metric like space (X, b_σ) such that $\lim_{n,m\to\infty} b_\sigma(r_n, r_m) = 0$ .Since Y is $b_\sigma-$closed in (X, b_σ) this means that there exists unique $r \in Y$ such that

$$b_\sigma(r,r) = b_\sigma(r_n, r) = \lim_{n,m\to\infty} b_\sigma(r_n, r_m) = 0. \tag{25}$$

Further, because $f(A_i) \subseteq A_{i+1}, A_{l+1} = A_1$ it follows that the sequence $\{r_n\}$ has infinitely many terms in each A_i for $i \in \{1,2,...,l\}$. Hence, we have the subsequences $\{r_{n_i}\}_{i=1}^{l}$ of $\{r_n\}$ where $\{r_{n_i}\}_{i=1}^{l} \subseteq A_i, i = 1,2,...,l$. It is clear that each r_{n_i} converges to r. From this it follows that $\bigcap_{i=1}^{l} A_i \neq \emptyset$ because it contains at least the element r. Obviously, $\left(\bigcap_{i=1}^{l} A_i, b_\sigma\right)$ is a $b_\sigma-$complete b-metric-like space and since $f : \bigcap_{i=1}^{l} A_i \to \bigcap_{i=1}^{l} A_i$, it is not hard to check that the restriction $f|_{\bigcap_{i=1}^{l} A_i}$ of f on $\bigcap_{i=1}^{l} A_i$ satisfies all conditions of Theorem 2.1. Hence, f has a unique fixed point in $\bigcap_{i=1}^{l} A_i$, that is, Theorem 2.1 implies Theorem 2 from [18]. The proof of the case $s = 1$ is similar. □

Remark 2.4. *It is not hard to check that all results in our paper hold true if instead of the set N we take the set M defined in the following manner:*

$$M(u,v) = \max\left\{ b_\sigma(u,v), b_\sigma(v,fu), \frac{(b_\sigma(u,fu))^2}{1+b_\sigma(u,v)}, \frac{b_\sigma(u,v)\, b_\sigma(v,fv)}{1+b_\sigma(u,fu)}, \right.$$
$$\left. \frac{b_\sigma(v,fv)\left[1+b_\sigma(u,fu)\right]}{1+b_\sigma(u,v)}, \frac{b_\sigma(u,fv)+b_\sigma(v,fu)}{4s}, \frac{b_\sigma(u,fu)+b_\sigma(fu,v)}{1+2s} \right\}. \tag{26}$$

3. Examples

Now, we shall consider some examples which support Theorems 2 and 3 from [18] as well as Theorem 2.1.

Example 3.1. *([18], Example 5.) Suppose that* $X = \mathbb{R}$ *is equipped with the metric like mapping* $\sigma(x,u) = |x| + |y|$ *for all* $x, y \in X$ *with* $s = 1$ *and* $q > 1$. *Suppose that* $B_1 = (-\infty, 0]$ *and* $B_2 = [0, +\infty)$ *and* $Y = B_1 \cup B_2$. *Define* $\Gamma : Y \to Y$ *and* $\eta : Y \times Y \to [0, +\infty)$ *by*

$$\Gamma k = \begin{cases} -2k, \text{ if } k < -1, \\ \frac{-k}{16}, \text{ if } k \in [-1,0], \\ \frac{-k^3}{8}, \text{ if } k \in [0,1], \\ -4k, \text{ if } k > 1 \end{cases} \quad \text{and } \eta(k,\mu) = \begin{cases} k^2+\mu^2+2, \text{ if } k,\mu \in [-1,1] \\ 0, \text{ otherwise.} \end{cases} \tag{27}$$

In addition, define $\theta : [0,+\infty) \to [0,+\infty)$ *by* $\theta(t) = \frac{1}{4}t$.

This example supports Theorem 2 from [18] only if we replace their third member $\frac{\omega(k,\Gamma k)\omega(\mu,\Gamma\mu)}{\omega(k,\mu)}$ in $N(k,\mu)$ on page 5 in [18] with $\frac{\omega(k,\mu)\omega(\mu,\Gamma\mu)}{1+\omega(k,\Gamma k)}$.

Instead of verification found in [18] we give a much shorter one by using Theorem 2.3. That is, we shall check that this example satisfies all conditions of Theorem 2.1 where $X = A_1 \cap A_2 = \{0\}$, $f = \Gamma|_{A_1 \cap A_2} = \Gamma|_{\{0\}}$. Since $\eta(0, f0)\eta(0, f0) = 2 \cdot 2 = 4 \geq 1$ as well as $2 \cdot 1^q b_\sigma(f0, f0) \leq \phi(N(0,0))$ that is, $2 \cdot b_\sigma(0,0) \leq \frac{1}{4} \cdot N(0,0)$ where

$$\begin{aligned} N(0,0) &= \max\left\{b_\sigma(0,0), b_\sigma(0,f0), \frac{b_\sigma(0,0)\cdot b_\sigma(0,f0)}{1+b_\sigma(0,f0)}, \frac{b_\sigma(0,f0)[1+b_\sigma(0,f0)]}{1+b_\sigma(0,0)}, \frac{b_\sigma(0,f0)+b_\sigma(0,f0)}{4\cdot 1}\right\} \\ &= \max\left\{0,0,\frac{0\cdot 0}{1+0}, \frac{0\cdot[1+0]}{1+0}, \frac{0+0}{4}\right\} = \max\{0,0,0,0,0\} = 0\ . \end{aligned} \tag{28}$$

Hence, we get that $2 \cdot 0 \leq \frac{1}{4} \cdot 0$ what is true, that is, all conditions of Theorem 2.1 are satisfied and $f = \Gamma|_{\{0\}}$ has a fixed point $r = 0 \in A_1 \cap A_2$, where $A_1 = B_1, A_2 = B_2$. □

One can check that also Example 6 from [18] is correct only if we take $\frac{\omega(k,\mu)\omega(\mu,\Gamma\mu)}{1+\omega(k,\Gamma k)}$ instead of the third member $\frac{\omega(k,\Gamma k)\omega(\mu,\Gamma\mu)}{\omega(k,\mu)}$ in $N(k,\mu)$ on page 12 in [18].

In the sequel follows two examples which support Theorem 2.1.

Example 3.2. *Suppose that* $X = [0,1]$ *is endowed with the b-metric-like mapping* $b_\sigma(u,v) = (u+v)^2$ *for all* $u, v \in X$ *with* $s = 2$ *and* $q > 1$. *Define* $f : X \to X$ *and* $\eta : X \times X \to [0,+\infty)$ *by*

$$fu = \frac{u}{6},\ \eta(u,v) = \begin{cases} 1, & \text{if } u = v = 0 \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

for all $u, v \in X$.

In addition, define $\phi : [0,\infty) \to [0,\infty)$ by $\phi(t) = \frac{t}{2}$.

We have that $\eta(u, fu)\eta(v, fv) \geq 1$ if and only if $(u,v) = (0,0)$. Now, for this pair $(0,0)$ we get

$2 \cdot s^q \cdot b_\sigma(f0, f0) = 0,$ while

$$\begin{aligned} &\phi\left(\max\left\{b_\sigma(0,0), b_\sigma(0,f0), \frac{b_\sigma(0,0)\cdot b_\sigma(0,f0)}{1+b_\sigma(0,f0)}, \frac{b_\sigma(0,f0)[1+b_\sigma(0,f0)]}{1+b_\sigma(0,0)}, \frac{b_\sigma(0,f0)+b_\sigma(0,f0)}{4\cdot 2}\right\}\right) \\ &= \frac{1}{2}\cdot\max\left\{b_\sigma(0,0), b_\sigma(0,f0), \frac{b_\sigma(0,0)\cdot b_\sigma(0,f0)}{1+b_\sigma(0,f0)}, \frac{b_\sigma(0,f0)[1+b_\sigma(0,f0)]}{1+b_\sigma(0,0)}, \frac{b_\sigma(0,f0)+b_\sigma(0,f0)}{8}\right\} \\ &= \frac{1}{2}\cdot\max\left\{0,0,\frac{0\cdot 0}{1+0}, \frac{0\cdot[1+0]}{1+0}, \frac{0+0}{8}\right\} = \frac{1}{2}\cdot 0 = 0 \end{aligned} \tag{30}$$

Hence, we obtained that this example supports Theorem 2.1. This means that $r = 0$ is a unique fixed point of f.

For some other functions $\eta : X \times X \to [0,+\infty)$ on can obtain the same conclusion.□

Example 3.3. *Let* $X = [0,\infty)$ *and* $b_\sigma(u,v) = u^2 + v^2 + |u-v|^2$ *for all* $u, v \in X$. *It is clear that* b_σ *is a b-metric-like on X with coefficient* $s = 2$ *and* (X, b_σ) *is* b_σ*−complete. Also,* b_σ *is not a metric-like or a b-metric (nor it is a metric on X). Define* $f : X \to X$ *and* $\eta : X \times X \to [0,\infty)$ *by* $fu = \frac{\ln(1+u)}{4}$ *and* $\eta(u,v) = u^2 + v^2 + 1$. *In addition define*

$\phi : [0, \infty) \to [0, \infty)$ by $\phi(t) = \frac{t}{2}$. Since the function f is η−admissible and $\eta(u, fu)\, \eta(v, fv) \geq 1$ for all $(u, v) \in X \times X$ then for all $u, v \in X$ with $q = 2$, we have

$$\begin{aligned}
2s^q b_\sigma(fu, fv) &= 2 \cdot 2^2(((fu)^2 + (fv)^2) + |fu - fv|^2) \\
&= 8\left(\left(\frac{\ln(1+u)}{4}\right)^2 + \left(\frac{\ln(1+v)}{4}\right)^2 + \left|\frac{\ln(1+u)}{4} - \frac{\ln(1+v)}{4}\right|^2\right) \\
&= 8\left(\left(\frac{\ln(1+u)}{4}\right)^2 + \left(\frac{\ln(1+v)}{4}\right)^2 + \frac{1}{16}\left|\ln(1+u) - \ln(1+v)\right|^2\right) \\
&= 8\left(\left(\frac{\ln(1+u)}{4}\right)^2 + \left(\frac{\ln(1+v)}{4}\right)^2 + \frac{1}{16}\left|\ln\left(\frac{1+u}{1+v}\right)\right|^2\right).
\end{aligned}$$

Suppose first that $u > v$. Then we have the following:

$$\begin{aligned}
&2s^q b_\sigma(fu, fv) = 8\left(\left(\frac{\ln(1+u)}{4}\right)^2 + \left(\frac{\ln(1+v)}{4}\right)^2 + \frac{1}{16}\left|\ln\left(1 + \frac{u-v}{1+v}\right)\right|^2\right) \leq 8\left(\frac{u^2}{16} + \frac{v^2}{16} + \frac{1}{16}|u - v|^2\right) \\
&= \frac{1}{2}\left(u^2 + v^2 + |u - v|^2\right) = \frac{1}{2} b_\sigma(u, v) \\
&\leq \frac{1}{2} \max\left\{b_\sigma(u, v), b_\sigma(v, fu), \frac{b_\sigma(u, v)\, b_\sigma(v, fv)}{1 + b_\sigma(u, fu)}, \frac{b_\sigma(v, fv)\,[1 + b_\sigma(u, fu)]}{1 + b_\sigma(u, v)}, \frac{b_\sigma(u, fv) + b_\sigma(v, fu)}{4 \cdot s}\right\} \\
&= \phi\left(\max\left\{b_\sigma(u, v), b_\sigma(v, fu), \frac{b_\sigma(u, v) \cdot b_\sigma(v, fv)}{1 + b_\sigma(u, fu)}, \frac{b_\sigma(v, fv) \cdot [1 + b_\sigma(u, fu)]}{1 + b_\sigma(u, v)}, \frac{b_\sigma(u, fv) + b_\sigma(v, fu)}{4 \cdot s}\right\}\right)
\end{aligned} \tag{31}$$

Therefore, we obtain that for $u > v$ the next condition is satisfied:

$$2s^q b_\sigma(fu, fv) \leq \phi(N(u, v)).$$

We also make an analogous conclusion when $u < v$.

Suppose now that $u = v$. Then

$$2s^q b_\sigma(fu, fv) = 16\left(\frac{\ln(1+u)}{4}\right)^2 \leq \frac{1}{2} \cdot 2u^2 = \frac{1}{2} b_\sigma(u, v) \leq \phi(N(u, v)).$$

Therefore, we obtain that

$$2s^q b_\sigma(fu, fv) \leq \phi(N(u, v)), \tag{32}$$

is satisfied for all $u, v \in X$ for which hold: $\eta(u, fu)\, \eta(v, fv) \geq 1$.

Hence, all conditions of our Theorem 2.1 are satisfied and therefore f has a unique fixed point $r = 0 \in X$. □

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