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Some Fixed Point Theorems via Asymptotic Regularity

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Abstract. In this article, we introduce some generalized contractive mappings over a metric space as extensions of various contractive mappings given by Kannan, Ćirić, Proinov and Górnicki. Some fixed point theorems have been proved for such new contractive type mappings via asymptotic regularity and some weaker versions of continuity. Supporting examples have been given in strengthening the hypothesis of our established theorems. As a by-product we explore some new answers to the open question posed by Rhoades.

1. Introduction and Preliminaries

In 1968, R. Kannan [9] proved a fixed point theorem for a mapping which was neither contraction nor contractive in nature. Also, a Kannan type contractive mapping may not always be continuous in the entire domain of definition.

Definition 1.1. *In a metric space (X, d), a mapping $T : X \to X$ is said to be*

(i) *Kannan type mapping [9] if there exists $A \in \left[0, \frac{1}{2}\right)$ such that for all $x, y \in X$*

$$d(Tx, Ty) \leq A \{d(x, Tx) + d(y, Ty)\}. \tag{1}$$

(ii) *Reich type mapping [17], [18] if there exist $a, b, c \in [0, 1)$ with $a + b + c < 1$ such that for all $x, y \in X$*

$$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty). \tag{2}$$

(iii) *Hardy-Rogers type mapping [8] if there exist $a, b, c, e, f \in [0, 1)$ with $a + b + c + e + f < 1$ such that for all $x, y \in X$*

$$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty) + ed(x, Ty) + fd(y, Tx). \tag{3}$$

Theorem 1.2. *In a complete metric space (X, d), a mapping $T : X \to X$ satisfying either Kannan or Reich or Hardy-Rogers type contractive condition, possesses a unique fixed point in X.*

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Recently in the year 2019, J. Górnicki [7] studied a new class of contractive mappings (see also [11, 16]) and proved a fixed point theorem for such mappings with assumption of continuity, which is as follows.

Theorem 1.3. *[7] In a complete metric space (X, d) a continuous and asymptotically regular mapping $T : X \to X$ satisfying*

$$d(Tx, Ty) \leq \alpha d(x, y) + K\{d(x, Tx) + d(y, Ty)\} \text{ for all } x, y \in X \tag{4}$$

for some $\alpha \in [0, 1)$ and for some $K \geq 0$ has a unique fixed point $u \in X$ and for each $x \in X$, $T^n x \to u$ as $n \to \infty$.

Now recall some basic definitions as follows.

Definition 1.4. *[1, 5] In a metric space (X, d), a mapping $T : X \to X$ is said to be asymptotically regular at $x \in X$, if $\lim_{n\to\infty} d(T^n x, T^{n+1} x) = 0$. If T is asymptotically regular at all $x \in X$, then T is said to be asymptotically regular.*

Definition 1.5. *For a self mapping T over a metric space (X, d) the set $O(x, T) := \{T^n x : n = 0, 1, 2, \cdots\}$, $x \in X$, is called an orbit of the mapping T.*

Definition 1.6. *[6] In a metric space (X, d), a mapping $T : X \to X$ is said to be orbitally continuous at a point $p \in X$ if for any sequence $\{x_n\} \subset O(x, T)$ (for some $x \in X$) $x_n \to p$ implies $Tx_n \to Tp$ as $n \to \infty$.*

Definition 1.7. *[12] In a metric space (X, d) a mapping $T : X \to X$ is called $k-$ continuous ($k = 1, 2, 3, \cdots$) if for some $p \in X$ and for any sequence $\{x_n\} \subset X$, $T^{k-1} x_n \to p$ implies $T^k x_n \to Tp$ as $n \to \infty$.*

Bisht [2] replaced the assumption of continuity in Theorem 1.3 by a weaker version of continuity condition, namely, orbital continuity or k-continuity.

In 1988, Rhoades [19] asked the question of the existence of a contractive condition which admits discontinuity at the fixed point as an existing open problem. Pant [14] resolved this problem in the setting of metric space. Several other solutions of this problem can be found in [2–4, 12, 13, 15, 20, 21].

In the following section we generalize the Górnicki type mapping (4) replacing the term $K\{d(x, Tx) + d(y, Ty)\}$ by an arbitrary function of $d(x, Tx)$ and $d(y, Ty)$, together with Reich, Hardy-Rogers and Ćirić type mappings and proved a fixed point theorem with the help of either orbitally continuity or k-continuity. We also provide some new answers to Rhoades open problem in the setting of metric space.

2. Main Result

We define an extended version of Kannan type contractive mappings, which is given below.

Definition 2.1. *(Kannan-Górnicki type mapping) In a metric space (X, d) a mapping $T : X \to X$ is said to be Kannan-Górnicki type contractive mapping if there exists some $\zeta \geq 0$ such that*

$$d(Tx, Ty) \leq \zeta\{d(x, Tx) + d(y, Ty)\} \text{ for all } x, y \in X. \tag{5}$$

Clearly any Kannan mapping is also Kannan-Górnicki type contractive mapping but one can find various Kannan-Górnicki type contractive mapping which are not Kannan contractive mapping.

There are several contraction mappings which are not Kannan mappings. See the following example.

Example 2.2. *Let $X = [0, 1]$ with usual metric, $T : X \to X$ be defined by $Tx = \frac{x}{2}$ for all $x \in X$. Then it can be easily checked that it is not usual Kannan contractive mapping.*

But any contraction mapping is also Kannan-Górnicki type contractive mapping. If (X, d) is a metric space and $T : X \to X$ is a contraction mapping with Lipschitz constant $\alpha \in [0, 1)$ then it can be easily verified that T is a Kannan-Górnicki type contractive mapping with Lipschitz constant $\frac{\alpha}{1-\alpha}$.

From Theorem 1.3 we can get the following obvious theorem.

Theorem 2.3. *In a complete metric space (X, d) a continuous and asymptotically regular Kannan-Górnicki type contractive mapping has a unique fixed point.*

It is known that if T is a Kannan mapping on a metric space (X, d) with constant $\xi \in [0, \frac{1}{2})$ then T^m is also a Kannan mapping with constant $\xi \left(\frac{\xi}{1-\xi}\right)^{m-1}$ for any positive integer $m \geq 2$. But in case of a Kannan-Górnicki contractive mapping it is not always true. See the following example.

Example 2.4. *Let $X = [\frac{1}{2}, 2]$ with discrete metric d_s defined by $d_s(x, y) = 0$ if $x = y$ and $d_s(x, y) = 1$ if $x \neq y$. Also let $T : X \to X$ be given by $Tx = \frac{1}{x}$ for all $x \in X$. Then T is a Kannan-Górnicki contractive mapping with Lipschitz constant $\zeta = 1$ but T^2 is the identity mapping which can not be Kannan-Górnicki contractive mapping for any $\zeta \geq 0$.*

From Theorem 1.2 we see that any Kannan mapping has a unique fixed point in a complete metric space. But there are Kannan-Górnicki contractive mappings which have no fixed point in a complete metric space. The following examples show this.

Example 2.5. *Let $X = \mathbb{R}$ with discrete metric d_s defined by $d_s(x, y) = 0$ if $x = y$ and $d_s(x, y) = 1$ if $x \neq y$.*

(i) Let $T : X \to X$ be given by $Tx = x + 1$ for all $x \in X$. Then T is a Kannan-Górnicki contractive mapping with Lipschitz constant $\zeta = \frac{1}{2}$. Clearly T has no fixed point in X. Actually here all the conditions of Theorem 2.3 are satisfied except for T is asymptotically regular.

(ii) Let $T : X \to X$ be given by $Tx = \sqrt{3}$ if $x \in \mathbb{Q}$ and $Tx = 1$ if $x \in \mathbb{R}\backslash\mathbb{Q}$. Then T is a Kannan-Górnicki contractive mapping with Lipschitz constant $\zeta = \frac{1}{2}$. It is clear that T has no fixed point in X. In this example also all the conditions of Theorem 2.3 are satisfied for T except for T is asymptotically regular.

Now we give some more general versions of Kannan contractive mappings and Górnicki contractive mappings (the contractive condition used in Theorem 1.3), see the following definitions.

First we define, the class $\mathfrak{F}$ of such functions $F : [0, \infty) \times [0, \infty) \to [0, \infty)$ satisfying the following conditions:

(i) $F(0, 0) = 0$

(ii) F is continuous at $(0, 0)$.

Definition 2.6. *In a metric space (X, d), a mapping $T : X \to X$ is said to be*

(i) *Ćirić-Proinov-Górnicki type mapping if there exists $\alpha \in [0, 1)$ such that*

$$d(Tx, Ty) \leq \alpha \max\{d(x, y), d(x, Ty), d(y, Tx)\} + F(d(x, Tx), d(y, Ty)) \tag{6}$$

(ii) *Hardy-Rogers-Proinov-Górnicki type mapping if there exist $\alpha, \beta, \gamma \in [0, 1)$ with $\alpha + \beta + \gamma < 1$ such that*

$$d(Tx, Ty) \leq \alpha d(x, y) + \beta d(x, Ty) + \gamma d(y, Tx) + F(d(x, Tx), d(y, Ty)) \tag{7}$$

(iii) *Reich-Proinov-Górnicki type mapping if there exist $\alpha \in [0, 1)$ such that*

$$d(Tx, Ty) \leq \alpha d(x, y) + F(d(x, Tx), d(y, Ty)) \tag{8}$$

for all $x, y \in X$ and for some $F \in \mathfrak{F}$.

We begin with the following result:

Theorem 2.7. *Let (X, d) be a complete metric space and $T : X \to X$ be asymptotically regular Ćirić-Proinov-Górnicki type mapping. Then T has a unique fixed point provided either T is k-continuous for $k \geq 1$ or T is orbitally continuous.*

Proof. Let $x_0 \in X$. Construct the iteration $x_{n+1} = Tx_n$ for all $n = 0, 1, 2, \cdots$. For $n, m \in \mathbb{N}, m > n$ we have,

$$\begin{aligned} d(x_n, x_m) &= d(T^n x_0, T^m x_0) \\ &\leq \alpha \max\left\{d(T^{n-1}x_0, T^{m-1}x_0), d(T^{n-1}x_0, T^m x_0), d(T^{m-1}x_0, T^n x_0)\right\} \\ &\qquad\qquad + F\left(d(T^{n-1}x_0, T^n x_0), d(T^{m-1}x_0, T^m x_0)\right) \\ &= \alpha C_{n,m} + F(a_n, b_m) \ \text{, (say)} \end{aligned}$$

where $C_{n,m} = \max\left\{d(T^{n-1}x_0, T^{m-1}x_0), d(T^{n-1}x_0, T^m x_0), d(T^{m-1}x_0, T^n x_0)\right\}$ and $a_n = d(T^{n-1}x_0, T^n x_0)$, $b_m = d(T^{m-1}x_0, T^m x_0)$.

Case 1: If $C_{n,m} = d(T^{n-1}x_0, T^{m-1}x_0)$ then,

$$\begin{aligned} d(x_n, x_m) &\leq \alpha d(T^{n-1}x_0, T^{m-1}x_0) + F(a_n, b_m) \\ &\leq \alpha\left[d(T^{n-1}x_0, T^n x_0) + d(T^n x_0, T^m x_0) + d(T^m x_0, T^{m-1}x_0)\right] + F(a_n, b_m), \end{aligned}$$

which implies

$$d(x_n, x_m) \leq \frac{\alpha}{1-\alpha}\left[d(T^{n-1}x_0, T^n x_0) + d(T^{m-1}x_0, T^m x_0)\right] + \frac{1}{1-\alpha}F(a_n, b_m). \tag{9}$$

Case 2: If $C_{n,m} = d(T^{n-1}x_0, T^m x_0)$ then,

$$\begin{aligned} d(x_n, x_m) &\leq d(T^{n-1}x_0, T^m x_0) + F(a_n, b_m) \\ &\leq \alpha\left[d(T^{n-1}x_0, T^n x_0) + d(T^n x_0, T^m x_0)\right] + F(a_n, b_m), \end{aligned}$$

implying that

$$d(x_n, x_m) \leq \frac{\alpha}{1-\alpha}d(T^{n-1}x_0, T^n x_0) + \frac{1}{1-\alpha}F(a_n, b_m). \tag{10}$$

Case 3: If $C_{n,m} = d(T^{m-1}x_0, T^n x_0)$ then,

$$\begin{aligned} d(x_n, x_m) &\leq \alpha d(T^{m-1}x_0, T^n x_0) + F(a_n, b_m) \\ &\leq \alpha\left[d(T^{m-1}x_0, T^m x_0) + d(T^m x_0, T^n x_0)\right] + F(a_n, b_m), \end{aligned}$$

which yields

$$d(x_n, x_m) \leq \frac{\alpha}{1-\alpha}d(T^{m-1}x_0, T^m x_0) + \frac{1}{1-\alpha}F(a_n, b_m). \tag{11}$$

So in any case from (9), (10) and (11) we can write

$$\begin{aligned} d(x_n, x_m) &\leq \frac{\alpha}{1-\alpha}\left[d(T^{n-1}x_0, T^n x_0) + d(T^{m-1}x_0, T^m x_0)\right] + \frac{1}{1-\alpha}F(a_n, b_m) \\ &= \frac{\alpha}{1-\alpha}(a_n + b_m) + \frac{1}{1-\alpha}F(a_n, b_m). \end{aligned} \tag{12}$$

Now since T is asymptotically regular, so $a_n \to 0$ and $b_m \to 0$ as $m > n \to \infty$. Then using the continuity of F, from (12) we have that $\{x_n\}$ is a Cauchy sequence in X. Since X is complete so $\{x_n\}$ is convergent and let $\lim_{n\to\infty} x_n = p \in X$.

Suppose T is k-continuous: Since $\lim_{n\to\infty} x_{n+1} = p$, so $\lim_{n\to\infty} Tx_n = p$. Moreover, for each $k \geq 1$ we have $\lim_{n\to\infty} T^k x_n = p$. Since $\lim_{n\to\infty} T^{k-1}x_n = p$ due to k-continuity of T we get $\lim_{n\to\infty} T^k x_n = Tp$. Thus $p = Tp$. *i.e.*, $p \in X$ is a fixed point of T.

Next suppose T is orbitally continuous: We have $\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} Tx_n = p$. Again from orbitally continuity $\lim_{n\to\infty} x_n = p$ implies $\lim_{n\to\infty} Tx_n = Tp$. Hence $p = Tp$. *i.e.*, $p \in X$ is a fixed point of T.

For uniqueness let us suppose that T has two fixed points $p \in X$ and $q \in X$. *i.e.*, $Tp = p$ and $Tq = q$. Then,

$$\begin{aligned} d(p,q) &= d(Tp,Tq) \\ &\leq \alpha \max\{d(p,q), d(p,Tq), d(q,Tp)\} + F(d(u,Tu), d(v,Tv)) \\ &= \alpha d(p,q) + F(0,0) \end{aligned}$$

Since $F(0,0) = 0$, we have $(1-\alpha)d(p,q) \leq 0$ which yields that $d(p,q) = 0$, *i.e.* $p = q$ and consequently fixed point is unique. □

Example 2.8. *Let $X = [0,2]$ equipped with the usual metric d. Define $T : X \to X$ by*

$$T(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ x-1 & \text{if } 1 < x \leq 2. \end{cases} \tag{13}$$

Then T satisfies all the conditions of Theorem 2.7 and has a unique fixed point $x = 1$.

Explanation: Take $F(x,y) = x^2 + y^2$. Then $F \in \mathfrak{F}$. Also take $\alpha = \frac{1}{2}$. Now we consider three cases.

Case 1: When $x, y \in [0,1]$ then $d(Tx,Ty) = 0$ and therefore the relation (6) holds trivially.

Case 2: Let $x, y \in (1,2]$ and let $\Gamma_{x,y} = \max\{d(x,y), d(x,Ty), d(y,Tx)\}$. We are not worried about the case $x = y$, because in that case $d(Tx,Ty) = 0$ and then we are done. So for $x \neq y$ we have, $d(Tx,Ty) \leq 1$ and $\alpha\Gamma_{x,y} + F(1,1) \geq 2$. Thus the relation (6) clearly holds whatever the value of $\Gamma_{x,y}$.

Case 3: Finally let $x \in [0,1]$ and $y \in (1,2]$. Then $F(d(x,Tx), d(y,Ty)) = 1 + (1-x)^2 > 1$ and $d(Tx,Ty) = |y-2| < 1$ for all $x \in [0,1]$ and $y \in (1,2]$.

Corollary 2.9. *In a complete metric space (X,d), an asymptotically regular Hardy-Rogers-Proinov-Górnicki type self mapping T has a unique fixed point provided either T is k-continuous for $k \geq 1$ or T is orbitally continuous.*

Proof. Since T is Hardy-Rogers-Proinov-Górnicki type mapping we have for any $x, y \in X$,

$$\begin{aligned} d(Tx,Ty) &\leq \alpha d(x,y) + \beta d(x,Ty) + \gamma d(y,Tx) + F(d(x,Tx), d(y,Ty)) \\ &\leq (\alpha+\beta+\gamma)\max\{d(x,y), d(x,Ty), d(y,Tx)\} + F(d(x,Tx), d(y,Ty)). \end{aligned}$$

Since $\alpha + \beta + \gamma < 1$ it follows that T satisfies contractive condition (6). Therefore the conclusion follows from the Theorem 2.7. □

Corollary 2.10. *In (7) if we take $\beta = 0 = \gamma$ and $F(x,y) = K(x+y)$ for some $K \geq 0$ then we can get the Górnicki type mapping (Contractive condition (4)) and thus Theorem 1.3 follows from Corollary 2.9.*

Contractive conditions (6), (7) and (8) do not always implies contractive condition (4). Some examples of nonlinear mappings are given below which prove our assertion.

Example 2.11. *Consider, $X = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\} \subset \mathbb{R}$ equipped with the usual metric on $\mathbb{R}$. Define a mapping $T : X \to X$ by $T(0) = 0$ and $T(\frac{1}{n}) = \frac{1}{n+1}$ for all $n \geq 1$.*
We claim that T does not satisfy the contractive condition (4). If it is, then by taking $x = \frac{1}{n}$ and $y = 0$ we have, $1 \leq \alpha\frac{n+1}{n} + \frac{M}{n}$, and for sufficiently large values of n, we can get $\alpha \geq 1$, arrives at a contradiction.
But here T satisfies all the three contractive conditions (6), (7) and (8) by taking $\alpha = \frac{1}{2}$ and $F(x,y) = \sqrt{x} + \sqrt{y}$ for all $x, y \in [0,\infty)$. Moreover all the conditions of the Theorem 2.7 are satisfied and $x = 0$ is the unique fixed point of T in X.

Example 2.12. *Consider the space $X = [0,\infty) \subset \mathbb{R}$ endowed with the usual metric on $\mathbb{R}$. Define a mapping $T : X \to X$ by $T(x) = \frac{x}{x^2+1}$ for all $x \in X$.*
First we claim that T does not satisfy the contractive condition (4). If it is, then by taking $x = 0$ and $y = \frac{1}{n}$ we have $1 \leq \alpha\frac{n^2+1}{n^2} + M.\frac{1}{n^2}$ and if we take n sufficiently large then we get $\alpha \geq 1$, which is a contradiction.
But here T satisfies all the three contractive conditions (6), (7) and (8) by taking $\alpha = \frac{1}{2}$ and $F(x,y) = \sqrt[3]{x} + \sqrt[3]{y}$ for all $x, y \in [0,\infty)$. Moreover all the conditions of the Theorem 2.7 are satisfied and it is clear that $x = 0$ is the unique fixed point of T in X.

In the next theorem, we assume asymptotic regularity of the mapping T at some point $x_0 \in X$ instead of for all $x \in X$ [10].

Theorem 2.13. *Let (X, d) be a complete metric space and $T : X \to X$. Suppose that there exists $x_0 \in X$ such that T is asymptotically regular at x_0 satisfying Ćirić-Proinov-Górnicki type mapping. Then T has a unique fixed point $p \in X$ and for each $x \in X$, $T^n x \to p$ as $n \to \infty$ for all $x \in O(x_0, T)$ provided either T is k-continuous for $k \geq 1$ or T is orbitally continuous.*

Proof. Let $x \in O(x_0, T)$. Since T is asymptotically regular at x_0, T is also asymptotically regular at x. The rest of the proof follows from Theorem 2.7. □

Here we give some examples of mappings in support of Theorem 2.13, each of which is asymptotically regular only at one point instead of everywhere in a metric space (X, d).

Example 2.14. *(i) Let $X = \{1, 2, 3\}$ equipped with the usual metric. Let $T : X \to X$ be defined by $T1 = 1$, $T2 = 3$ and $T3 = 2$. Then T is a Ćirić-Proinov-Górnicki type mapping for suitable choice of $\alpha \in (0, 1)$ and $F \in \mathfrak{F}$ and satisfies all the conditions of Theorem 2.13. Here it is to be noted that T is not asymptotically regular at 2 and 3 and 1 is the unique fixed point of T in X.*

(ii) Let $X = \mathbb{N} \cup \{0\}$ endowed with the usual metric. Let $T : X \to X$ be defined by

$$T(x) = \begin{cases} 0 & \text{if, } x = 0 \\ 2n & \text{if, } x = n \geq 1. \end{cases}$$

Then T is a Ćirić-Proinov-Górnicki type mapping for proper choice of $\alpha \in (0, 1)$ and $F \in \mathfrak{F}$ and satisfies all the conditions of Theorem 2.13. Clearly T is not asymptotically regular at any $n \geq 1$ and 0 is the unique fixed point of T in X.

(iii) Let $X = [0, \infty)$ endowed with the discrete metric d_s given by $d_s(x, y) = 0$ if $x = y$ and $d_s(x, y) = 1$ if $x \neq y$. Let $T : X \to X$ be defined by

$$T(x) = \begin{cases} 0 & \text{if, } x = 0 \\ x + 1 & \text{if, } x \neq 0. \end{cases}$$

Then it can be easily checked that T is a Ćirić-Proinov-Górnicki type mapping and also satisfies all the conditions of Theorem 2.13. Clearly T is not asymptotically regular at any $x \neq 0$ and 0 is the unique fixed point of T in X.

Corollary 2.15. *Let (X, d) be a complete metric space and $T : X \to X$. Suppose that there exists $x_0 \in X$ such that T is asymptotically regular at x_0 satisfying Hardy-Rogers-Proinov-Górnicki type mapping. Then T has a unique fixed point $p \in X$ and for each $x \in X$, $T^n x \to p$ as $n \to \infty$ for all $x \in O(x_0, T)$ provided either T is k-continuous for $k \geq 1$ or T is orbitally continuous.*

Corollary 2.16. *Let (X, d) be a complete metric space and $T : X \to X$. Suppose that there exists $x_0 \in X$ such that T is asymptotically regular at x_0 satisfying Reich-Proinov-Górnicki type mapping. Then T has a unique fixed point $p \in X$ and for each $x \in X$, $T^n x \to p$ as $n \to \infty$ for all $x \in O(x_0, T)$ provided either T is k-continuous for $k \geq 1$ or T is orbitally continuous.*

Remark 2.17. *Above proved theorems provide some new answers to the once open question (see Rhoades [19], p.242) on the existence of contractive mappings which admit discontinuity at the fixed point.*

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