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Hypersurfaces of trans-S-manifolds

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Abstract. We study hypersurfaces isometrically immersed in a trans-S-manifolds in order to find out under what conditions they could inherit the structure of the ambient manifold and so, to obtain new examples of such trans-S-manifolds. Mainly, we investigate this situation depending the behaviour of the second fundamental form of the immersion.

1. Introduction

Riemannian manifolds with a complementary structure adapted to the metric have been widely studied, for instance, almost complex and almost contact manifolds. More in general, K. Yano [12] introduced the notion of f-structure on a $(2n+s)$-dimensional manifold as a tensor field f of type (1,1) and rank $2n$ satisfying $f^3 + f = 0$. Almost complex ($s = 0$) and almost contact ($s = 1$) structures are well-known examples of f-structures. Riemannian manifolds endowed with an f-structure compatible with the metric satisfying certain additional conditions are called metric f-manifolds. In this context, almost-Hermitian manifolds ($s = 0$) and almost contact metric manifolds ($s = 1$) are metric f-manifolds.

For metric f-manifolds, D.E. Blair [3] defined K-manifolds (and particular cases of S-manifolds and C-manifolds). Then, K-manifolds are the analogue of Kaehlerian manifolds in the almost complex geometry and S-manifolds (resp., C-manifolds) of Sasakian manifolds (resp., cosymplectic manifolds) in the almost contact geometry.

Recently, P. Alegre, L.M. Fernández and A. Prieto-Martín [1] have introduced a new class of metric f-manifolds called trans-S-manifolds because, when $s = 1$ they actually are trans-Sasakian manifolds (see [10]). This class contains many types of metric f-manifolds studied in the literature: S-manifolds and C-manifolds of D.E. Blair, homothetic s-th Sasakian manifolds of I. Hasegawa, Y. Okuyama and T. Abe [7] f-manifolds of Kenmotsu type introduced by M. Falcitelly and A.M. Pastore [5] and generalized Kenmotsu manifolds of L. Bhatt and K.K. Dube [2] and A. Turgut Vanli and R. Sari [11]. In [1] more examples of trans-S-manifolds are given by using generalized D-conformal deformations and warped products as tools.

The purpose of this paper is to study if an oriented and isometrically immersed hypersurface in a trans-S-manifold inherits the property of being trans-S. In general, the answer is negative. Thus, we consider the particular case of hypersurfaces tangent to all the structure vector fields and we investigate the situation depending the behaviour of the second fundamental form of the immersion, that is, if the hypersurface is totally geodesic, totally umbilical, totally f-geodesic, totally f-umbilical or pseudo-umbilical. Finally, we study the case of hypersurfaces normal to one of the structure vector fields.

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2. Trans-S-manifolds

A $(2n+s)$-dimensional Riemannian manifold (M, g) endowed with an f-structure f (that is, a tensor field of type (1,1) and rank $2n$ satisfying $f^3 + f = 0$ [12]) is said to be a *metric f-manifold* if, moreover, there exist s global vector fields $\xi_1, \dots, \xi_s$ on M (called *structure vector fields*) such that, if $\eta_1, \dots, \eta_s$ are the dual 1-forms of $\xi_1, \dots, \xi_s$, then

$$f\xi_i = 0;\ \eta_i \circ f = 0;\ f^2 = -I + \sum_{i=1}^{s} \eta_i \otimes \xi_i;$$

$$g(X, Y) = g(fX, fY) + \sum_{i=1}^{s} \eta_i(X)\eta_i(Y), \tag{1}$$

for any $X, Y \in \mathcal{X}(M)$ and $i = 1, \dots, s$. The distribution on M spanned by the structure vector fields is denoted by $\mathcal{M}$ and its complementary orthogonal distribution is denoted by $\mathcal{L}$. Consequently, $TM = \mathcal{L} \oplus \mathcal{M}$. Moreover, if $X \in \mathcal{L}$, then $\eta_i(X) = 0$, for any $i = 1, \dots, s$ and if $X \in \mathcal{M}$, then $fX = 0$.

Let F be the 2-form on M defined by $F(X, Y) = g(X, fY)$, for any $X, Y \in \mathcal{X}(M)$. Since f is of rank $2n$, then

$$\eta_1 \wedge \cdots \wedge \eta_s \wedge F^n \neq 0$$

and, particularly, M is orientable. A metric f-manifold is said to be a *metric f-contact manifold* if $F = \mathrm{d}\eta_i$, for any $i = 1, \dots, s$.

The f-structure f is said to be *normal* if

$$[f, f] + 2\sum_{i=1}^{s} \xi_i \otimes \mathrm{d}\eta_i = 0,$$

where $[f, f]$ denotes the Nijenhuis tensor of f. If f is normal, then [6]

$$[\xi_i, \xi_j] = 0, \tag{2}$$

for any $i, j = 1, \dots, s$.

A metric f-manifold is said to be a K-manifold [3] if it is normal and $\mathrm{d}F = 0$. In a K-manifold M, the structure vector fields are Killing vector fields [3]. A K-manifold is called an S-manifold if $F = \mathrm{d}\eta_i$, for any i and a C-manifold if $\mathrm{d}\eta_i = 0$, for any i. Note that, for $s = 0$, a K-manifold is a Kaehlerian manifold and, for $s = 1$, a K-manifold is a quasi-Sasakian manifold, an S-manifold is a Sasakian manifold and a C-manifold is a cosymplectic manifold. When $s \geq 2$, non-trivial examples can be found in [3, 7]. Moreover, a K-manifold M is an S-manifold if and only if

$$\nabla_X \xi_i = -fX,\ X \in \mathcal{X}(M),\ i = 1, \dots, s,$$

where ∇ denotes the Riemannian connection associated with g and it is a C-manifold if and only if:

$$\nabla_X \xi_i = 0,\ X \in \mathcal{X}(M),\ i = 1, \dots, s.$$

It is easy to show that in an S-manifold,

$$(\nabla_X f)Y = \sum_{i=1}^{s} \left\{ g(fX, fY)\xi_i + \eta_i(Y) f^2 X \right\}, \tag{3}$$

for any $X, Y \in \mathcal{X}(M)$ and in a C-manifold:

$$\nabla f = 0. \tag{4}$$

A $(2n+s)$-dimensional metric f-manifold M is said to be an *almost trans-S-manifold* if it satisfies

$$(\nabla_X f)Y = \sum_{i=1}^{s} \Big[\alpha_i\{g(fX,fY)\xi_i + \eta_i(Y)f^2X\} + \beta_i\{g(fX,Y)\xi_i - \eta_i(Y)fX\}\Big], \tag{5}$$

for certain smooth functions α_i, β_i, $i = 1....s$, on M and any $X, Y \in \mathcal{X}(M)$. If, moreover, M is normal, then it is said to be a *trans-S-manifold*.

So, if $s = 1$, a trans-S-manifold is actually a trans-Sasakian manifold.

Observe that condition (5) does not imply normality. In fact, in [1] it is proved that an almost trans-S-manifold M is a trans S-manifold if and only if

$$\nabla_X \xi_i = -\alpha_i fX - \beta_i f^2X, \tag{6}$$

for any $X \in \mathcal{X}(M)$ and any $i = 1, \ldots, s$.

3. Hypersurfaces of trans-S-manifolds.

Though all this section, let $(\widetilde{M}, \widetilde{f}, \widetilde{\xi}_1, \ldots, \widetilde{\xi}_s, \widetilde{\eta}_1, \ldots, \widetilde{\eta}_s, g)$ be a metric f-manifold and let M be an oriented and isometrically immersed hypersurface in $\widetilde{M}$ such that N denotes the unit normal vector field of M in $\widetilde{M}$ and the structure vector fields $\widetilde{\xi}_1, \ldots, \widetilde{\xi}_{s-1}$ are tangent to M. Put $\widetilde{\xi}_s = \lambda\xi_s + \mu N$, where ξ_s is a unit tangent vector field to M. Then,

$$\lambda = \widetilde{\eta}_s(\xi_s);\ \mu = \widetilde{\eta}_s(N);\ \lambda^2 + \mu^2 = 1. \tag{7}$$

Firstly, we are going to suppose that $\lambda \neq 0$. Then,

$$\xi_{s+1} = -\frac{1}{\lambda}\widetilde{f}N$$

is a unit vector field tangent to M such that $\widetilde{f}\xi_s = \mu\xi_{s+1}$. Now, we define $s-1$ vector fields on M by $\xi_{i_x} = \widetilde{\xi}_{i_x}$, for any $x \in M$ and

$$\begin{aligned} &\eta_i(X) = \widetilde{\eta}_i(X),\ i = 1, \ldots, s-1, \\ &\eta_s(X) = -\frac{1}{\lambda}\widetilde{\eta}_s(X),\ \eta_{s+1}(X) = g(X, \xi_{s+1}), \\ &fX = \widetilde{f}X - \lambda\eta_{s+1}(X)N + \mu\eta_{s+1}(X)\xi_s - \mu\eta_s(X)\xi_{s+1}, \end{aligned} \tag{8}$$

for any $X \in \mathcal{X}(M)$. Then, it is straightforward to prove that

$$(M, f, \xi_1, \ldots, \xi_{s+1}, \eta_1, \ldots, \eta_{s+1}, g)$$

is also a metric f-manifold with $rank(f) = rank(\widetilde{f}) - 2$. Moreover, given $X, Y \in \mathcal{X}(M)$, we have that

$$\widetilde{f}^2X = f^2X - \eta_{s+1}(X)\xi_{s+1} - \mu^2\eta_s(X)\xi_s + \lambda\mu\eta_s(X)N \tag{9}$$

and:

$$g(\widetilde{f}X, \widetilde{f}Y) = g(fX, fY) + \mu^2\eta_s(X)\eta_s(Y) + \eta_{s+1}(X)\eta_{s+1}(Y). \tag{10}$$

For M, the Gauss-Weingarten formulas are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y);\ \widetilde{\nabla}_X N = -AX, \tag{11}$$

for any $X, Y \in \mathcal{X}(M)$, where now $\widetilde{\nabla}$ (resp., ∇) denotes the Riemannian connection of $\widetilde{M}$ (resp., M) and σ and A are the second fundamental form of the immersion and the shape operator associated with N, respectively, related by $\sigma(X, Y) = g(AX, Y)N$.

By using (8) and (11), we compute that, given $X, Y \in \mathcal{X}(M)$:

$$\begin{aligned}(\widetilde{\nabla}_X \widetilde{f})Y =&(\nabla_X f)Y - \lambda\eta_{s+1}(Y)AX\\ &- \mu\eta_{s+1}(Y)\nabla_X\xi_s + \mu\eta_s(Y)\nabla_X\xi_{s+1}\\ &- \{X(\mu\eta_{s+1}(Y)) + \mu\eta_{s+1}(\nabla_X Y)\}\xi_s\\ &+ \{X(\mu\eta_s(Y)) + \mu\eta_s(\nabla_X Y) + \lambda g(AX, Y)\}\xi_{s+1}\\ &+ \{X(\lambda\eta_{s+1}(Y)) + \lambda\eta_{s+1}(\nabla_X Y)\}N + \sigma(X, fY)\\ &- \mu\eta_{s+1}(Y)\sigma(X, \xi_s) + \mu\eta_s(Y)\sigma(X, \xi_{s+1}).\end{aligned} \tag{12}$$

Now, suppose that $\widetilde{M}$ is an almost trans-S-manifold with characteristic functions $(\widetilde{\alpha}_1, \ldots, \widetilde{\alpha}_s, \widetilde{\beta}_1, \ldots, \widetilde{\beta}_s)$. If $\mu \neq 0$, we observe that M can not be an almost trans-S-manifold too. Thus, we are going to consider that $\mu = 0$, that is, that $\lambda = 1$ and all the structure vector fields are tangent to the hypersurface. Moreover, we denote by α_i and β_i the restrictions of the characteristic functions to M. Then, from (12) and taking into account (8), (9) and (10), we deduce

$$\begin{aligned}(\nabla_X f)Y = \sum_{i=1}^{s}&\{\alpha_i g(fX, fY)\xi_i + \eta_{s+1}(X)\eta_{s+1}(Y)\xi_i\\ &+ \eta_i(Y)f^2X - \eta_{s+1}(X)\eta_i(Y)\xi_{s+1})\\ &+ \beta_i(g(fX, Y)\xi_i - \eta_i(Y)fX)\}\\ &+ \eta_{s+1}(Y)AX - g(AX, Y)\xi_{s+1},\end{aligned} \tag{13}$$

for any $X, Y \in \mathcal{X}(M)$ and consequently, M is neither, in general, an almost trans-S-manifold.

Regarding the normality condition (6), from (8) and (11) we obtain, for any $X \in \mathcal{X}(M)$,

$$\begin{aligned}\nabla_X\xi_i &= -\alpha_i fX + \beta_i(\eta_{s+1}(X)\xi_{s+1} - f^2X),\\ \eta_i(AX) &= -\alpha_i\eta_{s+1}(X),\end{aligned} \tag{14}$$

for any $i = 1, \ldots, s$ and, by using (5):

$$\nabla_X\xi_{s+1} = fAX - \sum_{i=1}^{s}\beta_i\eta_{s+1}(X)\xi_i. \tag{15}$$

Therefore, it seems interesting to study if M can be an almost trans-S-manifold depending on the behaviour of the shape operator. Firstly, we consider that the hypersurface M is totally geodesic in $\widetilde{M}$, that is, $A \equiv 0$. Then, from (13), we can prove the following theorem.

Theorem 3.1. *Let M be a totally geodesic hypersurface tangent to the structure vector fields of an almost trans-S-manifold $\widetilde{M}$, with characteristic functions $\widetilde{\alpha}_i$, $\widetilde{\beta}_i$, $i = 1, \ldots, s$. Then, M is an almost trans-S-manifold if and only if $\widetilde{\alpha}_i = 0$, for any i and, in such a case, its characteristic functions are $\alpha_i = 0$, β_i, for $i = 1, \ldots, s$ and $\alpha_{s+1} = \beta_{s+1} = 0$.*

Now, if $\widetilde{M}$ is a trans-S-manifold, from (14) and (15) we deduce:

Corollary 3.2. *A totally geodesic hypersurface M tangent to the structure vector fields of a trans-S-manifold $\widetilde{M}$ is a trans-S-manifold if and only if $\widetilde{M}$ is a C-manifold. In this case, M is also a C-manifold.*

If M is totally umbilical, that is, if $A = hI$, being h a differentiable function, a direct expansion of (13) shows that it is not an almost trans-S-manifold. Therefore, it seems necessary to use a variation of these concepts concerning the shape operator, more related to the structure. In this context, following the ideas introduced by Ornea [9] for S-manifolds, we say that an hypersurface M of a (almost) trans-S-manifold $\widetilde{M}$ is *totally f-geodesic* (resp., *totally f-umbilical*) if the distribution $\mathcal{L} = \{X \in \mathcal{X}(M)/\eta_i(X) = 0, i = 1, \dots, s\}$ is totally geodesic (resp., totally umbilical), that is, if $\sigma(X,Y) = 0$ (resp., $\sigma(X,Y) = g(X,Y)V$, being

$$V = \left(1 + \frac{s}{2n-1}\right)H),$$

where H denotes the mean curvature vector field, for any $X, Y \in \mathcal{L}$.

In other words, since for any $X, Y \in \mathcal{X}(M)$ we have that $\widetilde{f}^2X, \widetilde{f}^2Y \in \mathcal{L}$, by using (1) the hypersurface is totally f-geodesic if and only if

$$\begin{aligned}\sigma(X,Y) = &\sum_{i=1}^{s}(\eta_i(X)\sigma(Y,\xi_i) + \eta_i(Y)\sigma(X,\xi_i))\\ &- \sum_{i,j=1}^{s}\eta_i(X)\eta_j(Y)\sigma(\xi_i,\xi_j)\end{aligned} \tag{16}$$

and, by using (8), it is totally f-umbilical if and only if

$$\begin{aligned}\sigma(X,Y) = &\sum_{i=1}^{s}(\eta_i(X)\sigma(Y,\xi_i) + \eta_i(Y)\sigma(X,\xi_i))\\ &- \sum_{i,j=1}^{s}\eta_i(X)\eta_j(Y)\sigma(\xi_i,\xi_j) + g(\widetilde{f}X,\widetilde{f}Y)V\\ = &\sum_{i=1}^{s}(\eta_i(X)\sigma(Y,\xi_i) + \eta_i(Y)\sigma(X,\xi_i))\\ &- \sum_{i,j=1}^{s}\eta_i(X)\eta_j(Y)\sigma(\xi_i,\xi_j)\\ &+ (g(fX,fY) + \eta_{s+1}(X)\eta_{s+1}(Y))V.\end{aligned} \tag{17}$$

Then, a totally f-umbilical submanifold is totally f-geodesic if and only if it is minimal. On the other hand, it is easy to show that any totally f-geodesic submanifold is minimal and totally f-umbilical.

Now, suppose that $\widetilde{M}$ is a trans-S-manifold. From (6) and the second equation of (14), we get that

$$\sigma(X,\xi_i) = -\alpha_i\eta_{s+1}(X)N,$$

for any $X \in \mathcal{X}(M)$ and $i = 1, \dots, s$. Consequently, $\sigma(\xi_i, \xi_j) = 0$, for any i, j and formulas (16) and (17) become to

$$AX = -\sum_{i=1}^{s}\alpha_i(\eta_i(X)\xi_{s+1} + \eta_{s+1}(X)\xi_i) \tag{18}$$

and

$$AX = -\sum_{i=1}^{s}\alpha_i(\eta_i(X)\xi_{s+1} + \eta_{s+1}(X)\xi) - h\widetilde{f}^2X, \tag{19}$$

respectively, for any $x \in \mathcal{X}(M)$, where $h = g(V,N)$.

Therefore, for totally f-geodesic hypersurfaces of a trans-S-manifold, we have:

Theorem 3.3. *Let M be a totally f-geodesic hypersurface tangent to the structure vector fields of a trans-S-manifold $\widetilde{M}$ with characteristic functions $\widetilde{\alpha}_i$, $\widetilde{\beta}_i$, $i = 1,\dots,s$. Then, M is an almost trans-S-manifold with characteristic functions α_i, β_i, for $i = 1,\dots,s$ and $\alpha_{s+1} = \beta_{s+1} = 0$. Moreover, M is a trans-S-manifold if and only if $\beta_i = 0$, for any $i = 1,\dots,s$.*

Proof. From (13), a direct expansion using (18) gives that M is an almost trans-S-manifold with such characteristic functions. Now, from (14) and (15) we conclude the proof. □

For totally f-umbilical hypersurfaces of a trans-S-manifold, by using a similar reasoning from (19), we can prove:

Theorem 3.4. *Let M be a totally f-umbilical hypersurface tangent to the structure vector fields of a trans-S-manifold $\widetilde{M}$ whit characteristic functions $\widetilde{\alpha}_i$, $\widetilde{\beta}_i$, $i = 1,\dots,s$. Then, M is an almost trans-S-manifold with characteristic functions α_i, β_i, for $i = 1,\dots,s$ and $\alpha_{s+1} = -h$, $\beta_{s+1} = 0$. Moreover, M is a trans-S-manifold if and only if $\beta_i = 0$, for any $i = 1,\dots,s$.*

Next, let $\widetilde{M}$ an S-manifold, that is, a trans-S-manifold with characteristic functions $\widetilde{\alpha}_i = 1$ and $\widetilde{\beta}_i = 0$, for $i = 1,\dots,s$. An hypersurface M tangent to the structure vector fields of $\widetilde{M}$ is said to be *pseudo-umbilical* [4] if its shape operator satisfies

$$AX = -h_1\widetilde{f}^2X + h_2\eta_{s+1}(X)\xi_{s+1} - \sum_{i=1}^{s}(\eta_i(X)\xi_{s+1} + \eta_{s+1}(X)\xi_i), \tag{20}$$

for any $X \in \mathcal{X}(M)$, where h_1 and h_2 are differentiable functions on M. Pseudo-umbilical hypersurfaces of S-manifolds correspond to η-umbilical real hypesurfaces of Kaehlerian manifolds [8] (for more details and examples, the mentioned paper [4] can be consulted). Then, by using (13), (20) and the properties of the structures, we have the following theorem.

Theorem 3.5. *Any pseudo-umbilical hypersurface of an S-manifold is a trans-S-manifold with characteristic functions $\alpha_i = 1$, $\beta_i = 0$, for $i = 1,\dots,s$ and $\alpha_{s+1} = -h_1$, $\beta_{s+1} = 0$.*

Now, we are going to consider the case $\lambda = 0$ and so, $\mu = 1$. Consequently, the structure vector field ξ_s is normal to the hypersurface. In this context, we define $s-1$ vector fields on M by $\xi_{i_x} = \widetilde{\xi}_{i_x}$, for any $x \in M$ and

$$\eta_i(X) = \widetilde{\eta}_i(X), \ fX = \widetilde{f}X. \tag{21}$$

for any i=1,. . . , s and $X \in \mathcal{X}(M)$. Then,

$$(M, f, \xi_1, \dots, \xi_{s-1}, \eta_1, \dots, \eta_{s-1}, g)$$

is also a metric f-manifold with $rank(f) = rank(\widetilde{f})$. Moreover, given $X, Y \in \mathcal{X}(M)$, we have that

$$\widetilde{f}^2X = f^2X \text{ and } g(\widetilde{f}X, \widetilde{f}Y) = g(fX, fY). \tag{22}$$

It is necessary to point out that any hypersurface of $\widetilde{M}$ normal to the structure vector field $\widetilde{\xi}_s$ is an invariant submanifold because $\widetilde{\eta}_s(fX) = 0$, for any $X \in \mathcal{X}(M)$. On the other hand, if $d\widetilde{\eta}_s = F$ (for instance, if $\widetilde{M}$ is a metric f-contact manifold or, in particular, an S-manifold) it can be proved that an hypersurface M normal to ξ_s should be an anti-invariant submanifold. So, in such a case, there are no hypersurfaces satisfying the required condition.

By using the same notations as above. we can prove:

Theorem 3.6. *Let M be an hypersurface normal to the structure vector field ξ_s of a (almost) trans-S-manifold with characteristic functions $\widetilde{\alpha}_1,\dots,\widetilde{\alpha}_s,\widetilde{\beta}_1,\dots,\widetilde{\beta}_s$. Then, M is a (almost) trans-S-manifold with characteristic functions $\alpha_1,\dots,\alpha_{s-1},\beta_1,\dots,\beta_{s-1}$.*

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