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On the Schwarzschild-de Sitter metric of nonlocal de Sitter gravity

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Abstract. Earlier constructed a simple nonlocal de Sitter gravity model has a cosmological solution in a very good agreement with astronomical observations. In this paper, we continue the investigation of the nonlocal de Sitter model of gravity, focusing on finding an appropriate solution for the Schwarzschild-de Sitter metric. We succeeded to solve the equations of motion in a certain approximation. The obtained approximate solution is of particular interest for examining the possible role of non-local de Sitter gravity in describing the effects in galactic dynamics that are usually attributed to dark matter.

1. Introduction

For more than 100 years, General Relativity (GR) has been considered as one of the most beautiful and successful physical theories – from a phenomenological point of view, several remarkable predictions have been confirmed [1]. The Standard Model of Cosmology (SMC) assumes that GR is valid and applicable not only in the Solar system but also at the galactic and cosmological scale. However, GR has not been verified at the galactic and cosmological scales without the use of Dark Matter (DM) and Dark Energy (DE). Furthermore, the SMC assumes that the universe contains about 68 % of DE, 27 % of DM and only 5 % of ordinary matter. Also, GR solutions for the black holes as well as for the beginning of the universe contain singularity and it means that GR should be modified in the vicinity of these singularities. It should be also mentioned that GR is nonrenormalizable theory from the quantum point of view. Hence, in spite of significant successes of GR, it is reasonable to doubt in its validity in description and understanding of all astrophysical and cosmological gravitational phenomena. Keeping all this in mind, it follows that general relativity is not a final theory of gravitation and that its expansion should be considered.

Since, there is still no physical principle that could suggest us in which direction we should search for an extension of GR, there are many approaches to its modification, see [2–7] as some reviews. Despite many attempts, there is not yet generally accepted modification of general relativity. One of the current and attractive approaches is nonlocal modified gravity, see, e.g. [8–12]. In an analytic nonlocal modification, the Einstein-Hilbert action is extended by a term that contains all higher order degrees of d'Alembert-Beltrami operator $\Box = \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$. Usually, $\Box$ is used in the form of an analytic expression

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$F(\Box) = \sum_{n=0}^{+\infty} f_n \Box^n$, see [13–17, 31]. Recently, we used nonlocal operator in the form $F(\Box) = 1 + \mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$ [34]. Some other typical nonlocal models can be seen in [18–21].

So far we have considered the nonlocal de Sitter models with analytic nonlocality based on the action given by

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\Big(R - 2\Lambda + P(R)\,\mathcal{F}(\Box)\,Q(R)\Big), \tag{1}$$

where Λ is the cosmological constant, $P(R)$ and $Q(R)$ are some differentiable functions of the Ricci scalar R, see [22–33] and references therein. The case $P(R) = Q(R) = R$ and $\Lambda = 0$ has attracted a lot of attention, e.g. nonlocal R^2 gravity, see [8, 13] and references therein. In [31] a very special case $P(R) = Q(R) = \sqrt{R - 2\Lambda}$ was considered:

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\Big(R - 2\Lambda + \sqrt{R - 2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R - 2\Lambda}\Big), \tag{2}$$

where $\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} f_n \Box^n$. Importance of this model is not only in its simple form but also in its cosmological solutions in flat space-time: (i) $a_1(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$ and (ii) $a_2(t) = Ae^{\frac{\Lambda}{6}t^2}$. Solution $a_1(t)$ mimics interplay of dark matter and dark energy in a good agreement with cosmological observations [31]. Solution $a_2(t)$ is the nonsingular bounce one. Nonlocal gravity model (2) also contains several other vacuum cosmological solutions in flat, closed and open space, not only with $\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} f_n \Box^n$ but also when $\mathcal{F}(\Box) = \sum_{n=1}^{+\infty}\Big(f_n \Box^n + f_{-n}\Box^{-n}\Big)$, see [34]. After very successful application of model (2) with respect to dark energy and dark matter at the cosmological scale, it is natural to be interested how this model works at smaller scales.

In this paper we investigate model (2) with static metric around spherically symmetric body (stellar, galactic). In other words, we are interested in the Schwarzschild-type metric in nonlocal de Sitter gravity given by (2). The corresponding nonlocal operator has the form

$$\mathcal{F}(\Box) = \sum_{n=1}^{+\infty}\Big(f_n \Box^n + f_{-n}\Box^{-n}\Big). \tag{3}$$

Here we want to find metric solution around a massive, non-rotating, without charge, and spherically symmetric object. It is known that in GR with the cosmological constant Λ (de Sitter gravity) such solution is related to the Schwarzschild-de Sitter metric,

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + A(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{4}$$

where $A(r) = 1 - \frac{2MG}{r} - \frac{\Lambda r^2}{3}$ (the speed of light in the vacuum is taken $c = 1$).

Structure of the paper is as follows. Section 2 contains an introduction to our very simple nonlocal de Sitter gravity model, $P(R) = Q(R) = \sqrt{R - 2\Lambda}$. In Section 3, we look for the Schwarzschild-type metric and we find a solution using the corresponding eigenvalue problem. In Section 4, we discuss obtained solution of this nonlocal de Sitter gravity and give some concluding remarks with our plan for future investigation. There is also an appendix which contains derivation of some useful formulas.

2. On nonlocal de Sitter gravity model

Let us mention some facts of our nonlocal gravity model (see [34]) which is given by the action

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\,\sqrt{R - 2\Lambda}\,F(\Box)\,\sqrt{R - 2\Lambda}, \tag{5}$$

where $F(\Box)$ is the following formal expansion in terms of the d'Alemberian $\Box$:

$$F(\Box) = 1 + \mathcal{F}(\Box) = 1 + \mathcal{F}_{+}(\Box) + \mathcal{F}_{-}(\Box), \quad \mathcal{F}_{+}(\Box) = \sum_{n=1}^{+\infty} f_n \, \Box^n, \; \mathcal{F}_{-}(\Box) = \sum_{n=1}^{+\infty} f_{-n} \, \Box^{-n}. \tag{6}$$

When $F(\Box) = 1$, i.e. $\mathcal{F}(\Box) = 0$, then model (5) becomes local and coincides with Einstein-Hilbert action with cosmological constant Λ:

$$S_0 = \frac{1}{16\pi G} \int d^4x \; \sqrt{-g} \; \sqrt{R - 2\Lambda} \; \sqrt{R - 2\Lambda} = \frac{1}{16\pi G} \int d^4x \; \sqrt{-g} \, (R - 2\Lambda). \tag{7}$$

It is worth pointing out that action (5) can be obtained in a very simple and natural way from action (7) by embedding nonlocal operator (6) in symmetric product form of $R - 2\Lambda$, that is $\sqrt{R - 2\Lambda} \; \sqrt{R - 2\Lambda}$. Action (5) does not contain matter term and this is intentionally done to better see possible role of this nonlocal model in effects usually assigned to dark matter and dark energy.

The equations of motion (EoM) for model (1), for $P(R) = Q(R)$ are given by (for more detail, see [30, 34]):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} P(R)\mathcal{F}(\Box)P(R) + R_{\mu\nu}W - K_{\mu\nu}W + \frac{1}{2}\Omega_{\mu\nu} = 0, \tag{8}$$

$$W = 2P'(R) \, \mathcal{F}(\Box) \, P(R), \quad K_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu}\Box, \tag{9}$$

$$\Omega_{\mu\nu} = \sum_{n=1}^{+\infty} f_n \sum_{\ell=0}^{n-1} S_{\mu\nu}(\Box^{\ell} P, \Box^{n-1-\ell} P) - \sum_{n=1}^{+\infty} f_{-n} \sum_{\ell=0}^{n-1} S_{\mu\nu}(\Box^{-(\ell+1)} P, \Box^{-(n-\ell)} P). \tag{10}$$

If $P(R)$ is an eigenfunction of the corresponding d'Alembert-Beltrami operator $\Box$, and consequently also of its inverse $\Box^{-1}$, i.e. holds

$$\Box P(R) = qP(R), \qquad \Box^{-1} P(R) = q^{-1} P(R), \qquad \mathcal{F}(\Box)P(R) = \mathcal{F}(q)P(R), \qquad q \neq 0, \tag{11}$$

then

$$W = 2\mathcal{F}(q)P'P, \qquad \mathcal{F}(q) = \sum_{n=1}^{+\infty} f_n \, q^n + \sum_{n=1}^{+\infty} f_{-n} \, q^{-n}, \tag{12}$$

$$S_{\mu\nu}(\Box^{\ell} P, \Box^{(n-1-\ell)} P) = q^{n-1} S_{\mu\nu}(P, P), \tag{13}$$

$$S_{\mu\nu}(\Box^{-(\ell+1)} P, \Box^{-(n-\ell)} P) = q^{-n-1} S_{\mu\nu}(P, P), \tag{14}$$

$$S_{\mu\nu}(P, P) = g_{\mu\nu}\Big(\nabla^\alpha P \, \nabla_\alpha P + P\Box P\Big) - 2\nabla_\mu P \, \nabla_\nu P, \tag{15}$$

$$\Omega_{\mu\nu} = \mathcal{F}'(q) S_{\mu\nu}(P, P), \qquad \mathcal{F}'(q) = \sum_{n=1}^{+\infty} n \, f_n \, q^{n-1} - \sum_{n=1}^{+\infty} n \, f_{-n} \, q^{-n-1}, \tag{16}$$

and

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) P^2 + 2\mathcal{F}(q) R_{\mu\nu} PP' - 2\mathcal{F}(q) K_{\mu\nu} PP' + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(P, P) = 0. \tag{17}$$

The last equation becomes

$$\Big(G_{\mu\nu} + \Lambda g_{\mu\nu}\Big)(1 + 2\mathcal{F}(q)PP') + \mathcal{F}(q) g_{\mu\nu} \Big(-\frac{1}{2}P^2 + PP'(R - 2\Lambda)\Big) - 2\mathcal{F}(q) K_{\mu\nu} PP' + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(P, P) = 0. \tag{18}$$

If $P(R) = \sqrt{R-2\Lambda}$, then $P(R)P'(R) = \frac{1}{2}$ and

$$\Box\sqrt{R-2\Lambda} = q\,\sqrt{R-2\Lambda} = \eta\,\Lambda\,\sqrt{R-2\Lambda}, \qquad \eta\,\Lambda \neq 0, \tag{19}$$

where $q = \eta\,\Lambda$ and $q^{-1} = \eta^{-1}\Lambda^{-1}$ (η – dimensionless) follows from dimensionality of equalities (19). Since $P(R) = \sqrt{R-2\Lambda}$, EoM (18) simplify to

$$\Big(G_{\mu\nu} + \Lambda g_{\mu\nu}\Big)(1+\mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R-2\Lambda}, \sqrt{R-2\Lambda}) = 0. \tag{20}$$

It is clear that EoM (20) are satisfied if

$$\mathcal{F}(q) = -1 \qquad \text{and} \qquad \mathcal{F}'(q) = 0. \tag{21}$$

It is worth pointing out that not only nonlocal de Sitter model (5) is very simple and natural but also such are corresponding EoM (20) with respect to all other models and their EoM that can be derived from (1) with $\Lambda \neq 0$.

Let us remark that nonlocal operator $\mathcal{F}(\Box)$, which satisfies conditions (21) in model (5), can be taken in the symmetric form

$$F(\Box) = 1 + \mathcal{F}(\Box), \qquad \text{where} \qquad \mathcal{F}(\Box) = \sum_{n=1}^{+\infty} \tilde{f}_n\Big[\Big(\frac{\Box}{q}\Big)^n + \Big(\frac{q}{\Box}\Big)^n\Big], \tag{22}$$

where $\tilde{f}_n$ are dimensionless coefficients. It is easy to prove that $\mathcal{F}(\Box)$ presented in the following symmetric form:

$$\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} \tilde{f}_n\Big[\Big(\frac{\Box}{q}\Big)^n + \Big(\frac{q}{\Box}\Big)^n\Big] = -\frac{1}{2e}\Big(\frac{\Box}{q}e^{\frac{\Box}{q}} + \frac{q}{\Box}e^{\frac{q}{\Box}}\Big), \qquad q \neq 0, \tag{23}$$

satisfies conditions (21). We can take the eigenvalue q of d'Alembertian $\Box$ to be proportional to Λ, i.e. $q = \eta\,\Lambda$, where $\eta \neq 0$ (since $\Lambda \neq 0$) is a definite dimensionless constant. Moreover, it has to be $q = \eta\,\Lambda$, since there is no other parameter than Λ which is of the same dimension as $\Box$ in this nonlocal gravity model. Hence, nonlocal operator (23) can be rewritten as

$$\mathcal{F}(\Box) = \sum_{n=1}^{+\infty} \tilde{f}_n\Big[\Big(\frac{\Box}{\eta\Lambda}\Big)^n + \Big(\frac{\eta\Lambda}{\Box}\Big)^n\Big] = -\frac{1}{2e}\Big(\frac{\Box}{\eta\Lambda}e^{\frac{\Box}{\eta\Lambda}} + \frac{\eta\Lambda}{\Box}e^{\frac{\eta\Lambda}{\Box}}\Big), \qquad \eta\,\Lambda \neq 0, \tag{24}$$

where for some specific $\Box$ holds $\Box\sqrt{R-2\Lambda} = \eta\,\Lambda\,\sqrt{R-2\Lambda}$.

3. Schwarzschild-de Sitter-type metric

We want to investigate our model outside the spherically symmetric massive body. Since this model is a nonlocal generalization of general relativity with the cosmological constant Λ, it is natural to consider a generalization of the Schwarzschild-de Sitter (SdS) metric starting from the standard Schwarzschild expression

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2. \tag{25}$$

The corresponding scalar curvature R of above metric (25) is

$$R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)} \tag{26}$$

where $'$ denotes derivative with respect to r. In this paper we will consider the case $B(r) = A(r)^{-1}$, and formula (26) becomes

$$R = \frac{2 - 2A(r) - 4rA'(r) - r^2 A''(r)}{r^2}. \tag{27}$$

Note that (27) can be rewritten in the more compact form

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \Big[r^2 \Big(1 - A(r) \Big) \Big]. \tag{28}$$

According to Section 2, to find a solution of equations of motion it is necessary to solve an eigenvalue problem (19), that is $\Box \sqrt{R - 2\Lambda} = q \sqrt{R - 2\Lambda}$. Note that here d'Alembertian $\Box$ acts in the following way:

$$\Box u(r) = A(r)\, u''(r) + (A'(r) + \frac{2}{r} A(r))\, u'(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \Big[r^2 A(r) \frac{\partial u}{\partial r} \Big], \tag{29}$$

where $u(r)$ is any differentiable scalar function.

Let us now consider function $A(r)$ in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{6} r^2 - f(r), \tag{30}$$

where μ and ν are some parameters to be discussed later. Then one can show that for $A(r)$ given by (30) holds

$$R(r) = \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \Big[r^2 \Big(1 - A(r) \Big) \Big] = 2\Lambda + \frac{1}{r^2} \frac{\partial^2}{\partial r^2} \Big[r^2 f(r) \Big]. \tag{31}$$

Denoting $u(r) = \sqrt{R - 2\Lambda}$ and using expression (29) for d'Alembertian, the corresponding equation $\Box \sqrt{R - 2\Lambda} = q \sqrt{R - 2\Lambda}$ becomes

$$\Box \sqrt{R - 2\Lambda} = \frac{1}{r^2} \frac{\partial}{\partial r} \Big[r^2 A(r) \frac{\partial}{\partial r} \sqrt{R - 2\Lambda} \Big] = q \sqrt{R - 2\Lambda}. \tag{32}$$

Since unknown function $A(r)$ is contained also in the scalar curvature $R(r)$, equation (32) is very complicated and difficult to find exact solution. To get an approximative solution we take $A(r) \approx 1$ in (32), what is applicable when

$$\Big| \frac{\mu}{r} \Big| \ll 1, \quad \Big| \frac{\nu}{r^2} \Big| \ll 1, \quad |\Lambda r^2| \ll 1, \quad |f(r)| \ll 1, \tag{33}$$

in units $c = 1$. Under conditions (33), equation (32) becomes

$$\triangle \sqrt{R - 2\Lambda} = \frac{1}{r^2}\frac{\partial}{\partial r}\Big[r^2 \frac{\partial}{\partial r} \sqrt{R - 2\Lambda}\Big] = q \sqrt{R - 2\Lambda}, \tag{34}$$

where

$$\triangle = \frac{1}{r^2}\frac{\partial}{\partial r}\Big[r^2 \frac{\partial}{\partial r}\Big] = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} \tag{35}$$

is the Laplace operator (Laplacian) in spherical coordinates.

One can easily check that the general solution of equation

$$\triangle u(r) = \frac{\partial^2 u(r)}{\partial r^2} + \frac{2}{r}\frac{\partial u(r)}{\partial r} = q\, u(r) \tag{36}$$

is

$$u(r) = \frac{C_1}{r}\, e^{\sqrt{q}\, r} + \frac{C_2}{r}\, e^{-\sqrt{q}\, r}. \tag{37}$$

Since $u(r)$ should tend to 0 at large distances, in the sequel we will use only solution

$$u(r) = \frac{C_2}{r}\, e^{-\sqrt{q}\, r}. \tag{38}$$

According to equation (31) we have

$$R(r) - 2\Lambda = 2\Lambda + \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\Big[r^2 f(r)\Big] = u^2(r). \tag{39}$$

Equation (39) can be rewritten in the form

$$r^2 f''(r) + 4r f'(r) + 2f(r) = -2\Lambda r^2 + C_2^2\, e^{-2\sqrt{q}\, r}. \tag{40}$$

General solution of equation (40) is

$$f(r) = -\frac{\Lambda}{6} r^2 + \frac{C_2^2}{4q}\,\frac{1}{r^2}\, e^{-2\sqrt{q}\, r} + \frac{C_3}{r} + \frac{C_4}{r^2}. \tag{41}$$

Replacing $f(r)$ in (30) by expression (41) one obtains

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4qr^2}\, e^{-2\sqrt{q}\, r} - \frac{C_3}{r} - \frac{C_4}{r^2}. \tag{42}$$

Since all parameters μ, ν, C_3, C_4 have been so far arbitrary we can take $C_3 = C_4 = 0$ and maintain only μ and ν, i. e. in the sequel we have

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_2^2}{4qr^2}\, e^{-2\sqrt{q}\, r}. \tag{43}$$

Then the corresponding scalar curvature becomes

$$R(r) = 2\Lambda + \frac{C_2^2}{r^2}\, e^{-2\sqrt{q}\, r}. \tag{44}$$

4. Discussion and conclusion

In the previous sections we considered nonlocal de Sitter gravity model defined by its action (5) at scales smaller than the cosmological one, i.e. related to stars, galaxies, and clusters of galaxies. Cosmological solutions of model (5) are presented in [31, 34], and it was shown that the cosmological solution in flat space-time, $a_1(t) = At^{\frac{2}{3}}e^{\frac{\Lambda}{14}t^2}$ gives good agreement with observational data. Consequently, it is natural to see how this nonlocal model works at the smaller scales.

It is well known that $A(r)$ of the standard Schwarzschild-de Sitter metric (see [4] and references therein), i.e. in the case of local de Sitter gravity, is

$$A_\ell(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3c^2}, \qquad r \geq r_0, \tag{45}$$

where r_0 is the radius of spherically symmetric massive body (M–mass), and G is the Newton gravitational constant and c is speed of light. Note that (45) is written in the international system of units (SI). The nonlocal version of $A(r)$ (42) can be rewritten as

$$A_{n\ell}(r) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2}\Big(1 - e^{-2\sqrt{q}\,r}\Big), \qquad q = \frac{\eta\Lambda}{c^2}, \tag{46}$$

where we joined terms $\frac{\nu}{r^2}$ and $\frac{C_2^2}{4qr^2}\, e^{-2\sqrt{q}\,r}$ taking $\delta^2 = \frac{C_2^2}{4}, \nu = -\frac{\delta^2}{q}$. It is worth noting that δ and η are dimensionless parameters, and their values should be determined by experiments (astronomical observations). Comparing (45) and (46) follows formula

$$A_{n\ell}(r) - A_\ell(r) = \frac{\Lambda r^2}{6c^2} + \frac{\delta^2}{qr^2}\Big(1 - e^{-2\sqrt{q}\,r}\Big). \tag{47}$$

Testing formula (46) in the Solar and other astronomical systems is one of our main next tasks, and the results will be presented elsewhere.

At the end, it is worth noting that in paper [35] was found connection between cosmological constant Λ and mass of a scalar particle which has its origin in p-adic string theory [36]. About cosmological research with application of nonlocal fields in the matter sector of general relativity one can see [37, 38] and references therein.

Appendix

In the appendix we present curvature tensors for the Schwarzschild-de Sitter-type metric (25)

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + B(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{48}$$

The Christoffel symbols are given as follows (unlisted ones are equal to zero and ′ denotes derivative with respect to r):

$$\begin{aligned}
&\Gamma^0_{01} = \frac{1}{2}\frac{A'}{A}, \quad \Gamma^1_{00} = \frac{1}{2}\frac{A'}{B}, \quad \Gamma^1_{11} = \frac{1}{2}\frac{B'}{B}, \quad \Gamma^1_{22} = -\frac{r}{B}, \quad \Gamma^1_{33} = -\frac{r\sin^2\theta}{B},\\
&\Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \cot\theta.
\end{aligned} \tag{49}$$

Curvature tensor components with three or four different indices are all equal to zero. Only nonzero components are with two different indices and they are listed below:

$$\begin{aligned} R_{0101} &= \frac{A}{4}\left(-\left(\frac{A'}{A}\right)^2 - \frac{A'}{A}\frac{B'}{B} + 2\frac{A''}{A}\right), \quad R_{0202} = \frac{r}{2}\frac{A'}{B}, \quad R_{0303} = \frac{r}{2}\frac{A'}{B}\sin^2\theta, \\ R_{1212} &= \frac{r}{2}\frac{B'}{B}, \quad R_{1313} = \frac{r}{2}\frac{B'}{B}\sin^2\theta, \quad R_{2323} = r^2\frac{B-1}{B}\sin^2\theta. \end{aligned} \tag{50}$$

The Ricci tensor is diagonal and its components are:

$$\begin{aligned} R_{00} &= \frac{A''}{2B} - \frac{A'B'}{4B^2} - \frac{A'^2}{4AB} + \frac{A'}{rB}, \quad R_{11} = -\frac{A''}{2A} + \frac{A'B'}{4A(r)B(r)} + \frac{A'^2}{4A^2} + \frac{B'}{r}, \\ R_{22} &= -\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1, \quad R_{33} = \left(-\frac{rA'}{2AB} + \frac{rB'}{2B^2} - \frac{1}{B} + 1\right)\sin^2\theta. \end{aligned} \tag{51}$$

The scalar curvature is

$$R = -\frac{A''}{AB} + \frac{A'B'}{2AB^2} + \frac{A'^2}{2A^2B} - \frac{2A'}{rAB} + \frac{2B'}{rB^2} - \frac{2}{r^2B} + \frac{2}{r^2}. \tag{52}$$

The Einstein tensor is diagonal and components are presented as

$$\begin{aligned} G_{00} &= \frac{AB'}{rB^2} - \frac{A}{r^2B} + \frac{A}{r^2}, \quad G_{11} = \frac{A'}{rA} - \frac{B}{r^2} + \frac{1}{r^2}, \\ G_{22} &= \frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}, \quad G_{33} = \left(\frac{r^2A''}{2AB} - \frac{r^2A'B'}{4AB^2} - \frac{r^2A'^2}{4A^2B} + \frac{rA'}{2AB} - \frac{rB'}{2B^2}\right)\sin^2\theta. \end{aligned} \tag{53}$$

Case $B = \frac{1}{A}$.

In particular, for $B = \frac{1}{A}$ we have

$$ds^2 = -A(r)dt^2 + \frac{1}{A(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2. \tag{54}$$

The Christoffel symbols are:

$$\begin{aligned} \Gamma^0_{01} &= \frac{1}{2}\frac{A'}{A}, \quad \Gamma^1_{00} = \frac{1}{2}AA', \quad \Gamma^1_{11} = -\frac{1}{2}\frac{A'}{A}, \quad \Gamma^1_{22} = -rA, \quad \Gamma^1_{33} = -rA\sin^2\theta, \\ \Gamma^2_{12} &= \frac{1}{r}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \cot\theta. \end{aligned} \tag{55}$$

Curvature tensor components are:

$$\begin{aligned} R_{0101} &= \frac{1}{2}A'', \quad R_{0202} = \frac{r}{2}AA', \quad R_{0303} = \frac{r}{2}AA'\sin^2\theta, \\ R_{1212} &= -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} = -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} = r^2(1-A)\sin^2\theta. \end{aligned} \tag{56}$$

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \quad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A}, \quad R_{22} = 1 - A - rA', \quad R_{33} = (1 - A - rA')\sin^2\theta. \tag{57}$$

The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}. \tag{58}$$

The Einstein tensor is presented as follows:

$$\begin{aligned} G_{00} &= -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \quad & G_{11} &= \frac{A'(r)}{rA(r)} - \frac{1}{r^2 A(r)} + \frac{1}{r^2}, \\ G_{22} &= \frac{1}{2}r^2 A''(r) + rA'(r), & G_{33} &= \Big(\frac{1}{2}r^2 A''(r) + rA'(r)\Big)\sin^2\theta. \end{aligned} \tag{59}$$

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