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Decomposition of corona graph

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Abstract. Let $G = (V, E)$ be a finite and connected graph. The corona $G_m \odot G_n$ of two graphs G_m and G_n is defined as the graph created by taking one copy of G_m and $|V(G_m)|$ copies of G_n and attaching the i^{th} vertex of G_m to every vertex in the i^{th} copy of G_n. In this paper, we initiate to decompose the corona $G_m \odot G_n$ into cycles, paths, and claws of varying lengths.

1. Introduction

A graph $G = (V, E)$ is finite simple connected graph with n vertices and m edges. A path graph P_n with n vertices consists of vertices v_1, v_2, ..., v_n and edges $\{v_i, v_{i+1}\}$, where $i = 1, 2, \ldots, n-1$. The length of this path graph is $n - 1$ which is the number of edges in the graph. A cycle graph is a graph with only one cycle and denoted by C_n with length n. A star S_n is a tree with one internal vertex and n leaves / pendent vertex, or the complete bipartite graph $K_{1,n}$. The claw is a tree which is also a complete bipartite graph $K_{1,3}$ or star graph S_4. For term and notation not defined here refer in [4, 6]. For positive integrals m and n, see the corona in [10, 12, 20], where $G_m \odot G_n$ of two graphs G_m and G_n is the graph created by taking one copy of G_m and $|V(G_m)|$ copies of G_n and attaching the i^{th} vertex of G_m to every vertex in the i^{th} copy of G_n. Additionally, it has vertices of the form $V(G_m \odot G_n) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and edges of the form $E(G_m \odot G_n) = \{e_1, e_2, e_3, \ldots, e_{mn}\}$ with $m + n(m + 1) + 1$ vertices and $2mn + 2n - 1$ edges.
In [11, 17], the term "decomposition" refers to the grouping of subgraphs $H_1, H_2, \ldots, H_k$, of G such that each edge of G belongs to precisely one H_i, where $i = 1, 2, 3, \ldots, k$. Several authors have explored different sorts of decompositions and associated factors by placing restrictions on the decomposition's constituents [9, 13–15, 17]. These decompositions include claw decomposition, path decomposition, and cycle decomposition. A path decomposition of graph is the decomposition of its edges into subgraphs, where each subgraph represents a path or a union of paths [2, 7, 13, 14, 18] and a cycle decomposition is a decomposition of the graph such that every member of the subgraph is a cycle [1, 5, 8, 15, 16]. Finally, a claw decomposition is a decomposition where each subgraph represents a claw or union of claws [3, 9, 17, 19, 21]. Further, the

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corona graphs $S_4 \odot P_3$, $K_4 \odot P_3$, and $C_3 \odot P_3$ are depicted in Figures 1, 2, and 3. In this paper, we determine the decomposition of the corona $G_m \odot G_n$ into cycles, paths, and claws with respect to different length, where the graph G is S_m, C_m, K_m and P_n.

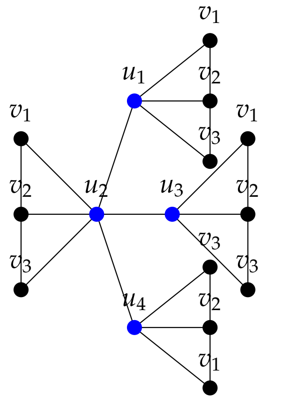

Figure 1: Corona Graph $S_4 \odot P_3$

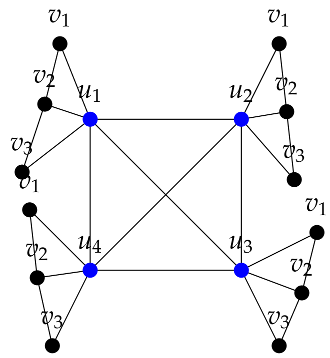

Figure 2: Corona Graph $K_4 \odot P_3$

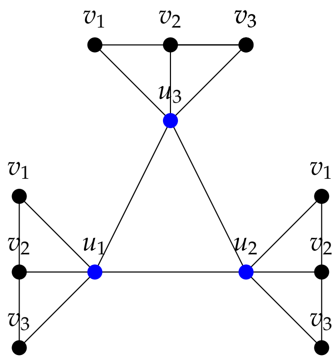

Figure 3: Corona Graph $C_3 \odot P_3$

2. Main Results

2.1. *Decomposition of* $S_m \odot P_n$

Here, we define the corona $S_m \odot P_n$ of star graph S_m and path graph P_n, where it is described as the graph created by taking one copy of S_m and $|V(P_m)|$ copies of P_n and attaching the i^{th} vertex of S_m to every vertex in the i^{th} copy of P_n with vertices of the form $V(S_m \odot P_n) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and edges of the form $E(S_m \odot P_n) = \{e_r = u_1u_{r+1}, e_s^1 = u_1v_s, e_s^2 = u_2v_s, \ldots, e_s^m = u_mv_s, e_t = v_tv_{t+1}\}$, where $s = \{1, 2, \ldots, n-1\}$, $t = r = \{1, 2, \ldots, m-1\}$, and has $m + n(m+1) + 1$ vertices and $2mn + 2n - 1$ edges.

Now, we start with the following result and proof.

Theorem 2.1. *Let* m, n *be positive integers and* $m, n \geq 4$, *then there exists a decomposition of* $S_m \odot P_n$ *into*
(1) $\lfloor \frac{n-2}{2} \rfloor m + 1$ *copies of* P_2, $\lfloor \frac{m-1}{2} \rfloor$ *copies of* P_3, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if* m, n *is even.*
(2) $\lfloor \frac{n-2}{2} \rfloor m$ *copies of* P_2, $\lfloor \frac{m-1}{2} \rfloor + m$ *copies of* P_3, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if* m, n *is odd.*
(3) $\lfloor \frac{n-2}{2} \rfloor m + 1$ *copies of* P_2, $\lfloor \frac{m-1}{2} \rfloor + m$ *copies of* P_3, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if* m *is even and* n *is odd.*
(4) $\lfloor \frac{n-2}{2} \rfloor m$ *copies of* P_2, $\lfloor \frac{m-1}{2} \rfloor$ *copies of* P_3, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if* m *is odd and* n *is even.*

Proof. Let $V(S_m \odot P_n) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and $E(S_m \odot P_n) = \{e_r = u_1u_{r+1}, e_s^1 = u_1v_s, e_s^2 = u_2v_s, \ldots, e_s^m = u_mv_s, e_t = v_tv_{t+1}\}$, denotes the vertex and edges of $S_m \odot P_n$. The theorem's proof consists of four cases:

Case 1: When $m, n \geq 4$ and m, n are even.
For P_2, let $E = \{e_r = u_1u_{r+1}\}$, where $r = \{3, 5, 7, \ldots, m-1\}$ and $F_t = \{e_t = v_tv_{t+1}\}$, where $t = \{2, 4, 6, \ldots, n-2\}$.
For P_3, let $G_r = \{e_r, e_{r+1}\}$, where $r = \{1, 3, 5, 7, \ldots, m-3\}$.
For C_3, let $H_s^1 = \{e_s^1, e_{s+1}^1, e_t\}, H_s^2 = \{e_s^2, e_{s+1}^2, e_t\}, H_s^3 = \{e_s^3, e_{s+1}^3, e_t\}, \ldots, H_s^m = \{e_s^m, e_{s+1}^m, e_t\}$, where $s = \{1, 3, 5, 7, \ldots, n-1\}$ and $t = \{1, 3, 5, \ldots, n-1\}$.
Now take one copy of path P_2 with length one create a subgraph $< E >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create as subgraphs $< F_2 >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< F_4 >$, and this process of decomposition continue until $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< F_{n-2} >$. Again, take $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two creates a subgraph $< G_r >$ and finally $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 with length three creates a subgraph $< H_s^1 >, < H_s^2 >$, till $< H_s^m >$. In this process, the corona graph of $S_m \odot P_n$ can be decomposed into $\{\lfloor \frac{n-2}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor + \ldots + \lfloor \frac{n-2}{2} \rfloor\} + 1 = \lfloor \frac{n-2}{2} \rfloor m + 1$ copies of path P_2 of length one, $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two, and $\{\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + \ldots + \lfloor \frac{n}{2} \rfloor\} = \lfloor \frac{n}{2} \rfloor m$ copies of cycle C_3 with length three.
Case 2: When $m, n \geq 4$ and m, n are odd.
For P_2, let $E_t = \{e_t = v_tv_{t+1}\}$, where $t = \{2, 4, 6, \ldots, n-3\}$.
For P_3, let $F_q = \{e_s^q, e_t\}$, where $q = \{1, 2, \ldots, m\}, s = \{m\}, t = \{m-1\}$ $G_r = \{e_r, e_{r+1}\}$, where $r = \{1, 3, 5, 7, \ldots, m-2\}$.
For C_3, let $H_s^1 = \{e_s^1, e_{s+1}^1, e_t\}, H_s^2 = \{e_s^2, e_{s+1}^2, e_t\}, H_s^3 = \{e_s^3, e_{s+1}^3, e_t\}, \ldots, H_s^m = \{e_s^m, e_{s+1}^m, e_t\}$, where $s = \{1, 3, 5, 7, \ldots, n-2\}$ and $t = \{1, 3, 7, \ldots, n-2\}$.
Now, take $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_2 >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_4 >$, and this process continues until $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_{n-3} >$, m copies of path P_3 with length two creates a subgraph F_q and $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two creates a subgraph $< G_r >$, and finally $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 with length three creates a subgraph $< H_s^1 >, < H_s^2 >$, till $< H_s^m >$. Hence, by the above process the corona graph $S_m \odot P_n$ can be decomposed into $\{\lfloor \frac{n-2}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor + \ldots + \lfloor \frac{n-2}{2} \rfloor\} = \lfloor \frac{n-2}{2} \rfloor m$ copies of path P_2 of length one, $\lfloor \frac{m-1}{2} \rfloor + m$ copies of path P_3 with length two, and $\{\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + \ldots + \lfloor \frac{n}{2} \rfloor\} = \lfloor \frac{n}{2} \rfloor m$ copies of cycle C_3 with length three.
Case 3: When $m, n \geq 4$ and m is even and n is odd.
For P_2, let $E = \{e_r = u_1u_{r+1}\}$, where $r = \{3, 5, 7, \ldots, m-1\}$ and $F_t = \{e_t = v_tv_{t+1}\}$, where $t = \{2, 4, 6, \ldots, n-2\}$.
For P_3, let $F_q' = \{e_s^q, e_t\}$, where $q = \{1, 2, \ldots, m\}, s = \{m\}, t = \{m-1\}$ $G_r = \{e_r, e_{r+1}\}$, where $r = \{1, 3, 5, 7, \ldots, m-2\}$.
For C_3, let $H_s^1 = \{e_s^1, e_{s+1}^1, e_t\}, H_s^2 = \{e_s^2, e_{s+1}^2, e_t\}, H_s^3 = \{e_s^3, e_{s+1}^3, e_t\}, \ldots, H_s^m = \{e_s^m, e_{s+1}^m, e_t\}$, where $s = \{1, 3, 5, 7, \ldots, n-2\}$ and $t = \{1, 3, 7, \ldots, n-2\}$.
Now, take one copy of path P_2 with length one create a subgraph $< E >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< F_2 >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< F_4 >$, and this process continues until $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< F_{n-2} >$, m copies of path P_3 with length two create a subgraph F_q' and $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two creates a subgraph $< G_r >$, and finally $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 with length three creates a subgraph $< H_s^1 >$, $< H_s^2 >$, till $< H_s^m >$. Hence, by the above process the corona graph $S_m \odot P_n$ can be decomposed into $\{\lfloor \frac{n-2}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor + \ldots + \lfloor \frac{n-2}{2} \rfloor\} + 1 = \lfloor \frac{n-2}{2} \rfloor m + 1$ copies of path P_2 of length one, $\lfloor \frac{m-1}{2} \rfloor + m$ copies of path P_3 with length two, and $\{\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + \ldots + \lfloor \frac{n}{2} \rfloor\} = \lfloor \frac{n}{2} \rfloor m$ copies of cycle C_3 with length three.
Case 4: When $m, n \geq 4$ and m is odd and n is even.
For P_2, let $E_t = \{e_t = v_tv_{t+1}\}$, where $t = \{2, 4, 6, \ldots, n-3\}$.
For P_3, let $G_r = \{e_r, e_{r+1}\}$, where $r = \{1, 3, 5, 7, \ldots, m-3\}$.
For C_3, let $H_s^1 = \{e_s^1, e_{s+1}^1, e_t\}, H_s^2 = \{e_s^2, e_{s+1}^2, e_t\}, H_s^3 = \{e_s^3, e_{s+1}^3, e_t\}, \ldots, H_s^m = \{e_s^m, e_{s+1}^m, e_t\}$, where $s = \{1, 3, 5, 7, \ldots, n-1\}$ and $t = \{1, 3, 7, \ldots, n-1\}$.
Now, take $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_2 >$, $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_4 >$, and this process continues until $\lfloor \frac{n-2}{2} \rfloor$ copies of path P_2 with length one create a subgraph $< E_{n-3} >$, $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two creates a subgraph $< G_r >$ and finally $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 with length three creates a subgraph $< H_s^1 >, < H_s^2 >$, till $< H_s^m >$. Hence, by the above process the corona graph $S_m \odot P_n$ can be decomposed into $\{\lfloor \frac{n-2}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor + \ldots + \lfloor \frac{n-2}{2} \rfloor\} = \lfloor \frac{n-2}{2} \rfloor m$ copies of path P_2 of length one, $\lfloor \frac{m-1}{2} \rfloor$ copies of path P_3 with length two, and $\{\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + \ldots + \lfloor \frac{n}{2} \rfloor\} = \lfloor \frac{n}{2} \rfloor m$ copies of cycle C_3 with length three. $\square$

From Theorem 2.1, we have the following observations:

Observation 2.2. *When $m, n \geq 4$ and m, n be positive integers, then $S_m \odot P_n$ can be decomposed into*
(1) $(m-1) + (\frac{n-2}{2})m$ *copies of* P_2, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if n is even.*
(2) $(m-1) + (\frac{n+1}{2})m$ *copies of* P_2, *and* $\lfloor \frac{n}{2} \rfloor m$ *copies of* C_3, *if n is odd.*

Observation 2.3. *When $m, n \geq 3$, and m, n be a positive integers, then $S_m \odot P_n$ can be decomposed into*
(1) *m copies of P_2, and $\lfloor \frac{m-1}{2} \rfloor + (n-1)m$ copies of P_3, if m is odd.*
(2) *$m+1$ copies of P_2, and $\lfloor \frac{m-1}{2} \rfloor + (n-1)m$ copies of P_3, if m is even.*
(3) *$(n-2)m+1$ copies of P_2, and $\lfloor \frac{m-1}{2} \rfloor$ copies of P_3, and m copies of cycle C_{n+1} of length $n+1$, if m is even.*
(4) *$(n-2)m$ copies of P_2, and $\lfloor \frac{m-1}{2} \rfloor$ copies of P_3, and m copies of cycle C_{n+1} of length $n+1$, if m is odd.*
(5) *$(m-1)$ copies of P_2, m copies of P_n, and $\frac{mn}{2}$ copies of claw $K_{1,3}$, if $m = n = 3q$, where $q = 1, 2, \ldots$*
(6) *$(2m-1)$ copies of P_2, m copies of P_n, and $\frac{n^2-n}{3}$ copies of claw $K_{1,3}$, if $m = n = 3q+1$, where $q = 1, 2, \ldots$*
(7) *$(m-1)$ copies of P_2, m copies of P_3, m copies of P_n, and $\frac{nm-2m}{3}$ copies of claw $K_{1,3}$, if $m = n = 3q+2$, where $q = 1, 2, \ldots$*

2.2. Decomposition of $C_m \odot P_n$

The corona $C_m \odot P_n$ of cycle C_m and path P_n is defined as the graph obtained by taking one copy of C_m and $|V(C_m)|$ copies of P_n and joining the i^{th} vertex of C_m to every vertex in the i^{th} copy of P_n.
The graph formed by the corona product $C_m \odot P_n$ has vertices of the form $V(C_m \odot P_n) = \{u_1, u_2, u_3, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and edges of the form $E(C_m \odot P_n) = \{e_l = u_l u_{l+1}, e_{l'} = u_{l'} u_1, e_i = u_1 v_i, e_i^{'} = u_2 v_i, e_i^{''} = u_3 v_i, \ldots, e_i^{m-1} = u_m v_i, e_j = v_j v_{j+1}\}$, where $i = \{1, 2, \ldots, n\}$, $j = l = \{1, 2, \ldots, m-1\}$, $l^{'} = \{m\}$. There are $m + mn$ vertices and $m + m(n-1) + mn$ edges in the corona product $C_m \odot P_n$.

Now, we are calculating the decomposition of corona graph of $C_m \odot P_n$.

Theorem 2.4. *Let m, n be positive integers. If $n \geq 2, m \geq 3$, then there exists a decomposition of $C_m \odot P_n$ into a single copy of cycle C_m of length m, m copies of path P_{n+1} of length n, and $m(n-1)$ copies of path P_2 of length one.*

Proof. Let $V(C_m \odot P_n) = \{u_1, u_2, u_3, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ be the vertex set and edges-set consisting of all edges of the form $E(C_m \odot P_n) = \{e_l = u_l u_{l+1}, e_{l'} = u_{l'} u_1, e_i = u_1 v_i, e_i^{'} = u_2 v_i, e_i^{''} = u_3 v_i, \ldots, e_i^{m-1} = u_m v_i, e_j = v_j v_{j+1}\}$, where $i = \{1, 2, \ldots, n\}$, $j = \{1, 2, \ldots, n-1\}$, $l = \{1, 2, \ldots, m-1\}$, $l^{'} = \{m\}$.
Since m and n are positive integers that can be either an odd number or an even number.
Case 1: When m, n are even and it can be written as $m = n = 2q$, where $q = 1, 2, 3, \ldots$
Let $F = \{e_l, e_{l'}\}$, where $l = \{1, 2, \ldots, m-1\}$ and $l^{'} = \{m\}$, $E_1 = \{e_i, e_j\}$, $E_2 = \{e_i^{'}, e_j\}$, $E_3 = \{e_i^{''}, e_j\}, \ldots, E_m = \{e_i^{m-1}, e_j\}$ where $i = \{1\}$, $j = \{1, 2, 3, 4, \ldots, n-1\}$. Also $F_i = \{e_i\}$, $F_i^{'} = \{e_i^{'}\}$, $F_i^{''} = \{e_i^{''}\}, \ldots, F_i^{m-1} = \{e_i^{m-1}\}$ where $i = \{1, 2, \ldots, n-1\}$.
Then, the subgraph $< F >$ create a single cycle C_m of length m, the subgraph $< E_1 >$ creates 1 copy of path P_{n+1} of length n, the subgraph $< E_2 >$ creates 1 copy of path P_{n+1} of length n , the subgraph $< E_3 >$ creates 1 copy of path P_{n+1} of length n, and this process continues until the subgraph $< E_m >$ creates 1 copy of path P_{n+1} of length n.
Again the subgraph $< F_i >$ creates $n-1$ copies of path P_2 of length one, the subgraph $< F_i^{'} >$ creates $n-1$ copies of path P_2 of length one, the subgraph $< F_i^{''} >$ creates $n-1$ copies of path P_2 of length one, and this process continues until the subgraph $< F_i^{m-1} >$ creates $n-1$ copies of path P_2 of length one. Hence $C_m \odot P_n$ of cycle C_m and path P_n can be decomposed into one copy of cycle C_m of length m, and $(1 + 1 + \ldots + 1) = m$ copies of path P_{n+1} of length n and $\{(n-1) + (n-1) + \ldots + (n-1)\} = m(n-1)$ copies of path P_2 of length 1.
Case 2: When m, n are odd and it can be written as $m = n = 2q+1$, where $q = 1, 2, 3, \ldots$.
Let $F = \{e_l, e_{l'}\}$, where $l = \{1, 2, \ldots, m-1\}$ and $l^{'} = \{m\}$, $E_1 = \{e_i, e_j\}$, $E_2 = \{e_i^{'}, e_j\}$, $E_3 = \{e_i^{''}, e_j\}, \ldots, E_m = \{e_i^{m-1}, e_j\}$ where $i = \{1\}$, $j = \{1, 2, 3, 4, \ldots, n-1\}$. Also $F_i = \{e_i\}$, $F_i^{'} = \{e_i^{'}\}$, $F_i^{''} = \{e_i^{''}\}, \ldots, F_i^{m-1} = \{e_i^{m-1}\}$ where $i = \{1, 2, \ldots, n-1\}$.
Then, the subgraph $< F >$ creates a single cycle C_m of length m, the subgraph $< E_1 >$ creates 1 copy of path

P_{n+1} of length n, the subgraph $< E_2 >$ creates 1 copy of path P_{n+1} of length n , the subgraph $< E_3 >$ creates 1 copy of path P_{n+1} of length n, and this process continues until the subgraph $< E_m >$ creates 1 copy of path P_{n+1} of length n.
Again the subgraph $< F_i >$ creates $n-1$ copies of path P_2 of length one, the subgraph $< F_i^{'} >$ creates $n-1$ copies of path P_2 of length one, the subgraph $< F_i^{''} >$ creates $n-1$ copies of path P_2 of length one, and this process continues until the subgraph $< F_i^{m-1} >$ creates $n-1$ copies of path P_2 of length one. Hence the corona product $C_m \odot P_n$ of cycle C_m and path P_n can be decomposed into one copy of cycle C_m of length m, and $(1+1+\ldots+1) = m$ copies of path P_{n+1} of length n and $\{(n-1)+(n-1)+\ldots+(n-1)\} = m(n-1)$ copies of path P_2 of length 1.
□

By employing Theorem 2.4, we have the following observation:

Observation 2.5. *When $n \geq 2, m \geq 3$ and m, n be positive integers, then $C_m \odot P_n$ can be decompose into a single copy of path P_m of length $m-1$, m copies of path P_{n+1} of length n and $(mn-m+1)$ copies of path P_2 of length one.*

Theorem 2.6. *Let m, n be a positive integers and $m, n \geq 4$, then there exist a decomposition of $C_m \odot P_n$ into*
(1) One copy of C_m, $\frac{nm}{2}$ copies of C_3, and $(\frac{n-2}{2})m$ copies of P_2, if m, n is even.
(2) One copy of C_m, $\lfloor \frac{n}{2} \rfloor m$ copies of C_3, m copies of P_3, and $(\frac{n-3}{2})m$ copies of P_2, if m, n is odd.

Proof. Let $V(C_m \odot P_n) = \{u_1, u_2, u_3, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ be the vertex and $E(C_m \odot P_n) = \{e_l = u_l u_{l+1}, e_{l'} = u_{l'} u_1, e_i = u_1 v_i, e_i^{'} = u_2 v_i, e_i^{''} = u_3 v_i, \ldots, e_i^{m-1} = u_m v_i, e_j = v_j v_{j+1}\}$, where $i = \{1, 2, \ldots, n\}$, $j = \{1, 2, \ldots, n-1\}$, $l = \{1, 2, \ldots, m-1\}$, $l' = \{m\}$ be the edges set.
Since m and n are positive integers that can be either an odd number or an even number.
Case 1: When $n > 2, m > 4$ and $m = n =$ even number.
For C_m, let $F = \{e_l, e_{l+1}, \ldots, e_{l+m-2}, e_{l'}\}$, where $l = \{1\}$,$l' = \{m\}$.
For C_3, let $E_i = \{e_i, e_{i+1}, e_j\}$, $E_i^{'} = \{e_i^{'}, e_{i+1}^{'} e_j\}$, $E_i^{''} = \{e_i^{''}, e_{i+1}^{''}, e_j\}$, …, $E_i^{m-1} = \{e_i^{m-1}, e_{i+1}^{m-1}, e_j\}$, where $i = j = \{1, 3, 5, \ldots, n-1\}$.
For P_2, let $H_j = \{e_j\}$, $H_j^{'} = \{e_i\}$, …, $H_j^{m-1} = \{e_j\}$, where $j = \{2, 4, 6, \ldots, n-2\}$. Then, the subgraph $< F >$ creates one copy of cycle C_m of length m, the subgraph $< E_i >$ creates $\frac{n}{2}$ copies of cycle C_3 of length three, the subgraph $< E_i^{'} >$ creates $\frac{n}{2}$ copies of cycle C_3 of length three, the subgraph $< E_i^{''} >$ creates $\frac{n}{2}$ copies of cycle C_3 of length three, and this process continues until subgraph E_i^{m-1} creates $\frac{n}{2}$ copies of cycle C_3 of length three. Finally, the subgraph $< H_j >$, $< H_j^{'} >$, …, $< H_j^{m-1} >$ creates $(\frac{n-2}{2})$ copies of path P_2 of length one. Hence, $C_m \odot P_n$ of cycle C_m and path P_n can be decomposed into a single cycle C_m of length m, $\{(\frac{n}{2}) + (\frac{n}{2}) + \ldots + (\frac{n}{2})\} = \frac{nm}{2}$ copies of cycle C_3 of length three and $\{(\frac{n-2}{2}) + (\frac{n-2}{2}) + \ldots + (\frac{n-2}{2})\} = (\frac{n-2}{2})m$ copies of path P_2 of length one.
Case 2: When $n > 3, m > 3$ and $m = n =$ odd number.
For C_m, let $E = \{e_l, e_{l+1}, \ldots, e_{l+m-2}, e_{l'}\}$, where $l = \{1\}$, $l' = \{m\}$.
For C_3, let $F_i = \{e_i, e_{i+1}, e_j\}$, $F_i^{'} = \{e_i^{'}, e_{i+1}^{'}, e_j\}$, $F_i^{''} = \{e_i^{''}, e_{i+1}^{''}, e_j\}$, …, $F_i^{m-1} = \{e_i^{m-1}, e_{i+1}^{m-1}, e_j\}$ where $i = j = \{1, 3, 5, \ldots, n-2\}$.
For P_3, let $H = \{e_i, e_j\}$, $H_1 = \{e_i^{'}, e_j\}$, …, $H_{m-1} = \{e_i^{m-1}, e_j\}$, where $i = n$, $j = n-1$.
For P_2, let $G_j = \{e_j\}$, $G_j^{'} = \{e_j\}$, …, $G_j^{m-1} = \{e_j\}$, where $j = \{2, 4, 6, \ldots, n-3\}$.
Then, the subgraph $< E >$ creates a single cycle C_m of length m, the subgraph $< F_i >$ creates $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 of length three, the subgraph $< F_i^{'} >$ creates $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 of length three, the subgraph $< F_i^{''} >$ creates $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 of length three, and this process continues until the subgraph $< F_i^{m-1} >$ creates $\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3 of length three, the subgraph $< H >$ creates a single path P_3 of length two, the subgraph $< H_1 >$ creates a single path P_3 of length two, the subgraph $< H_2 >$ creates a single path P_3 of length two, the subgraph $< H_3 >$ creates a single path P_3 of length two, and by the above process continues until the subgraph $< H_{m-1} >$ creates a single path P_3 of length two. Furthermore the subgraph $< G_j >$, $< G_j^{'} >$, …, $< G_j^{m-1} >$ creates $\frac{n-3}{2}$ copies of path P_2 of length one. Hence, $C_m \odot P_n$ of cycle C_m and path P_n can be decomposed into one copy of cycle C_m of length m, $\{(\lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + \ldots + \lfloor \frac{n}{2} \rfloor)\} = m\lfloor \frac{n}{2} \rfloor$ copies of cycle C_3

of length three, $(1+1+...+1) = m$ copies of path P_3 of length two and $\{(\frac{n-3}{2}) + (\frac{n-3}{2}) + \ldots + (\frac{n-3}{2})\} = (\frac{n-3}{2})m$ copies of path P_2 of length one. □

By employing Theorem 2.6, we have the following observations:

Observation 2.7. *When $m, n \geq 4$ and m, n be positive integers, then $C_m \odot P_n$ can be decompose into*
(1) *One copy of path P_m, $\frac{nm}{2}$ copies of cycle C_3, and $\frac{m(n-2)+2}{2}$ copies of path P_2, if m, n is even.*
(2) *One copy of path P_m, $\lfloor \frac{n}{2} \rfloor m$ copies of cycle C_3, m copies of path P_3, and $\frac{m(n-3)+2}{2}$ copies of path P_2, if m, n is odd.*

Observation 2.8. *When $m, n \geq 4$ and m, n be positive integers, then $C_m \odot P_n$ can be decomposed into*
(1) *One copy of cycle C_m, $\frac{mn}{3}$ copies of claw $K_{1,3}$, and m copies of P_n, if $n = 3q$, where $q = 1, 2, 3, \ldots$*
(2) *One copy of cycle C_m, $m(\frac{n-1}{3})$ copies of claw $K_{1,3}$, m copies of path P_n, and m copies of path P_2, if $n = 3q + 1$, where $q = 1, 2, 3, \ldots$*
(3)*One copy of cycle C_m, $(\frac{n-2}{3})m$ copies of claw $K_{1,3}$, m copies of path P_n, and m copies of P_3, if $n = 3q + 2$, where $q = 1, 2, 3, \ldots$*
(4). *One copy of path P_m, one copy of path P_2, $\frac{mn}{3}$ copies of claw $K_{1,3}$, and m copies of path P_n, if $n = 3q$, where $q = 1, 2, 3, \ldots$*
(5) *One copy of path P_m, $m(\frac{n-1}{3})$ copies of claw $K_{1,3}$, m copies of path P_n, and $(m+1)$ copies of path P_2, if $n = 3q + 1$, where $q = 1, 2, 3, \ldots$*
(6) *One copy of path P_m, one copy of path P_2, $(\frac{n-2}{3})m$ copies of claw $K_{1,3}$, m copies of path P_n, and m copies of path P_3, if $n = 3q + 2$, where $q = 1, 2, 3, \ldots$*

2.3. Decomposition of $K_m \odot P_n$

Here, we decompose the $K_m \odot P_n$ of the complete graph K_m and path graph P_n. It is obtained by taking one copy of K_m and $|V(P_n)|$ copies of P_n and joining the $i-$th vertex of K_m to every vertex in the i^{th} copy of P_n. Let the vertex set be $V(K_m \odot P_n) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and edge set be $E(K_m \odot P_n) = \{e_i = v_i v_{i+1}, e_j^1 = u_1 v_j, e_j^2 = u_2 v_j, \ldots, e_j^m = u_m v_j, e_k = u_k u_{k+1}, e_m = u_m u_1, e_{l^1}^1 = u_1 u_{l^1+2}, e_{l^2}^2 = u_2 u_{l^2+3}, \ldots, e_{l^{m-2}}^{m-2} = u_{m-2} u_{l^{m-2}+(m-1)}\}$, where $i = \{1, 2, 3, \ldots, n-1\}$, $j = \{1, 2, 3, \ldots, n\}$, $k = \{1, 2, 3, \ldots, m-1\}$, $l^1 = \{1, 2, 3, \ldots, m-3\}$, $l^2 = \{1, 2, 3, \ldots, m-3\}$, $l^3 = \{1, 2, 3, \ldots, m-4\}$, $l^4 = \{1, 2, 3, \ldots, m-5\}$, ..., $l^{m-2} = \{1, 2, 3, \ldots, m-(m-1)\}$.

Now, we are calculating the decomposition of corona graph of $K_m \odot P_n$.

Theorem 2.9. *Let m, n be positive integers and $m, n \geq 4$, then there exists a decomposition of $K_m \odot P_n$ into*
(1) *One copy of complete graph K_m, $\frac{mn}{2}$ copies of cycle C_3, and $m(\frac{n-2}{2})$ copies of path P_2, if m, n are even.*
(2) *One copy of the complete graph K_m, $m(\frac{n-1}{2})$ copies of cycle C_3, m copies of path P_3, and $(\frac{n-3}{2})m$ copies of path P_2, if m, n are odd.*

Proof. Let $V(K_m \odot P_n) = \{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}$ and, $E(K_m \odot P_n) = \{e_i = v_i v_{i+1}, e_j^1 = u_1 v_j, e_j^2 = u_2 v_j, \ldots, e_j^m = u_m v_j, e_k = u_k u_{k+1}, e_m = u_m u_1, e_{l^1}^1 = u_1 u_{l^1+2}, e_{l^2}^2 = u_2 u_{l^2+3}, \ldots, e_{l^{m-2}}^{m-2} = u_{m-2} u_{l^{m-2}+(m-1)}\}$, where $i = \{1, 2, 3, \ldots, n-1\}$, $j = \{1, 2, 3, \ldots, n\}$, $k = \{1, 2, 3, \ldots, m-1\}$, $l^1 = \{1, 2, 3, \ldots, m-3\}$, $l^2 = \{1, 2, 3, \ldots, m-3\}$, $l^3 = \{1, 2, 3, \ldots, m-4\}$, $l^4 = \{1, 2, 3, \ldots, m-5\}$, ..., $l^{m-2} = \{1, 2, 3, \ldots, m-(m-1)\}$, denotes the vertex and edges of $K_m \odot P_n$. The proof of the theorem consists of two cases:
Case 1. When m, n is even and $m \geq 4, n \geq 4$. Let the subgraph $E = \{e_k, e_m, e_{l^1}^1, e_{l^2}^2, \ldots, e_{l^{m-2}}^{m-2}\}$, where $k = \{1, 2, 3, \ldots, m-1\}$, $l^1 = \{1, 2, 3, \ldots, m-3\}$, $l^2 = \{1, 2, 3, \ldots, m-3\}$, $l^3 = \{1, 2, 3, \ldots, m-4\}$, $l^4 = \{1, 2, 3, \ldots, m-5\}$, ..., $l^{m-2} = \{1, 2, 3, \ldots, m-(m-1)\}$, $F_j^1 = \{e_j^1, e_{j+1}^1, e_i\}$, $F_j^2 = \{e_j^2, e_{j+1}^2, e_i\}$, ..., $F_j^m = \{e_j^m, e_{j+1}^m, e_i\}$, where $j = \{1, 3, 5, \ldots, n-1\}$, and $G_i = \{e_i\}$, where $i = \{2, 4, 6, \ldots, n-2\}$.
Then the subgraph $< E >$ generates a single complete graph K_m, the subgraph $< F_j^1 >$ generates $\frac{n}{2}$ copies of cycle C_3 with length three, the subgraph $< F_j^2 >$ generates $\frac{n}{2}$ copies of cycle C_3 with length three, and this process continue until the subgraph $< F_j^m >$ generates $\frac{n}{2}$ copies of cycle C_3 with length three, the subgraph G_2 generates $\frac{n-2}{2}$ copies of P_2 with length one, the subgraph G_4 generates $\frac{n-2}{2}$ copies of P_2 with length one, and this process continue until the subgraph G_{n-2} generates $\frac{n-2}{2}$ copies of P_2 with length one.

Therefore, the $K_m \odot P_n$ of K_m and P_n contains one copy of K_m, $\{\frac{n}{2} + \frac{n}{2} + \frac{n}{2} + ... + \frac{n}{2}\} = (\frac{n}{2})m$ copies of C_3, $\{\frac{n-2}{2} + \frac{n-2}{2} + \ldots + \frac{n-2}{2}\} = (\frac{n-2}{2})m$ copies of P_2.

Case 2. When m, n is odd and $m \geq 4, n \geq 4$. Let the subgraph $E = \{e_k, e_m, e^1_{l^1}, e^2_{l^2}, \ldots, e^{m-2}_{l^{m-2}}\}$, where $k = \{1, 2, 3, \ldots, m-1\}$, $l^1 = \{1, 2, 3, \ldots, m-3\}$, $l^2 = \{1, 2, 3, \ldots, m-3\}$, $l^3 = \{1, 2, 3, \ldots, m-4\}$, $l^4 = \{1, 2, 3, \ldots, m-5\}$, $\ldots$, $l^{m-2} = \{1, 2, 3, \ldots, m-(m-1)\}$, $F^1_j = \{e^1_j, e^1_{j+1}, e_i\}$, $F^2_j = \{e^2_j, e^2_{j+1}, e_i\}$, $\ldots$, $F^m_j = \{e^m_j, e^m_{j+1}, e_i\}$, where $j = \{1, 3, 5, \ldots, n-2\}$, $G_i = \{e_i\}$, where $i = \{2, 4, 6, \ldots, n-1\}$, and $H^t = \{e^t_j, e_i\}$, where $t = \{1, 2, \ldots, m\}$, $j = \{n\}$, and $i = \{n-1\}$.

Then, the subgraph $< E >$ generates a single complete graph K_m, the subgraph $< F^1_j >$ generates $\frac{n-1}{2}$ copies of cycle C_3 with length three, the subgraph $< F^2_j >$ generates $\frac{n-1}{2}$ copies of cycle C_3 with length three, and this process continue until the subgraph $< F^m_j >$ generates $\frac{n-1}{2}$ copies of cycle C_3 with length three, the subgraph G_2 generates $\frac{n-3}{2}$ copies of P_2 with length one, the subgraph G_4 generates $\frac{n-3}{2}$ copies of P_2 with length one, and this process continue until the subgraph G_{n-2} generates $\frac{n-3}{2}$ copies of P_2 with length one, the subgraph H^1 generates one copy of path P_3 of length 2, the subgraph H^2 generates one copy of path P_3 of length 2, and this process continue until the subgraph H^m generates one copy of path P_3 of length 2. Therefore, the $K_m \odot P_n$ of K_m and P_n contains one copy of K_m, $\{\frac{n-1}{2} + \frac{n-1}{2} + \frac{n-1}{2} + \ldots + \frac{n-1}{2}\} = (\frac{n-1}{2})m$ copies of C_3, $\{\frac{n-3}{2} + \frac{n-3}{2} + \ldots + \frac{n-3}{2}\} = (\frac{n-3}{2})m$ copies of P_2, $\{1 + 1 + 1 + \ldots + 1\} = m$ copies of P_3.
□

By utilizing Theorem 2.9, we have the following observation:

Observation 2.10. *When $m, n \geq 4$ and m, n be positive integers, then $K_m \odot P_n$ can be decomposed into*
(1) *One copy of cycle C_m, $\frac{mn}{2}$ copies of path C_3, and $\{2(m-3) + (m-4) + \ldots + (m-(m-1)) + m(\frac{n-3}{2})\}$ copies of path P_2, if m, n are even.*
(2) *One copy of cycle C_m, $(\frac{n-1}{2})m$ copies of path C_3, m copies of path P_3, and $\{2(m-3) + (m-4) + \ldots + (m-(m-1)) + m(\frac{n-3}{2})\}$ copies of path P_2, if m, n are odd.*
(3) *One copy of cycle C_m, $\frac{n}{3}m$ copies of claw $K_{1,3}$, and m copies of path P_n, if $n = 3d$, where $d = 1, 2, 3, \ldots$*
(4) *One copy of the complete graph K_m, $m(\frac{n-1}{3})$ copies of claw $K_{1,3}$, m copies of path P_n, and m copies of path P_2, if $n = 3d + 1$, where $d = 1, 2, 3, \ldots$*
(5) *One copy of cycle C_m, $(\frac{n-2}{3})m$ copies of claw $K_{1,3}$, m copies of path P_n, and m copies of path P_3, if $n = 3d + 2$, where $d = 1, 2, 3, \ldots$.*

3. Conclusion

In this paper, we decompose the corona graphs $S_m \odot P_n$, $C_m \odot P_n$, and $K_m \odot P_n$ into cycles, claws, and paths of different lengths. In particular, we observe that for any positive integers m, n, $S_m \odot P_n$ can be decomposed into $\lfloor\frac{n-2}{2}\rfloor m + 1$ copies of P_2, $\lfloor\frac{m-1}{2}\rfloor$ copies of P_3, and $\lfloor\frac{n}{2}\rfloor m$ copies of C_3. Similarly, $C_m \odot P_n$ can be decomposed into a single copy of cycle C_m of length m, m copies of path P_{n+1} of length n and $m(n-1)$ copies of path P_2 of length one. Moreover, $K_m \odot P_n$ can be decomposed into $\frac{m-2}{2}$ copies of complete graph K_m, $\frac{nm}{2}$ copies of cycle C_3, and $\frac{m+n-2}{2}$ copies of path P_2.

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