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A novel SIR-based model for containing misinformation on social media

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Abstract. The widespread dissemination of misinformation through social media poses significant challenges. With the increasing prevalence of social media, vast amounts of information - both accurate and inaccurate - can be rapidly shared among a large audience. This information often plays a critical role in shaping public opinion and influencing significant events, such as elections. This paper addresses the urgent need for more effective models to combat misinformation. We propose a novel approach based on the Susceptible-Infective-Removed (SIR) model, traditionally used in studying information diffusion. Our modified model, termed SIRMIS (SIR-based Misinformation Spreading Model), integrates stochastic differential equations (SDEs) to account for the inherent randomness and uncertainty in information diffusion processes on social networks. SIRMIS offers insights into the dynamics of misinformation propagation, the role of accurate information in counteracting falsehoods, and the estimation of the peak number of misinformed users. The stability of the stochastic equations within the SIRMIS model is rigorously proven using Lyapunov stability theory, ensuring that the model reliably predicts the conditions under which misinformation can be controlled or eradicated. Our results indicate that systematically increasing the exposure of accurate, verified information to key segments of the population can slow down or even completely combat the spread of misinformation. This effect is particularly pronounced when the true information is disseminated by trusted sources, highlighting the importance of credibility in combating falsehoods. Furthermore, we discuss the linear and global stability of the proposed model, emphasizing its potential in effectively mitigating the impact of misinformation on social networks.

1. Introduction

According to the statistics for the year 2024, approximately 4.95 billion individuals are active social media users, which is approximately 61 percent of the global population. Some well-known social network platforms include Twitter, LinkedIn, Facebook, and Instagram, with Facebook alone boasting a massive user base of 3.05 billion individuals [38].

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Social networks have become the easiest way to spread information and misinformation. Therefore, revealing the mechanisms of information spread and predicting the spreading process is of huge significance. Different information spread models [10] have been studied which are most of the time epidemic models [14, 40, 45, 51]. In addition to epidemic models, there are other models developed like models based on topological graphs [36, 37], or statistical inference-based models [28], or opinion model [21]. The spread of information in social networks is comparable to the spread of infectious diseases [13, 31], with epidemic models being more prominently applied [20, 46, 53].

The typical and very popular model used in epidemiology is the Susceptible - Infected - Removed (or Susceptible - Infected - Recovered) model, known as the SIR model. [16]. The SIR model in epidemiology describes the interactions between the three categories: Susceptible, Infected, and Recovered (or Removed). The desired outcome is that the infected category leads to zero or reaches a stable endemic level. When applied to the social networks, the SIR model puts users into different categories depending on their states S, I, and R [29]. S represents the social network user who has not receive the information yet, I represents the user who knows the information and can spread it. R represents the user who does not spread the information. Users from group S can transition to group I at rate β per unit of time. Also, they can recover and come to the R group at the rate γ per unit time. The number of users is denoted for the categories by S(t), I(t), and R(t). The average degree is denoted by k and the total number of users by N. The modifications of the SIR model can lead to non-linear differential equations which are based on mean-field theory [48, 58]. Further modifications are proposed by introducing new categories [32, 43] or changes in immune parameter and information diffusion [44, 52, 55] or developing new models to improve traditional models [5, 27, 42]. Many research papers contributed to improving the basic models and they enhanced understanding of the spread of information process. However, the differential equations are solved as continuous functions, while simulations are based on discrete time. By comparing numerical solutions and simulation outputs, it is clear that there is a significant time deviation between the results [22]. This is mainly because the model will make a faster diffusion process when it is studied in continuous time. Therefore, the basic SIR models can not easily be implemented in the simulations and predictions of information propagation. Contribution to the solutions of the above-mentioned problems is the model SPIR [34] where is added a set of potential spreaders to the susceptible group that is likely to become infected in the subsequent unit time. The SIR model is mostly improved in the following aspects: Introducing new categories [32, 43, 56], enhancing diffusion, the setting of the immune parameter [44, 52, 55] and optimizing diffusion models [5, 27, 42].

Many types of SIR model variables have been studied. For example, the population S (Susceptible) is categorized into subgroups where the infection rates differ [2, 3, 9, 25, 30]. Infections can be considered when divided into lethal and sublethal [4, 35] and reproductive rates can be analyzed in the infected or recovered population [1]. Of course, some of the complicated models can lead to instabilities. The similarity between epidemics and rumors using mathematical analysis had been proposed by Daley in 1964 [8]. Today we can find that many various epidemiological models are used to analyze the propagation of misinformation on social networks [9, 18, 26, 39]. Furthermore, the spreading of rumor was studied in homogeneous network model 2SIH2R where are considered online users who share the rumor and users who share the truth. The 2SIH2R model analyzed mechanisms of discernment and conflict in a heterogeneous network to explain the spread of the rumor and the truth [47]. Considering the control of the rumor diffusion, a rumor-spreading model called SIDRQ was developed with forgetting, immune, and suspicion mechanisms. This model guides a uniform network [54]. Furthermore, an investigation was conducted on delayed rumor propagation in the SEIR model that incorporates saturation on heterogeneous networks[6]. A novel information propagation Mb-SIR model [41] studied characteristics of the microblog considering the user's incomplete reading behavior. It is discovered that the most influential role in information diffusion is the user's reading rate. Sometimes, people do not know the authenticity of a rumor. Therefore, they will not spread the rumor immediately. This is called a hesitation state. The model with hesitation state is discussed in [49]. The results of forcing silence on users of social networks are studied in a rumor propagation SAIR model [59].

The dynamic of rumor propagation is studied in so-called small-world networks in [50]. The SIR model was modified to the Bass-SIR model [11] when propagation of new products is obtained within the model. Consumers who adopt the product enter the "recovered group", and they do not influence further adoption of the product. The SIR model dynamical evolution equations are developed in [57] at the moment of information explosion, which is considered to influence breath, influence strength, relaxation, and pick time. The extended version of the SIR model, known as the SIHR model, is specifically designed to analyze the propagation of rumors [56]. This model introduces a category of individuals named Hibernators. Due to the forgetting mechanism, the Hibenators come from the spreaders and they become spreaders due to the remembering mechanism, so the spread of rumors can be repeated. Hibernators directly transition from being uninformed to actively suppressing the spread of rumors. They can postpone the terminal time of a rumor, and can reduce the maximum influence of a rumor.

The aforementioned citations represent a synthesis of research on the spread of information across social networks. With the constant growth of social networks platforms there will remain a need to study diffusion of various information. Research in the field of social network information propagation is not only significant for science but also plays an important role in protecting the public from threading misinformation.

2. Materials and methods

We developed the SIR-based Misinformation Spreading model (SIRMIS model) by considering the SIR model. The stability of the SIRMIS model is discussed using the Lyapunov function and LaSalle's invariance principle. For a better understanding of the novel SIRMIS model, first we will explain the SIR model and its properties, and then we will explain the Lyapunov stability, and LaSalle's invariance principle, which are used to discuss the stability of the SIRMIS model.

2.1. SIR model

The SIR model was developed by Kermack and McKendrik in 1927 [16]. Today, the SIR model is well-known and widely used epidemiological model. It calculates the number of infected people with a contagious disease in a closed population over time. This model detects the dynamics of acute infections where once recovered individuals have permanent immunity. The model can be applied in the study of illnesses like measles, mumps, smallpox, chickenpox, typhoid fever, and diphtheria. The following conditions are applied:

1. The population number is fixed.
2. It is only possible to leave the susceptible group if a person becomes infected. If it is recovered from the disease, person can leave the infected group. Once recovered, the person has immunity.
3. The probability of a person being infected is not affected by age, race, sex, or social status.
4. There is no inherited immunity.
5. Everyone in the population mixes homogeneously (or has the same number of interactions with one another, well-mixed population).
6. Natural births and deaths are excluded.
7. A short time period is considered.

In the SIR model, there are 3 dependent variables as functions of time. The independent variable is time t. Time is measured in days. The constant value is N, and it represents the total population, given as N = S + I + R.

The dependent variables are detailed below (Table 1).

S(t) - Susceptible	The number of people susceptible to infection.
I(t) - Infected	The number of infected people with the ability to transmit the infection to other susceptible individuals.
R(t) - Recovered (or Removed)	The number of people who have recovered after being infected.

Table 1: Description of terms in the SIR model

The SIR model equations are:

$$\frac{dS}{dt} = -\beta SI \tag{1}$$

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

where β denotes the infection rate, while γ denotes recovery rate. The initial conditions are given by: $S(0) > 0, I(0) \geq 0$ and $R(0) \geq 0$.

The most important role is played by the essential reproductive number R_0. It is derived from the equation $R_0 = \frac{\beta}{\gamma}$. The value of R_0 signifies the number of secondary infections that occur from one infected person within a population that is entirely susceptible to the disease. The SIR model provides invaluable insights into the dynamics of epidemics, encompassing key factors such as the peak rate of infection, the overall number of infections, and the duration of the epidemic. It is important to highlight that the SIR model serves as a simplified representation and, as such, more intricate models may be employed to provide a more precise portrayal of real-world scenarios.

2.2. *Lyapunov stability*

The stability properties of equilibrium points of linear and non-linear systems are studied using Lyapunov function. Lyapunov's study on stability, initially published in 1892, was translated into English and made available a century later in a March 1992 edition of the International Journal of Control [24]. Lyapunov systematically studied the stability of solutions of nonlinear differential equations, which varies in time in near equilibrium point.

To ensure the global stability of the disease - free equilibrium point, the Lyapunov function should satisfy the following: At the disease - free equilibrium point the function should be zero.For all points other than the disease - free equilibrium point, the function should be positive and definite. The derivative of the Lyapunov function with respect to time, when multiplied by the derivative of the state variables, should be negative semi-definite at all points except the disease - free equilibrium point.

Prior to the introduction of non-linear variables, it is necessary to establish the stability of a system. Let $x = 0$ be an equilibrium point for $x = f(x)$, and $D \subset \mathbb{R}^n$ be a domain which contains $x = 0$.

1. Lyapunov Stability Theorem:
   It states that the origin is stable if, in the domain D that contains the origin, there is a continuously differentiable positive definite function $V(x)$ so that $\dot{V}$ is negative semi-definite, and is asymptotically stable if $\dot{V}$ is negative definite [17]. When V satisfies the condition for stability then V is known as the Lyapunov function.
2. Asymptotic Stability Theorem:
   If there exists a Lyapunov function $V(x)$ with the derivative $\frac{dV}{dt}$ that is negative definite for all X in the neighborhood U excluding $X = 0$, then the equilibrium point of an autonomous system is asymptotically stable [7]. To ensure the asymptotic stability of the zero solution, the total derivative must be strictly negative within the neighborhood of the origin.

2.3. *LaSalle's invariance principle*

The LaSalle's invariance principle [12, 19, 33] states that if a system possesses a Lyapunov function, which either diminishes or remains constant along its trajectories, then the system's trajectories will ultimately converge towards the most general invariant set within the region where the Lyapunov function attains zero.

By combining LaSalle's Principle with a Lyapunov function, one can not only identify the stability properties of the system but also determine the exact region in which the trajectories converge. This information is crucial for understanding the behavior and control of dynamical systems, especially in complex and non-linear systems where stability analysis can be challenging.

3. SIRMIS model

We present the SIR-based misinformation spreading model, SIRMIS model, in which we have false and true information sharing, and the suggested model is as follows:

$$\begin{aligned} \frac{dn}{dt} &= (1-p)\mu - \beta mn - \mu n \\ \frac{dm}{dt} &= \beta mn - \gamma m - \mu m \\ \frac{dr}{dt} &= \gamma m - \mu r \\ \frac{dg}{dt} &= p\mu - \mu g \end{aligned} \tag{4}$$

The variables for the SIRMIS model are defined as follows (Table 2):

t	Represents time.
n	Represents the social network users that have no information (for instance, about some particular event).
m	Represents the social network users that received the misinformation, and they are sharing it.
r	Represents the social network users that (received misinformation but later accepted the true information) stopped sharing the misinformation.
g	Represents the social network users that received the true information (despite they have true information, some of them will still share the misinformation after receiving it. This depends on the true information success sharing rate).
μ	Represents the rate of withdrawn social network users within the n group.
β	Represents the sharing rate of the misinformation.
γ	Represents the stop sharing rate of the misinformation.
p	Represents the sharing rate of the true information.

Table 2: Description of terms in the SIRMIS model

The equations given by Eq.(4) are system of differential equations that describes the dynamics of misinformation in a population with true information received. The equation describes how users with no information, misinformation, and true information accepted, change over time. The true information rate, p, affects the misinformation dynamics by reducing the no information population and the user reproduction number, which is the average number of misinformed users caused by one user who shares the misinformation.

The number of new misinformed users initiated by single user who shared the misinformation is given by

$$R_0 = \frac{\beta(1-p)}{\gamma + \mu} < 1 \tag{5}$$

The number of new misinformed users after the true information share parameter is p is introduced is given by

$$R_v = R_o(1-p) \tag{6}$$

The misinformation-free equilibrium is globally asymptotically stable, and sharing the misinformation eventually stops. When $R_0 > 1$, there is misinformation spread stable level equilibrium (endemic equilibrium). The system can be analyzed using linear stability analysis and Lyapunov functions to determine the stability of the misinformation-free equilibrium and endemic equilibrium.

$$n(t) + m(t) + r(t) + g(t) = 1 \tag{7}$$

$$r(t) = 1 - n(t) - m(t) - g(t) \tag{8}$$

We will obtain:

$$\frac{dn}{dt} = (1-p)\mu - \beta mn - \mu n \tag{9}$$

$$\frac{dm}{dt} = \beta mn - \gamma m - \mu m \tag{10}$$

$$\frac{dg}{dt} = p\mu - \mu g \tag{11}$$

The feasible region is given as:

$$\Omega_1 = \{(n(t), m(t), g(t)) \in \mathbb{R}^3_+ \,|\, n(t) + m(t) + g(t) \leq 1\}. \tag{12}$$

There are two equilibrium points:

1. Misinformation-free equilibrium point: $E_0(n = 1 - p, m = 0, g = p)$.

2. Misinformation spread stable level (endemic) equilibrium point:

$E_e\left(n = \frac{\gamma+\mu}{\beta}, m = \frac{\mu(\beta(1-p)-\gamma-\mu)}{\beta(\gamma+\mu)}, g = p\right)$

To show that E_e exist it is required to find the value of m from equation 9, and substitute it in equation 10, which results in

$$n^2 - n(1 - p + \frac{\gamma}{\beta} + \frac{\mu}{\beta}) + \left(\frac{(1-p)\gamma}{\beta} + \frac{(1-p)\mu}{\beta}\right) = 0 \tag{13}$$

The discriminant is given as:

$$\Delta = \left(1 - p + \frac{\gamma}{\beta} + \frac{\mu}{\beta}\right)^2 - 4\left(\frac{(1-p)\gamma}{\beta} + \frac{(1-p)\mu}{\beta}\right) \tag{14}$$

For $\Delta \geq 0$ we have:

$$\begin{gathered}
\left(1 - p + \frac{\gamma}{\beta} + \frac{\mu}{q} uad\beta\right)^2 \geq 4\left(\frac{(1-p)\gamma}{\beta} + \frac{(1-p)\mu}{\beta}\right) \\
\left(\frac{\beta(1-p)+(\gamma+\mu)}{\beta}\right)^2 \geq 4\left(\frac{(1-p)(\gamma+\mu)}{\beta}\right) \\
\left(\frac{\gamma+\mu}{\beta}\right)^2\left(1 + \frac{\beta(1-p)}{\gamma+\mu}\right)^2 \geq 4\frac{(1-p)(\gamma+\mu)}{\beta} \\
(1 + R_v)^2 \geq 4R_v \\
(R_v - 1)^2 \geq 0
\end{gathered} \tag{15}$$

which implies $R_v \geq 1$.

To show the linear stability of the misinformation-free equilibrium point and the misinformation spread stable level (endemic) equilibrium point, it is required to find the characteristic equation what corresponds to the misinformation-free equilibrium points as follows:

$$\begin{vmatrix} -\mu-\lambda & -\beta(1-p) & 0 \\ 0 & \beta(1-p)-\gamma-\mu-\lambda & 0 \\ 0 & 0 & -\mu-\lambda \end{vmatrix} = 0 \tag{16}$$

There are three eigenvalues:

$$\begin{aligned} \lambda_1 &= -\mu \\ \lambda_2 &= -\mu \\ \lambda_3 &= \beta(1-p)-\gamma-\mu \end{aligned} \tag{17}$$

The eigenvalues λ_1 and λ_2 are negative.
If $\lambda_3 > 0$ then $\beta(1-p) > \gamma+\mu$. Hence, $R_0(1-p) > 1$ or $R_v > 1$. This means that the misinformation-free equilibrium point is not locally asymptotically stable.
If $\lambda_3 < 0$ then $\beta(1-p) < \gamma+\mu$. Hence, $R_0(1-p) < 1$ or $R_v < 1$.

The model is linearly stable as all the eigenvalues are negative. There is no wide spread of misinformation and the trajectories will approach a misinformation-free equilibrium point.

To get a characteristic equation, the following determinant is obtained:

$$\begin{vmatrix} -\frac{\mu\beta(1-p)}{\gamma+\mu}-\lambda & -(\gamma+\mu) & 0 \\ \frac{\mu(\beta(1-p)-\gamma-\mu)}{\gamma+\mu} & -\lambda & 0 \\ 0 & 0 & -\mu-\lambda \end{vmatrix} = 0 \tag{18}$$

and we get:

$$(\mu+\lambda)\left(\lambda^2+\frac{\mu\beta(1-p)}{\gamma+\mu}\lambda+\mu[\beta(1-p)-\gamma-\mu]\right)=0 \tag{19}$$

The coefficients $\frac{\mu\beta(1-p)}{\gamma+\mu}$ and $\mu[\beta(1-p)-\gamma-\mu]$ are positive.

The eigenvalues are:

$$\begin{aligned} \lambda_1 &= -\mu \\ \lambda_2 &= -\frac{\mu\beta(1-p)}{\gamma+\mu} \pm \sqrt{\frac{\mu^2\beta^2(1-p)^2}{(\gamma+\mu)^2}-4\mu[\beta(1-p)-\gamma-\mu]} \\ \lambda_3 &= -\mu R_v \pm \sqrt{\mu^2R_v^2-4\mu(\gamma+\mu)(R_v-1)} \end{aligned} \tag{20}$$

Since $\mu[\beta(1-p)-\gamma-\mu]$ is positive, the value under the square root is either less than $\mu^2R_v^2$ or it is greater. If it is greater, the solutions are complex with negative real part $\frac{\mu\beta(1-p)}{\gamma+\mu}$.

Otherwise, the absolute value of the root must be less than $\mu^2 R_v^2$, but the real part of the eigenvalue is negative. In both cases the conclusion is that the misinformation spread stable level (endemic) equilibrium is stable because the real parts of both eigenvalues are negative and λ_1 is negative as well. Conclusion is that the endemic equilibrium point is locally stable. The misinformation sharing rate has reduced because of the true information sharing parameter.

Next, we will study the global stability of the misinformation-free equilibrium point and the endemic equilibrium point. To prove that the misinformation-free equilibrium point of the system is globally asymptotically stable on Ω_1 we will construct the Lyapunov function.

The Lyapunov function is:

$$V : \Omega_1 \to \mathbb{R}\,(V(n, m, g) = n(t) + m(t)) \tag{21}$$

The time derivative of V is given as:

$$\begin{aligned}\dot{V}(n, m, g) &= n(t) + m(t)\\ &= (1-p)\mu - \mu n - (\gamma+\mu)m\\ &= (\gamma+\mu)\left[\frac{(1-p)\mu}{\gamma+\mu} - \frac{\mu n}{\gamma+\mu} - m\right]\\ &= (\gamma+\mu)\left[\frac{\mu R_v}{\beta} - \frac{\mu R_0 n}{\beta} - m\right]\\ &= \frac{\gamma+\mu}{\beta}[\mu R_v - \mu R_0 n - \beta m]\end{aligned} \tag{22}$$

Thus, if $R_v < 1 \implies \dot{V}(n, m, g) < 0$. Moreover, at $(1-p, 0, g)\,, \dot{V}(n, m, g) = 0$

By LaSalle's invariance principle, the misinformation-free equilibrium point is globally asymptotically stable. To prove that the endemic equilibrium point $E^*(n^*, m^*, g^*)$ of the system is globally asymptotically stable on Ω_1 we use a Lyapunov function.

$L : \Omega_+ \to \mathbb{R}$, where $\Omega_+ = \{n(t), m(t) \in \Omega \,|\, n(t) > 0, m(t) > 0, g(t) > 0\}$ given by

$$L_1(n, m, g) = W_1\left[n - n^* \ln\left(\frac{n}{n^*}\right)\right] + W_2\left[m - m^* \ln\left(\frac{m}{m^*}\right)\right] + W_3\left[g - g^* \ln\left(\frac{g}{g^*}\right)\right] \tag{23}$$

where W_1,W_2,W_3 are positive constants. The time derivative of the Lyapunov function is:

$$\frac{dL_1}{dt} = W_1(n-n^*)\left[\frac{(1-p)\mu}{n} - \beta m - \mu\right] + W_2(m-m^*)\,[\beta n - \gamma - \mu] + W_3(g-g^*)\left[\frac{p\mu}{g} - \mu\right] \tag{24}$$

From considering equilibrium points, we get $\mu = \frac{(1-p)\mu}{n^*} - \beta^* m^*$, $\beta n^* = \gamma + \mu$, and $g^* = p$ and after substituting the values in the above equation, we get the following:

$$\frac{dL_1}{dt} = -\mu W_1(1-p)\frac{(n-n^*)^2}{nn^*} + \beta(W_2 - W_1)(m-m^*)(n-n^*) - W_3\mu(g-g^*)^2 \tag{25}$$

If $W_1 = W_2 = W_3 = 1$, then

$$\frac{dL_1}{dt} \leq -\mu W_1(1-p)\frac{(n-n^*)^2}{nn^*} - W_3\mu(g-g^*)^2 \leq 0 \tag{26}$$

and if $n = n^*$ and $g = g^*$ then $\frac{dL_1}{dt} = 0$.

Therefore, by the LaSalle invariance principle, the endemic equilibrium point (the spread level of misinformation) is globally asymptotically stable [33].

4. Discussion

The SIRMIS Model can be used to estimate the peak number of misinformed users, the duration of the sudden increase in the number of misinformed users, and the impacts of sharing the true information. The model simulates the transition of social network users between different categories over time.

The SIRMIS Model outline the rate of change of users in each category based on the misinformation sharing rate, true information acceptance rate, and non-informed social network users number where users can move between no-information, misinformation, true information accepted, and true information shared categories. The parameters μ, β, γ, and p represent the new users or stop sharing users rate, the misinformation sharing rate from misinformed user in a time period, true information acceptance rate, and true information sharing rate, respectively. The variables n, m, r and g represent the population sizes of non-informed, misinformed, truth accepted, and truth received users, respectively.

By understanding the dynamics of misinformation share researchers and social networks policymakers can assess the effectiveness of different control measures, predict the future course of the sudden increase in the number of misinformed users, and develop strategies to reduce the spread of misinformation within social networks.

We will discuss Figure 1. which is a graphical representation of the SIRMIS model.

The parameters in the model are $\mu = 0.5$, $\beta = 1.98$, $\gamma = 0.5$, and $p = 0.6$. The $R_v = 0.79 < 1$ point is stable.The eigenvalues corresponding to misinformation free equilibrium point are -0.5, -0.5 and -0.208. Therefore, it is linearly stable. After sharing true information, the users with misinformation decreased from 0.247 to 0.005 at the misinformation share rate $\beta = 1.98$ due to the effect of true information share. The results show that the group of users with no information decreased to a lower level due to share of true information and misinformation. Further, the number of misinformation-sharing users declined drastically because of the sharing of true information, and the number of users who accepted the true information increased. As we induce the true information on social network users with no information, the number of misinformed users suddenly decreases to a very low level. Furthermore, we can notice that the spread of misinformation is very fast, and for a particular example, it has a peak where it reaches around 8000 users, but with true information sharing it progressively decreases.

Therefore, we can conclude that the misinformation share rate and R_0 have a very important role in misinformation spreading and this can be controlled by true information impact.

Figure 2 illustrates the spread of misinformation without sharing the true information. We can conclude that the number of users n (with no information) decreases to a lower level because of effect of the misinformation sharing rate.

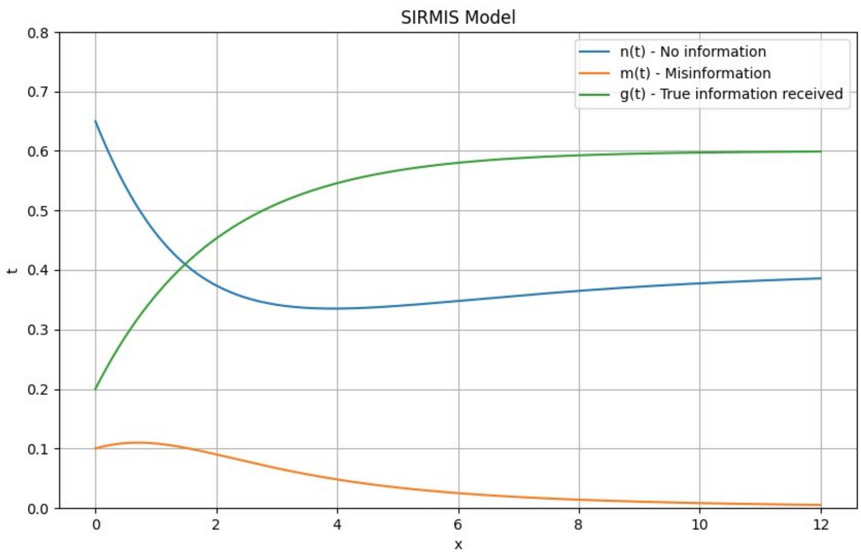

Figure 1: The spread of misinformation after sharing the true information

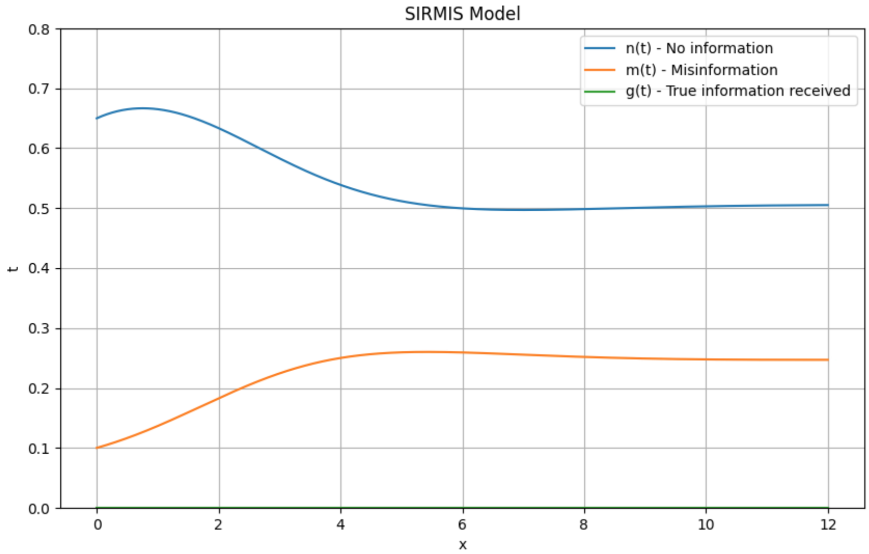

Figure 2: The spread of misinformation without sharing the true information

5. Conclusion

We have introduced a novel SIRMIS model based on SIR epidemic equations which can be effectively applied in the field of social networks misinformation spread control. The model detects the dynamics of misinformation sharing when users receive the true information. The system of equations is shown to be stable and allows one to predict the misinformation spread and potential prevention by sharing the true information. This system allows many predictions in time and outcomes of particular information sharing result in one closed group of social media users.

According to our SIRMIS model, the feasibility of controlling the misinformation spread on social networks critically depends on the values of R_0 and R_v. The more severe misinformation spreading (the greater value of R_0 or R_v), the more intensive must be the true information share in order to significantly reduce the misinformation spread on social networks.

In the next phase of this study, we intend to enhance our SIRMIS model by adopting an approach that involves the identification and engagement of targeted influential users, to whom will be shared the true information so that we can see the effect on the misinformation spreading control. These users will be meticulously selected based on their potential to effectively disseminate accurate information within their networks. By ensuring that true information is shared with these key individuals, we aim to observe and analyze its impact on controlling and mitigating the spread of misinformation. This extension of the model addresses the essential role of influential users in the dissemination of information and directly confronting the challenges posed by misinformation in today's digital landscape.

Our approach emphasizes the importance of targeted interventions in combating the proliferation of false information and underscores the critical potential of influential users in shaping the dynamics of information flow within online communities. The outcomes of this research are expected to make a contribution to the field of misinformation control, offering insights that can be applied across various contexts to strengthen the integrity of information ecosystems. In our future publication we will perform and present the additional evaluations for other network types, like small-word networks.

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