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A holistic perspective on soft separation axioms: Addressing open problems, rectifications, and introducing novel classifications

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Abstract. Soft topology establishes its importance as a frame of reference through numerous formulas derived for each classical topological concept, implying that classical topology is a special case obtained when the set of parameters is a singleton. In this work, we successfully solve two open problems concerning the relationships between two types of soft separation axioms in two categories. Then, we amend existing example showing that soft $T_0^B \nrightarrow$ soft T_0^S. In this context, we clarify that soft $T_0^B \rightarrow$ soft T_0^S if the set of parameters is finite. In contrast, we construct a soft topological structure with infinite set of parameters to illustrate that soft $T_0^B \nrightarrow$ soft T_0^S. Finally, we define a new form of soft points inspired by fuzzy points. Surprisingly, the new definition results in a spectrum of soft points that starts at ε_x and ends at $(x, \mathcal{P})$ for every $x \in \mathcal{U}$, where $\mathcal{P}$ is the set of parameters and $\mathcal{U}$ is the universe. We make use of this sort of soft points to create two classes of separation axioms via soft topologies: {soft T^0, soft T^1, soft T^2, soft T^3, soft T^4} and {soft T^{00}, soft T^{01}, soft T^{02}, soft T^{03}, soft T^{04}}. The master features of these axioms are scrutinized and the relationships between them as well as their relationships with the foregoing ones are revealed with the help of interesting counterexamples. Especially, we clarify that the axioms of soft T^S and soft T^E structures are special case of the current classes. Among the interesting results that we obtain are the identity between soft T^3 and classical T_3 structures and the equivalence between soft T^1 and soft T_1^B structures.

1. Introduction

In the real world, managing and modeling various sorts of vagueness is essential for addressing challenging issues across multiple areas, such as medicine, engineering, economics, environmental science, and social sciences. While present frameworks such as rough sets and fuzzy sets offer valuable instruments for treating ambiguity and uncertainty, each has limitations. A common shortcoming among these mathematical tools is the absence of adequate parameterization capabilities. To overcome this loophole, Molodtsov [28] put forward the idea of soft sets with various applications in different areas such as probability theory,

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game theory and operations research in 1999. Then, in 2002, Maji et al. [26] invested soft sets to cope with decision-making problems. In 2003, the notions of soft subset, complement, union and intersections were defined and studied by Maji et al. [27]. Ali et al. [6] suggested a new definition of the complement of a soft set and introduced the concepts of restricted union, and extended and restricted intersections between soft sets. To keep the properties of classical set theory in the frame of soft set theory, new types of soft subsets and soft unions were adopted by Abbas et al. [1] and Al-shami and El-Shafei [10].

In 2011, soft topology was declared by [32] and [19]. Soft separation axioms were first introduced in [23]. They followed different methodologies to set up topological structures via soft settings. Topology definition displayed by [32] is more consistent in keeping symmetry and periodicity of topological concepts. So, the discussion is conducted through this definition in this article. Following that, researchers, scholars, and intellectuals navigated the notions of basis [2], separation axioms [20, 31], covering [15, 24], connectedness [5], continuity [16, 38], cluster operator, homogeneity [4], expandable spaces [7, 30], embedding theorem [29], and metric structures [34] through soft topologies. To emphasize the role of soft topology as an applied framework, it has been employed some soft topological concepts to tackle practical problems in real life; see, [11].

Soft separation axioms are still one of the most active subjects in soft topologies, it has been defined several types such as those introduced in [13, 14, 31]. It is known that all separation axioms (except normality) separate points or points and closed sets, so the concept of point is essential in separation axioms definition. In soft set theory, the concept of soft points does not have a unified definition. The soft points of the form ε_F were introduced by [37], soft points of the form ε_x were given by [34] and soft points $(x, \mathcal{P})$ were presented by [32]. The concepts of soft belonging and soft none belonging, also, do not have a unified definition. The concepts $\widetilde{\in}$ and $\widetilde{\notin}$ (known as total belonging and partial none belonging) were initiated by [37], the concepts $\tilde{\in}_\varepsilon$ and $\tilde{\notin}_\varepsilon$ were familiarised by [22], and the concepts $\Subset$ and $\not\Subset$ (known as partial belonging and total none belonging) were explored by [21]. The diversity of definitions of soft points and belonging and none belonging relations produces different classes of soft separation axioms. Some of these classes are soft T_i^S [32], soft T_i^E [21], soft T_i^G [22], soft T_i^B [17], soft T_i^H [23], and soft T_i^T [35], where $i = 0, 1, 2, 3, 4$. Some of these classes behave similarly as the separation axioms do in general topology; for example, they have a soft version of the following set of implications in general topology:

$$T_4 \Rightarrow T_3 \Rightarrow T_2 \Rightarrow T_1 \Rightarrow T_0$$

Some authors ignored the uncertainty of soft topology and overlooked the factors that control separation axioms; types of soft points and relations. This resulted in generalized some classical results and relationships of topological concepts in mistake. Therefore, the authors of [8, 9, 25, 33] demonstrated these errors with some examples and counterexamples and derived the necessary conditions to preserve these results and relationships via soft topology. However, some errors are still existed. On the other hand, some counterexamples that show the failure of some implications have not been built. Moreover, some relationships between classes of soft separation axioms have not been discovered and are still open problems. The existence of the aforementioned obstacles encouraged us to write this manuscript, which we devoted to solving open problems and mending some invalid results of [8] as well as displaying novel classes of soft separation axioms with fuzzy flavor.

It is organized this work as following. After this introduction, it is recalled the different sorts of soft separation axioms and the results that are required to make the content is self-contained in Section 2. Then, we devoted Section 3 to answer the question "Is every soft T_i^B-structure soft T_i^E for $i = 2, 4$?", which exhibited as an open problem by monograph [8]. After that, in Section 4, we build a counterexample to show that the relationship between soft T_0^B and soft T_0^S structures investigated in [8] is not always true. We also prove that soft T_0^B structure is soft T_0^S providing that the set of parameters is finite. In Section 5, we offer a new version of soft points and applied to initiate two families of soft separation axioms. We examine the connections between them, as well as their connections to other classes of soft separation axioms. We discuss and analyze the results presented herein and propose future road for the upcoming works in Section 6.

2. Preliminaries

Some core definitions and primary results in the realms of soft sets and soft topologies are presented in this section aiming to comprehend the paper's findings.

Definition 2.1. ([28]) Consider the parameters set $\mathcal{P}$ and the universe $\mathcal{U}$, and let $2^{\mathcal{U}}$ be all subsets of $\mathcal{U}$. The *soft set F* on $\mathcal{U}$ is the function $F : \mathcal{P} \to 2^{\mathcal{U}}$ or, equivalently, $F = \{(\varepsilon, F(\varepsilon)); \varepsilon \in \mathcal{P}\}$. We will use the symbol $SS(\mathcal{U}, \mathcal{P})$ to refer the class of all soft sets on $\mathcal{U}$.

Definition 2.2. ([6, 18]) Consider the parameters set $\mathcal{P}$ and the universe $\mathcal{U}$, and take F and G as soft sets on $\mathcal{U}$. Then:

1. F is a *soft subset* of G (in symbols $F \widetilde{\subseteq} G$) if $F(\varepsilon) \subset G(\varepsilon)$ for all $\varepsilon \in \mathcal{P}$.
2. The *soft complement* of F is the soft set$F^{\widetilde{c}}$ s.t. $F^{\widetilde{c}}(\varepsilon) = \mathcal{U} - Fe)$.
3. The *soft difference* of F and G is the soft set $F - G$ s.t. $(F - G)(\varepsilon) = F(\varepsilon) - G(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$.
4. F and G are said *disjoint iff* $F(\varepsilon) \cap G(\varepsilon) = \emptyset$ for every $\varepsilon \in \mathcal{P}$.
5. $1_{\mathcal{P}}$ stands for the *universal soft set F* where $F(\varepsilon) = \mathcal{U}$ for every $\varepsilon \in \mathcal{P}$.
6. $0_{\mathcal{P}}$ stands for the *empty soft set F* where $F(\varepsilon) = \emptyset$ for every $\varepsilon \in \mathcal{P}$.

Definition 2.3. ([6]) If F_α is a soft set for every $\alpha \in \Delta$, then

1. their *soft union* is the soft set $G = \widetilde{\bigcup}_{\alpha\in\Delta} F_\alpha$ s.t. $G(\varepsilon) = \bigcup_{\alpha\in\Delta} F_\alpha(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$.
2. their *soft intersection* is the soft set $G = \widetilde{\bigcap}_{\alpha\in\Delta} F_\alpha$ s.t. $G(\varepsilon) = \bigcap_{\alpha\in\Delta} F_\alpha(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$.

Definition 2.4. ([19]) A sub-collection $\mathcal{O}$ from $SS(\mathcal{U}, \mathcal{P})$ is named a soft topology if the next stipulations are satisfied:

1. $0_{\mathcal{P}}, 1_{\mathcal{P}} \in \mathcal{O}$.
2. For every two (or finite) elements from $\mathcal{O}$, their soft intersection is an element of $\mathcal{O}$.
3. For arbitrary elements from $\mathcal{O}$, their soft union is an element of $\mathcal{O}$.

The 3-tuple $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is said a *soft topological structure*. F is called *open soft* if $F \in \mathcal{O}$, and it is called *closed soft* if $F^{\widetilde{c}} \in \mathcal{O}$.

Theorem 2.5. ([32]) *Let $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ be a soft topological structure. Then for every $\varepsilon \in \mathcal{P}$ the collection $\mathcal{O}_\varepsilon = \{U(\varepsilon); U \in \mathcal{O}$ and $\varepsilon \in \mathcal{P}\}$ is a topology for X.*

Soft points have different definitions, which lead to different approaches for soft separation axioms. We will exhibit the existing definitions of soft points.

Definition 2.6. ([37]) $F \in SS(\mathcal{U}, \mathcal{P})$ is said to be a *para-soft point* in $\mathcal{U}$ if there exists $\varepsilon \in \mathcal{P}$ s.t. $F(\acute{\varepsilon}) = \emptyset$ for every $\acute{\varepsilon} \neq \varepsilon$; para-soft points will be denoted by ε_F. ε_F is said to be in the soft set G (In symbols $\varepsilon_F \widetilde{\in} (G, B)$) if and only if $F(\varepsilon) \subset G(\varepsilon)$. Two para-soft points ε_F and ε'_G are said *distinct* (we write $\varepsilon_F \neq \varepsilon'_G$) if and only if $F(p) \cap G(p) = \emptyset$ for every $p \in \mathcal{P}$.

Proposition 2.7. ([37]) $\varepsilon_F \widetilde{\in} (G, B) \Longleftrightarrow \varepsilon_F \widetilde{\notin} (G, B)^{\widetilde{c}}$.

Another definition for soft points was introduced by [34] in his study for soft metric structures. We will call them strong-soft points.

Definition 2.8. ([34]) $F \in SS(\mathcal{U}, \mathcal{P})$ is said to be a *strong-soft point* in $\mathcal{U}$ if there exists $\varepsilon \in \mathcal{P}$ and $x \in \mathcal{U}$ s.t. $F(\acute{\varepsilon}) = \emptyset$ for every $\acute{\varepsilon} \neq \varepsilon$ and $F(\varepsilon) = \{x\}$. Strong-soft points will be denoted by ε_x. Two strong-soft point ε_x and $\acute{\varepsilon}_x$ are said distinct if and only if $x \neq y$ or $\varepsilon \neq \acute{\varepsilon}$.

Every strong-soft point ε_x is a para-soft point, since $\varepsilon_x = \varepsilon_F$ where $F(\varepsilon) = \{x\}$. The following is another definition for soft points.

Definition 2.9. ([32]) A soft set $F \in SS(\mathcal{U},\mathcal{P})$ is said to be a *whole-soft point* in $\mathcal{U}$ if there exists $x \in \mathcal{U}$ s.t. $F(\varepsilon) = \{x\}$ for every $\varepsilon \in \mathcal{P}$. Whole-soft points will be denoted by $(x,\mathcal{P})$.

$(x,\mathcal{P})\widetilde{\in}F$ if and only if $x \in F(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$. Two whole-soft points $(x,\mathcal{P})$ and $(y,\mathcal{P})$ are said distinct if and only if $x \neq y$.

Definition 2.10. Consider a soft set $F \in SS(\mathcal{U},\mathcal{P})$ and a whole-soft point $(x,\mathcal{P})$. Then

1. [21] $(x,\mathcal{P}) \Subset F$ if and only if $x \in F(\varepsilon)$ for some $\varepsilon \in \mathcal{P}$. And $(x,\mathcal{P}) \notin F$ if and only if $x \notin F(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$.
2. [22] For every $\varepsilon \in \mathcal{P}$, $(x,\mathcal{P}) \in_\varepsilon F$ if and only if $x \in F(\varepsilon)$. And $(x,\mathcal{P}) \notin_\varepsilon F$ if and only if $x \notin F(\varepsilon)$.

Soft separation axioms (briefly, SS-axioms) separate soft points, but we do not have a unified definition for soft points, so that we have different classes of SS-axioms. We will review the most important classes of SS-axioms.

SS-axioms first introduced in 2011.

Definition 2.11. ([32]) Let $(\mathcal{U},\mathcal{O},\mathcal{P})$ be a soft topological structure and let $(x,\mathcal{P})$ and $(y,\mathcal{P})$ be any two distinct whole-soft points in $\mathcal{U}$. Then

1. $(\mathcal{U},\mathcal{O},\mathcal{P})$ is called a soft T_0^S- *structure* providing that there is an open soft set F in $\mathcal{O}$ s.t. $(x,\mathcal{P})\widetilde{\in}F$ and $(y,\mathcal{P})\widetilde{\notin}F$, or $(y,\mathcal{P})\widetilde{\in}F$ and $(x,\mathcal{P})\widetilde{\notin}F$.
2. $(\mathcal{U},\mathcal{O},\mathcal{P})$ is called a soft T_1^S- *structure* providing that there is an open soft set F in $\mathcal{O}$ s.t. $(x,\mathcal{P})\widetilde{\in}F$ and $(y,\mathcal{P})\widetilde{\notin}F$.
3. $(\mathcal{U},\mathcal{O},\mathcal{P})$ is called a soft T_2^S- *structure* providing that there are disjoint open soft sets F and G in $\mathcal{O}$ s.t. $(x,\mathcal{P})\widetilde{\in}F$ and $(y,\mathcal{P})\widetilde{\in}G$.
4. $(\mathcal{U},\mathcal{O},\mathcal{P})$ is called a soft T_3^S- *structure* if and only if it is soft T_1^S and for any closed soft sets C with $(x,\mathcal{P})\widetilde{\notin}C$ there are disjoint open soft sets F and G in $\mathcal{O}$ s.t. $(y,\mathcal{P})\widetilde{\in}F$ and $C\widetilde{\subseteq}G$.
5. $(\mathcal{U},\mathcal{O},\mathcal{P})$ is called a soft T_4^S- *structure* if and only if it is soft T_1^S and for any soft disjoint closed sets C and K there are disjoint open soft sets F and G in $\mathcal{O}$ s.t. $C\widetilde{\subseteq}F$ and $K\widetilde{\subseteq}G$.

El-Shafei et al. [21] introduced T_i^E-structures for $i = 0,1,2,3,4$ be replacing $\widetilde{\in}$ in Definition 2.11 by $\Subset$. In [22], the authors displayed T_i^G-structures for $i = 0,1,2,3,4$ be replacing $\widetilde{\in}$ and $\widetilde{\notin}$ in Definition 2.11 by $\in_\varepsilon$ and $\notin_\varepsilon$, respectively and disjoint open sets F and G by $F(\varepsilon) \cap G(\varepsilon) = \emptyset$.

Another approach of SS-axioms is:

Definition 2.12. ([17]) Consider $(\mathcal{U},\mathcal{O},\mathcal{P})$ and let ε_x and $\acute{\varepsilon}_y$ be two distinct strong-soft points in $\mathcal{U}$. Then we say $(\mathcal{U},\mathcal{O},\mathcal{P})$ is:

1. A soft T_0^B-structure if and only if there is an open soft set U in $\mathcal{O}$ s.t. $\varepsilon_x\widetilde{\in}U$ and $\acute{\varepsilon}_y\widetilde{\notin}U$ or $\acute{\varepsilon}_y\widetilde{\in}U$ and $\varepsilon_x\widetilde{\notin}U$.
2. A soft T_1^B-structure if and only if there is an open soft set U in $\mathcal{O}$ with $\varepsilon_x\widetilde{\in}U$ and $\acute{\varepsilon}_y\widetilde{\notin}F$.
3. A soft T_2^B-structure if and only if there are disjoint open soft sets U and V in $\mathcal{O}$ with $\varepsilon_x\widetilde{\in}U$ and $\acute{\varepsilon}_y\widetilde{\notin}U$.
4. A soft T_3^B- *structure* if and only if it is soft T_1^B and for any closed soft sets C with $\varepsilon_x\widetilde{\notin}C$ there are disjoint open soft sets U and V in $\mathcal{O}$ s.t. $\varepsilon_x\widetilde{\in}U$ and $C\widetilde{\subseteq}V$.
5. A soft T_4^B- *structure* if and only if it is soft T_1^B and for any soft disjoint closed sets C and K there are disjoint open soft sets U and V in $\mathcal{O}$ s.t. $C\widetilde{\subseteq}U$ and $K\widetilde{\subseteq}V$.

In Definition 2.12, if the distinct strong-soft points ε_x and $\acute{\varepsilon}_y$ are replaced by ε_x and ε_y, then we will get a new class of SS-axioms T_i^T where $i = 0, 1, 2, 3, 4$; see [35], for their properties.

And if the distinct strong-soft points ε_x and $\acute{\varepsilon}_y$ are replaced by the whole-soft points ε_F and ε_G, then we will get a new class of SS-axioms T_i^H where $i = 0, 1, 2, 3, 4$; see [23], for their properties.

More classes of SS-axioms and about relations among them can be found in [8], where Al-Shami introduced a detailed study of this subject.

Theorem 2.13. ([8])

1. *soft* $T_3 \Rightarrow$ *soft* $T_2 \Rightarrow$ *soft* $T_1 \Rightarrow$ *soft* T_0 *where* $T = T^S, T^E, T^B, T^H$, T^G *and* T^T.
2. *soft* T_4*-structure* $\Rightarrow$ *soft* T_3*-structure only for* $T = T^E$ *and* T^B.
3. *soft* $T_2^E \Leftrightarrow$ *soft* T_2^S.
4. *soft* $T_0^T \Leftrightarrow$ *soft* T_0^G.
5. *soft* $T_1^H \Leftrightarrow$ *soft* $T_1^T \Leftrightarrow$ *Soft* T_1^G.
6. *soft* $T_1^B \Rightarrow$ *soft* T_1^H, *and soft* $T_1^B \Rightarrow$ *soft* T_1^S.

In topological structures: X is T_1 if and only if every singleton $\{x\}$ is a closed set in X. In soft topological structures, we have.

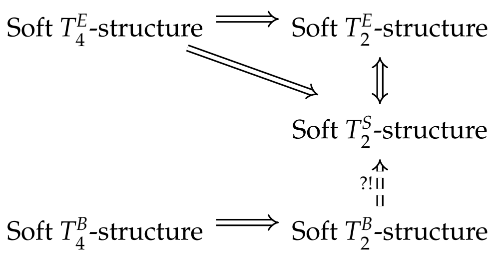
Theorem 2.14. *Let* $(\mathcal{U}, O, \mathcal{P})$ *be a soft topological structure. Then*

1. *([17])* $(\mathcal{U}, O, \mathcal{P})$ *is soft* T_1^B *if and only if* ε_x *is a closed soft set for every strong-soft point* ε_x *in* $\mathcal{U}$.
2. *([21])* $(\mathcal{U}, O, \mathcal{P})$ *is soft* T_1^E *if and only if* $(x, \mathcal{P})$ *is a closed soft set for every whole-soft point* $(x, \mathcal{P})$ *in* $\mathcal{U}$.
3. *([35]) If* ε_x *is a closed soft set for every strong-soft point* ε_x *in* $\mathcal{U}$, *then* $(\mathcal{U}, O, \mathcal{P})$ *is soft* T_1^T. *And the converse is not true.*
4. *([8]) If* ε_F *is a closed soft set for every par-soft point* ε_F *in* $\mathcal{U}$, *then* $(\mathcal{U}, O, \mathcal{P})$ *is soft* T_1^H. *And the converse is not true.*
5. *([32]) If* $(x, \mathcal{P})$ *is a closed soft set for every* $x \in \mathcal{U}$, *then* $(\mathcal{U}, O, \mathcal{P})$ *is soft* T_1^S. *And the converse is not true.*

3. Solve an open problem: "Is every soft T_i^B-structure soft T_i^E for $i = 2, 4$?"

The following problem appeared in [8].

Problem 1: Is every soft T_i^B-structure soft T_i^E for $i = 2, 4$?We will show that the answer of this problem is No! for $i = 2, 4$. Before we construct our example, see the following chart which shows relations between some SS-axioms (see Theorem2.13).



From the above chart a none soft T_2^S-structure is not a soft T_2^E-structure, a fortiori, not a soft T_4^E-structure. And any Soft T_4^B-structure is a soft T_2^B-structure. So to show that the answer of the above problem is No! for $i = 2, 4$, it suffices to construct a soft topological structure which is a soft T_4^B-structures but not a soft T_2^S-structure. See the following example.

Example 3.1. Consider the universe $\mathcal{U} = \{x, y\}$ and the set of parameters $\mathcal{P} = \{1, 2, 3, ...\}$. The strong-soft points will be denoted by n_x or n_y and will be considered as soft sets. Define $\mathcal{B}_1 = \{n_x, n_y; n = 2, 3, ...\}$, $\mathcal{B}_x = \{U_x^n; n = 2, 3, ...\}$ and $\mathcal{B}_y = \{U_y^n; n = 2, 3, ...\}$ s.t.:

$$U_x^n(m) = \begin{cases} x & \text{if } m = 1 \\ \emptyset & \text{if } m = 2, 3, ..., n \\ y & \text{if } m = n+1, n+2, ... \end{cases}$$

and

$$U_y^n(m) = \begin{cases} y & \text{if } m = 1 \\ \emptyset & \text{if } m = 2, 3, ..., n \\ x & \text{if } m = n+1, n+2, ... \end{cases}$$

Let $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_x \cup \mathcal{B}_y$. It easy to show that for any $U, V \in \mathcal{B}$ we have $U \widetilde{\cap} V \in \mathcal{B}$. Thus $\mathcal{B}$ is a soft base for soft topology O on $\mathcal{U}$ (O is all possible soft unions of elements form $\mathcal{B}$).

Claim 1: $(\mathcal{U}, O, \mathcal{P})$ is a soft T_2^B-structure.

Proof of Claim 1: Let n_z and m_w be two distinct strong-soft points. We have four cases:

1. $n \neq 1$ and $m \neq 1$: Then n_z and m_w are disjoint open soft sets containing n_z and m_w, respectively.
2. $n = 1$ and $m \neq 1$: Then U_z^{m+2} and m_w are disjoint open soft sets containing n_z and m_w, respectively.
3. $n \neq 1$ and $m = 1$: Then n_z and U_w^{n+2} are disjoint open soft sets containing n_z and m_w, respectively.
4. $n = 1$ and $m = 1$: Then U_z^2 and U_w^2 are disjoint open soft sets containing n_z and m_w, respectively.

which completes the proof of Claim 1.

Claim 2: $(\mathcal{U}, O, \mathcal{P})$ is a soft T_4^B-structure.

Proof of Claim 2: Let A and B be two disjoint closed soft sets in O. Again we have four cases:

1. $1_x \widetilde{\notin} A$ and $1_y \widetilde{\notin} A$: Then n_z ($n \neq 1$) is open soft set for each strong-soft point $n_z \in A$, which means A is an open soft set. But A is a closed soft set, thus $1_{\mathcal{U}} - A$ is an open soft set containing B. Hence A and $1_{\mathcal{U}} - A$ are disjoint open soft sets containing A and B respectively.
2. $1_x \widetilde{\notin} B$ and $1_y \widetilde{\notin} B$: Argue as we did in case 1 to get B and $1_{\mathcal{U}} - B$ are disjoint open soft sets containing B and A respectively.
3. $1_x \widetilde{\in} B$ and $1_y \widetilde{\in} A$: Then $1_x \widetilde{\in} (1_{\mathcal{U}} - A)$ and $1_y \widetilde{\in} (1_{\mathcal{U}} - B)$. Since $(1_{\mathcal{U}} - A)$ and $(1_{\mathcal{U}} - B)$ are open soft sets containing 1_x and 1_y respectively, there exist two open soft sets U_x^n and U_y^m s.t.

$$1_x \widetilde{\in} U_x^n \widetilde{\subset} (1_{\mathcal{U}} - A) \text{ and } 1_y \widetilde{\in} U_y^m \widetilde{\subset} (1_{\mathcal{U}} - B).$$

   It is clear that $U_x^n \widetilde{\cap} A = 0_{\mathcal{U}}$ and $U_y^m \widetilde{\cap} B = 0_{\mathcal{U}}$. Define

$$U_A = U_y^m \widetilde{\cup} A \quad \text{and} \quad U_B = U_x^n \widetilde{\cup} B.$$

   Then U_A is an open soft set, since each strong-soft point $\varepsilon_z \widetilde{\in} U_A$ is an open soft set except 1_y, but $1_y \widetilde{\in} U_y^m \widetilde{\subset} U_A$. Similarly we prove U_B is open soft. Now, since U_x^n and U_y^m are soft disjoint, and A and B are soft disjoint, we have U_A and U_B are disjoint open soft sets containing A and B, respectively.
4. $1_y \widetilde{\in} B$ and $1_x \widetilde{\in} A$: Argue as we did in case(3) to get two disjoint open soft sets U_A and U_B containing A and B, respectively.

And this proves the second claim.

Claim 3: $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is not a soft T_2^S-structure.
Proof of Claim 3: Consider the two distinct whole-soft points $(x, \mathcal{P})$ and $(y, \mathcal{P})$. Suppose, by contrapositive, there are disjoint open soft sets U and V s.t. $(x, \mathcal{P})\widetilde{\in}U$ and $(y, \mathcal{P})\widetilde{\in}V$. Then $x \in U(n)$ and $y \in V(n)$ for every $n = 1, 2, ...,$ and this implies (when $n = 1$) $1_x\widetilde{\in}U$ and $1_y\widetilde{\in}V$. But U and V are open soft, so there exist two open soft sets U_x^n and U_y^m s.t. $1_x\widetilde{\in}U_x^n\widetilde{\subset}U$ and $1_y\widetilde{\in}U_y^m\widetilde{\subset}V$. Since $U_x^n(n+1) = y \in U(n+1)$ and $y \in V(n+1)$, we have $U(n+1) \cap V(n+1) \neq \emptyset$. A contradiction with the soft disjointness of U and V. Thus $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is not a soft T_2^S-structure.

Now, since "Soft T_2^S-structure $\Leftrightarrow$ Soft T_2^E-structure", we have $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is not a soft T_2^E-structure. But T_4^E-structure $\Rightarrow$ T_2^E-structure, so $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is not a soft T_4^E-structure as well. And the answer of the above problem is no for $i = 2, 4$.

4. A correction about the relationship between soft T_0^B and soft T_0^S-structures

The following example was introduced by [8] as an example of a soft T_0^B-structure which is not soft T_0^S (see Figure 2 in page 1116 in [8]).

Example 4.1. ([8]) Consider the soft sets U and V over $\mathcal{U} = \{x, y\}$ under a set of parameters $\mathcal{P} = \{m_1, m_2\}$ defined as follows:
$U = \{(m_1, \{x\}), (m_2, \{y\})\}$ and $V = \{(m_1, \{y\}), (m_2, \{x\})\}$. Then the collection $\tau = \{0_{\mathcal{U}}, 1_{\mathcal{U}}, U, V\}$ forms a soft topology on $\mathcal{U}$.

According to Figure 2 in [8], $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ given the above example is a soft T_0^B-structure which is not soft T_0^S. This is incorrect, since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is not a soft T_0^B-structure, to see this, consider the two distinct soft points m_{1x} and m_{2y}. Any open soft set containing one of them contains the other.

We will show that in any soft topological structure $(\mathcal{U}, \mathcal{O}, \mathcal{P})$, if $\mathcal{P}$ is finite then $T_0^B \implies T_0^S$. Firstly, we prove this when $\mathcal{P}$ contains only two parameters.

Theorem 4.2. *Let $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ be a soft topological structure with $\mathcal{P} = \{a, b\}$. If $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure, then it is a soft T_0^S-structure.*

Proof. Suppose that $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure, and let $(x, \mathcal{P})$ and $(y, \mathcal{P})$ be distinct whole-soft points in $\mathcal{U}$. Consider the two strong-soft points a_x and a_y. Since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure, there is an open soft set U s.t. $a_x\widetilde{\in}U$ and $a_y\widetilde{\notin}U$, or $a_y\widetilde{\in}U$ and $a_x\widetilde{\notin}U$. We may assume that $a_x\widetilde{\in}U$ and $a_y\widetilde{\notin}U$. This leads to two cases

- Either $b_x\widetilde{\in}U$, then $(x, \mathcal{P})\widetilde{\in}U$ and $(y, \mathcal{P})\widetilde{\notin}U$ and we complete the proof.
- Or $b_x\widetilde{\notin}U$. Then, we consider two distinct strong-soft points b_x and a_y. Since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure, there is an open soft set V s.t. $b_x\widetilde{\in}V$ and $a_y\widetilde{\notin}V$, or $a_y\widetilde{\in}V$ and $b_x\widetilde{\notin}V$. That is,
  - If $b_x\widetilde{\in}V$ and $a_y\widetilde{\notin}V$, then $H = U\widetilde{\cup}V$ is an open soft set with $(x, \mathcal{P})\widetilde{\in}H$ and $(y, \mathcal{P})\widetilde{\notin}H$ (Since $a_y\widetilde{\notin}U$ and $a_y\widetilde{\notin}V$) which completes the proof.
  - If $b_x\widetilde{\notin}V$ and $a_y\widetilde{\in}V$, then $H = U\widetilde{\cup}V$ is an open soft set with $b_x\widetilde{\notin}H$.
    * If $b_y\widetilde{\in}H$, then H is an open soft set with $(y, \mathcal{P})\widetilde{\in}H$ and $(x, \mathcal{P})\widetilde{\notin}H$ (Since $b_x\widetilde{\notin}U$ and $b_x\widetilde{\notin}V$), and done.
    * And if $b_y\widetilde{\notin}H$, then we consider the two distinct strong-soft points b_x and b_y. Again, since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure, then there is an open soft set W s.t. $b_x\widetilde{\in}W$ and $b_y\widetilde{\notin}W$, or $b_y\widetilde{\in}W$ and $b_x\widetilde{\notin}W$. If $b_x\widetilde{\in}W$ and $b_y\widetilde{\notin}W$, then $S = H\widetilde{\cup}W$ is an open soft set with $(x, \mathcal{P})\widetilde{\in}S$ and $(y, \mathcal{P})\widetilde{\notin}S$. And if $b_y\widetilde{\in}W$ and $b_x\widetilde{\notin}W$, then $(y, \mathcal{P})\widetilde{\in}S$ and $(x, \mathcal{P})\widetilde{\notin}S$. Which completes the proof.

□

Now, we will prove the above theorem when $\mathcal{P}$ is any finite set of parameters. But we need to defined a new kind of soft point.

Definition 4.3. Let $\mathcal{U}$ and $\mathcal{P}$ be a universe and a set of a parameters, respectively. For any $E \subseteq \mathcal{P}$ let E^x to be the soft set F s.t. $F(\varepsilon) = \{x\}$ if $\varepsilon \in E$, and $F(x) = \emptyset$ otherwise.

It is clear that $E^x = \widetilde{\cup}\{\varepsilon_x; \varepsilon \in E\}$, and since $(x, \mathcal{P}) = \widetilde{\cup}\{\varepsilon_x; \varepsilon \in \mathcal{P}\}$, we have $E^x \widetilde{\subset} (x, \mathcal{P})$.

Lemma 4.4. *Let $(\mathcal{U}, O, \mathcal{P})$ be a soft topological structure. If $(\mathcal{U}, O, \mathcal{P})$ is a soft T_0^B-structure, then for any $x \neq y$ and any finite $E \subset \mathcal{P}$ there is an open soft set U s.t. $E^x \widetilde{\in} U$ and $E^y \widetilde{\notin} U$, or $E^x \widetilde{\notin} U$ and $E^y \widetilde{\in} U$.*

Proof. We will proceed by induction on $|E|$ (the number of elements of E). If E has only two elements, then the proof is similar to the proof of Theorem 4.2. Now, suppose that the theorem is true for any $E \subset \mathcal{P}$ with $|E| < n$. We shall show that the theorem is true when $|E| = n$. Let $a \in E$ and set $\acute{E} = E - \{a\}$. Then $|\acute{E}| < n$, so from the assumption, there is an open soft set U s.t. $\acute{E}^x \widetilde{\in} U$ and $\acute{E}^y \widetilde{\notin} U$, or $\acute{E}^x \widetilde{\notin} U$ and $\acute{E}^y \widetilde{\in} U$. We may assume that $\acute{E}^x \widetilde{\in} U$ and $\acute{E}^y \widetilde{\notin} U$. If the strong soft point $a_x \widetilde{\in} U$, then $E^x \widetilde{\in} U$ and $E^y \widetilde{\notin} U$, and we done. Suppose that $a_x \widetilde{\notin} U$, then for any $\varepsilon \in E$ with $\varepsilon_y \widetilde{\notin} U$ we have $\varepsilon_y \neq a_x$. Since $(\mathcal{U}, O, \mathcal{P})$ is soft T_0^B, there is an open soft set U_ε s.t. $a_x \widetilde{\in} U_\varepsilon$ and $\varepsilon_y \widetilde{\notin} U_\varepsilon$, or $a_x \widetilde{\notin} U_\varepsilon$ and $\varepsilon_y \widetilde{\in} U_\varepsilon$. That is, we have two cases:
Case I: $a_x \widetilde{\in} U_\varepsilon$ and $\varepsilon_y \widetilde{\notin} U_\varepsilon$ for some $\varepsilon \in E$ with $\varepsilon_y \widetilde{\notin} U$. Then $H = U \widetilde{\cup} U_\varepsilon$ is an open soft set with $E^x \widetilde{\in} U$ and $E^y \widetilde{\notin} U$.
Case II: $a_x \widetilde{\notin} U_\varepsilon$ and $\varepsilon_y \widetilde{\in} U_\varepsilon$ for ever $\varepsilon \in E$ with $\varepsilon_y \widetilde{\notin} U$. Then

$$H = U \widetilde{\cup} (\widetilde{\cup}\{U_\varepsilon; \varepsilon \in E, \varepsilon_y \widetilde{\notin} U\})$$

is an open soft set with $E^x \widetilde{\notin} H$ (since $a_x \notin H$) and $E^y \widetilde{\in} H$. And the proof is completed. □

Corollary 4.5. *Let $(\mathcal{U}, O, \mathcal{P})$ be a soft topological structure with $\mathcal{P}$ is finite. If $(\mathcal{U}, O, \mathcal{P})$ is a soft T_0^B-structure, then it is a soft T_0^S-structure.*

The following example shows that if $\mathcal{P}$ is infinite, then $T_0^B \nRightarrow T_0^S$.

Example 4.6. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{1, 2, 3, ...\}$. Since each soft set is the union of its strong-soft points, the following soft sets are well-defined where i_x and i_y denote strong-soft points in $\mathcal{U}$.

$$U_n = \widetilde{\cup}\{i_x, j_y; i \text{ is odd }, j \text{ is even and } i, j \leq n\}.$$

$$V_n = \widetilde{\cup}\{i_x, j_y; i \text{ is even }, j \text{ is odd and } i, j \leq n\}.$$

See Figure 1 to understand how the soft sets U_n and V_n are defined where U_3, U_6, V_2 and V_7 are presented.

Let $\beta = \{0_\mathcal{U}, 1_\mathcal{U}, U_n, V_n; n = 1, 2, ...\}$. Then β is a soft base for some soft topologies on $\mathcal{U}$, since the following

1. $U_n \widetilde{\cap} V_m = 0_\mathcal{U}$ for every n, m.
2. $U_n \widetilde{\cap} U_m = U_t$ where $t = \min\{n, m\}$.
3. $V_n \widetilde{\cap} V_m = V_t$ where $t = \min\{n, m\}$.

Let O be the soft topology generated on $\mathcal{U}$ by the soft base β. For sake of later applications we define

$$U = \widetilde{\bigcup}_{n=1}^{\infty} U_n \text{ and } V = \widetilde{\bigcup}_{n=1}^{\infty} V_n.$$

Note that U and V are open soft sets, $U \widetilde{\cup} V = 1_\mathcal{P}$ and $U \widetilde{\cap} V = 0_\mathcal{P}$.

$$\text{Since } (x, \mathcal{P}) = \widetilde{\cup}\{i_x; i = 1, 2, ...\} \text{ and } (y, \mathcal{P}) = \widetilde{\cup}\{i_y; i = 1, 2, ...\},$$

the only open soft set containing $(x, \mathcal{P})$ or $(y, \mathcal{P})$ is $1_\mathcal{P}$. Which implies $(\mathcal{U}, O, \mathcal{P})$ is not soft T_0^S.
It remains to show that $(\mathcal{U}, O, \mathcal{P})$ is soft T_0^B. Let i_a and j_b be two distinct strong-soft points. We have the following cases.

1. $i_a \in U$ and $j_b \in V$, or $i_a \in V$ and $j_b \in U$. Then we done, since U and V are disjoint open soft sets.
2. $i_a \in U$ and $j_b \in U$. Let $t = \min\{i, j\}$. Then U_t contains (from construction) only one of the strong soft points i_a and j_b.
3. $i_a \in V$ and $j_b \in V$. Let $t = \min\{i, j\}$. Then V_t contains (from construction) only one of the strong soft points i_a and j_b.

Hence $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T_0^B-structure.

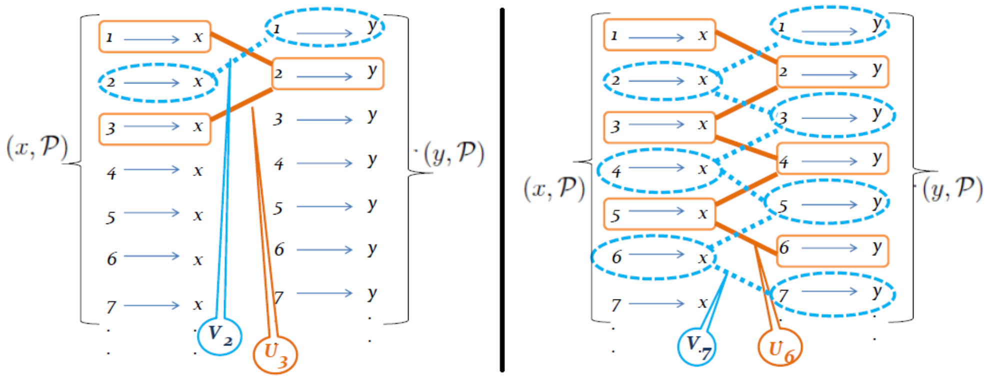


Figure 1: Basic open soft sets in Example 4.6

5. New approaches for soft points and soft separation axioms

Here, we will introduce a new version of soft points, which is inspired from Molodtsov first paper about soft sets [28] and the concept of fuzzy points. Fuzzy points first introduced by Wong in 1974 as follows.

Definition 5.1. ([36]) Let $x_\circ \in U$ and $\gamma \in (0, 1]$. Then a fuzzy point $P_{x_\circ}^{\gamma}$ is a fuzzy set A with the membership function $\mu_A : U \to [0, 1]$ s.t. $\mu_A(x_\circ) = \gamma$ and $\mu_A(x) = 0$ for every $x \neq x_\circ$. If $\gamma = 1$, then $P_{x_\circ}^{1}$ is called a crisp point.

In [28], Molodtsov showed that every fuzzy set A with a membership function $\mu_A : U \to [0, 1]$ is a soft set as follows:
For every $\alpha \in [0, 1]$ let $F_\alpha = \{x \in U; \mu_A(x) \geq \alpha\}$ be the α-level sets of A. We can redefine μ_A by means of the following definition:

$$\mu_A(x) = \sup_{\alpha \in [0,1]} \{\alpha; x \in F_\alpha\}$$

So the fuzzy set A is the soft set $G_A : [0, 1] \to 2^U$ s.t. $G_A(\alpha) = F_\alpha$ for every $\alpha \in [0, 1]$.
In the above discussion if we replace the fuzzy set A by the fuzzy point $A = P_{x_\circ}^{\gamma}$, then the α-level sets of A come

$$F_\alpha = \{x \in U; \mu_A(x) \geq \alpha\} = \begin{cases} \emptyset & \text{if } \alpha > \gamma \\ \{x_\circ\} & \text{if } \alpha \leq \gamma \end{cases}$$

Thus $P_{x_\circ}^{\gamma}$ is the soft set $G : [0, 1] \to 2^U$ s.t. $G(\alpha) = \emptyset$ if $\alpha > \gamma$ and $G(\alpha) = \{x_\circ\}$ if $\alpha \leq \gamma$.
If $\gamma = 1$, then $P_{x_\circ}^{1}$ is a crisp point, so it become the soft set $G : [0, 1] \to 2^U$ s.t. $G(\alpha) = \emptyset$ if $\alpha > 1$ and $G(\alpha) = \{x_\circ\}$ if $\alpha \leq 1$. Since there is no $\alpha > 1$, we have $G(\alpha) = \{x_\circ\}$ for every $\alpha \in [0, 1]$. The above discussion suggests the following new definition for soft points.

Definition 5.2. Let $\mathcal{U}$ be a universe and $\mathcal{P}$ be a set of parameters. A soft set G is said to be a soft point if and only if there exists $x \in \mathcal{U}$ and a nonempty subset E of $\mathcal{P}$ s.t.:

$$G(\varepsilon) = \begin{cases} x & \text{if } \varepsilon \in E \\ \emptyset & \text{if } e \notin E \end{cases}$$

Such soft points will be denoted by E^x where $E \subset \mathcal{P}$ is nonempty. If $E = \mathcal{P}$, then E^x is said a *crisp soft point* and will be denoted by $\mathcal{P}^x$. If $E = \{\varepsilon\}$ for some $\varepsilon \in \mathcal{P}$, then E^x will be called a *soft atom* and will be denoted by e^x.

Remarks about Definition 5.2:

1. Strong soft points ε_x (see Definition 2.8) are the soft atoms, since $\varepsilon_x = e^x$.
2. Whole soft points $(x, \mathcal{P})$ (see Definition 2.10) are the crisp soft points, since $\mathcal{P}^x = (x, \mathcal{P})$.
3. The new definition of soft points is formulated in Definition 4.3 and used in the proof of Lemma 4.4 which shows the significance of the new definition.

The following example is helpful to understand the new definition of soft points.

Example 5.3. Consider the set of parameters $\mathcal{P} = \{a, b, c\}$ and the universe $U = \{x, y, z\}$. Then

1. $A = \{(a, \{x\}), (b, \emptyset), (c, \{x\})\}$ is a soft point which is neither a strong soft point nor a whole soft point. If we let $E = \{a, c\}$, then $A = E^x$.
2. $A = \{(a, \{x\}), (b, \emptyset), (c, \{y\})\}$ is not a soft point, since $x \neq y$.
3. $A = \{(a, \{x\}), (b, \emptyset), (c, \emptyset)\}$ is a soft atom and we denote it by a^x.
4. $A = \{(a, \{z\}), (b, \{z\}), (c, \{z\})\}$ is a crisp soft point, and we denote it by $\mathcal{P}^z$.

Definition 5.4. Take $\mathcal{U}$ and $\mathcal{P}$ as a universe and a set of parameters, respectively.

1. We terminologies that a soft point E^x belongs to the soft set G (in symbols $E^x \widetilde{\in} G$) if and only if $E^x \widetilde{\subset} G$; that is $x \in G(\varepsilon)$ for every $\varepsilon \in E$.
2. Two soft points E^x and $\acute{E}^y$ are said to be distinct if and only if E^x and $\acute{E}^y$ are disjoint soft sets; that is $E^x(\varepsilon) \cap \acute{E}^y(\varepsilon) = \emptyset$ for every $\varepsilon \in \mathcal{P}$.

Proposition 5.5. *The disjointness of soft points E^x and $\acute{E}^y$ holds providing that $x \neq y$ or $E \cap \acute{E} = \emptyset$.*

Proof. Suppose E^x and $\acute{E}^y$ are distinct. If, by contrapositive, $x = y$ and $E \cap \acute{E} \neq \emptyset$, then thereare $\varepsilon \in E \cap \acute{E}$ with $x \in E^x(\varepsilon)$ and $y \in \acute{E}^y(\varepsilon)$. But $x = y$, so $x \in E^x(\varepsilon) \cap \acute{E}^y(\varepsilon)$. A contradiction with $E^x(\varepsilon) \cap \acute{E}^y(\varepsilon) = \emptyset$.
Conversely; suppose that $x \neq y$ or $E \cap \acute{E} = \emptyset$. Then, $E^x(\varepsilon) \cap \acute{E}^y(\varepsilon) = \emptyset$, which means E^x and $\acute{E}^y$ are distinct. □

Definition 5.6. Take $\mathcal{U}$ and $\mathcal{P}$ as a universe and a set of parameters, respectively. Then

1. $E^x \widetilde{\notin} G$ if and only if for some $\varepsilon \in E$ we have $x \notin G(\varepsilon)$.
2. $E^x \notin G$ if and only if for each $\varepsilon \in E$ we have $x \notin G(\varepsilon)$.

The following is obvious!

Theorem 5.7. *Let $\mathcal{U}$ be a universe and $\mathcal{P}$ be a set of parameters. For any soft set F we have $F = \bigcup\{E^x; E^x \widetilde{\in} F\}$. i.e. F consists of all soft points in it.*

The new definition of soft points leads to two new classes for SS-axioms. The first class is:

Definition 5.8. Let E^x and $\acute{E}^y$ be any two distinct soft points in $(\mathcal{U}, O, \mathcal{P})$.

1. If there is an open soft set F in O s.t. $E^x \widetilde{\in} F$ and $\acute{E}^y \widetilde{\notin} F$, or $E^x \widetilde{\notin} F$ and $\acute{E}^y \widetilde{\in} F$, then $(\mathcal{U}, O, \mathcal{P})$ is named a soft T^0-structure.
2. If there is an open soft set F in O s.t. $E^x \widetilde{\in} F$ and $\acute{E}^y \widetilde{\notin} F$, then $(\mathcal{U}, O, \mathcal{P})$ is named a soft T^1-structure.

3. If there are disjoint open soft sets F and H in $\mathcal{O}$ s.t. $E^x\widetilde{\in}F$ and $\acute{E}^y\widetilde{\in}H$, then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is named a soft T^2-structure.

And the other class is:

Definition 5.9. Let E^x and $\acute{E}^y$ be any two distinct soft points in $(\mathcal{U},\mathcal{O},\mathcal{P})$.

1. If there is an open soft set F in $\mathcal{O}$ s.t. $E^x\widetilde{\in}F$ and $\acute{E}^y \notin F$, or $E^x \notin F$ and $\acute{E}^y\widetilde{\in}F$, then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is named a soft T^{00}-structure.
2. If there is an open soft set F in $\mathcal{O}$ s.t. $E^x\widetilde{\in}F$ and $\acute{E}^y \notin F$, then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is named a soft T^{01}-structure.
3. If there are disjoint open soft sets F and H in $\mathcal{O}$ s.t. $E^x\widetilde{\in}F$ and $\acute{E}^y\widetilde{\in}H$, then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is named a soft T^{02}-structure.

The following are obvious.

Proposition 5.10. 1. *Every soft T^2-structure (T^{02}-structure) is a soft T^1 (T^{01}-structure), and every soft T^1-structure (T^{01}-structure) is a soft T^0-structure (T^{00}-structure).*
2. *Every soft T^i-structure is a soft T_i^B-structure for each $i = 0, 2$ and a soft T_i^S-structure for each $i = 0, 1, 2$.*

Examples of a soft T^0-structure (T^{00}-structure) which is not soft T^1 (T^{01}-structure), and a soft T^1-structure (T^{01}-structure) which is not soft T^2 (T^{02}-structure), can be obtained by letting the set of parameters to be a singleton, see the following example.

Example 5.11. Let τ_1 and τ_2 be two (classical) topological structures on X and Y, respectively, s.t. τ_1 is a T_0-structure but not T_1 and τ_2 is a T_1-structure but not T_2. Let $\mathcal{P} = \{\varepsilon\}$ be the set of parameters. Define $\mathcal{O}_1 = \{(\varepsilon, U); U \in \tau_1\}$ and $\mathcal{O}_2 = \{(\varepsilon, U); U \in \tau_2\}$. Then The soft topological structure $(X,\mathcal{O}_1,\mathcal{P})$ is a soft T^{00}-structure and a soft T^0-structure but not a soft T^{01}-structure nor a soft T^1-structure, and $(Y,\mathcal{O}_2,\mathcal{P})$ is a soft T^{01}-structure and a soft T^1-structure but not a soft T^{02}-structure nor a soft T^2-structure.

The following theorem shows that T^1 and T_1^B are equivalent.

Theorem 5.12. *Let $(\mathcal{U},\mathcal{O},\mathcal{P})$ be a soft topological structure. Then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is a soft T^1-structure if and only if it is a soft T_1^B-structure.*

Proof. The necessary part is obvious.

To prove the sufficient part, suppose that $(\mathcal{U},\mathcal{O},\mathcal{P})$ is a soft T_1^B-structure and let E_1^x and E_2^y be two distinct soft points in $\mathcal{U}$. Let $\varepsilon \in E_1$. We want to separate the soft points ε_x and E_2^y. For every $a \in E_2$, ε_x and a_y are distinct strong-soft points. Since $(\mathcal{U},\mathcal{O},\mathcal{P})$ is a soft T_1^B-structure, there exist two open soft sets U_a and V_a s.t. $\varepsilon_x \in U_a$, $\varepsilon_x \notin V_a$, $a_y \in V_a$ and $a_y \notin U_a$. Define

$$V = \bigcup_{a\in E_2} V_a$$

Pick any $a \in E_2$. Then U_a and V are open soft sets with $\varepsilon_x\widetilde{\in}U_a$, $\varepsilon_x\widetilde{\notin}V$, $E_2^y\widetilde{\in}V$ and $E_2^y\widetilde{\notin}U_a$ (since $a_y\widetilde{\notin}U_a$).

Similarly, let $\varepsilon' \in E_2$. Then there are open soft sets V_b and U s.t. $\varepsilon'_y\widetilde{\in}V_b$, $\varepsilon'_y\widetilde{\notin}U$, $E_1^x\widetilde{\in}U$ and $E_1^x\widetilde{\notin}V_b$ for some $b \in E_1$. It is clear that U and V are open soft sets with $E_1^x\widetilde{\in}U$, $E_1^x\widetilde{\notin}V$ (since $\varepsilon_x\widetilde{\notin}V$), $E_2^y\widetilde{\in}V$ and $E_2^y\widetilde{\notin}U$ (since $\varepsilon'_y\widetilde{\notin}U$). Thus $(\mathcal{U},\mathcal{O},\mathcal{P})$ is a soft T^1-structure. □

Theorem 2.14 part (1) and Theorem 5.12 implies the following.

Corollary 5.13. *Let $\mathcal{P}$ be a finite set, $(\mathcal{U},\mathcal{O},\mathcal{P})$ be a soft topological structure. Then $(\mathcal{U},\mathcal{O},\mathcal{P})$ is a soft T^1-structure if and only if every strong soft point ε_x is a closed soft set.*

Since $T^0 \to T_0^B$ and $T_0^S \nrightarrow T_0^B$, we have $T_0^S \nrightarrow T^0$. And Since $T^2 \to T_2^B$ and $T_2^S \nrightarrow T_2^B$, we have $T_2^S \nrightarrow T^2$.

Again, since $T^0 \to T_0^S$ and $T_0^B \nrightarrow T_0^S$, we have $T_0^B \nrightarrow T^0$. And Since $T^2 \to T_2^S$ and $T_2^B \nrightarrow T_2^S$, we have $T_2^B \nrightarrow T^2$.

The following example shows that $T_2^E \nrightarrow T^0$, which implies $T_2^E \nrightarrow T^2$, $T_2^E \nrightarrow T^{02}$, $T_1^E \nrightarrow T^1$, $T_1^E \nrightarrow T^{01}$, $T_0^E \nrightarrow T^0$ and $T_0^E \nrightarrow T^{00}$.

Example 5.14. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{a, b\}$. Then $O = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, (x, \mathcal{P}), (y, \mathcal{P})\}$ is a soft topology on $\mathcal{U}$. It is clear that O is soft T_2^E-structure, since the only two distinct soft-whole points are $(x, \mathcal{P})$ and $(y, \mathcal{P})$ which are open subsets of $\mathcal{U}$. But O is not soft T^0, since the distinct strong-soft points a_x and b_x belongs to the same open soft sets in O (which, also, shows that O is not even a T_0^B).

Easily, one obtain the proof of the following.

Proposition 5.15. 1. *Every T^{00}-structure is a T^0-structure, and every T^{01}-structure is a T^1-structure.*
2. $T^{02} \Longleftrightarrow T^2$.

The converse sides of the above proposition is false in general. The following example support this fact.

Example 5.16. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{a, b\}$. Then $O = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, U, V, H, K\}$ s.t. $U = \{(a, \{x, y\}), (b, \{y\})\}$, $V = \{(a, \{x, y\}), (b, \{x\})\}$, $H = U \widetilde{\cap} V = \{(a, \{x, y\}), (b, \emptyset)\}$ and , $K = \{(a, \{y\}), (b, \emptyset)\}$. It clear that O is a soft topology on $\mathcal{U}$. O is a soft T^0-structure (one can prove this by considering all possible cases of distinct soft points E^x and $\acute{E}^y$). To show that O is not a soft T^{00}-structure we consider the soft points E^x and E^y with $E = \mathcal{P}$ (that is $E^x = (x, \mathcal{P})$ and $E^y = (y, \mathcal{P})$). The only open soft set containing E^x not E^y is V, but it is not true that $E^y \notin V$. And the only open soft set containing E^y not E^x is U, but it is not true that $E^x \notin U$. Thus O is T^0 not T^{00}.

Example 5.17. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{1, 2, ...\}$. Define O s.t. $U \in O$ if and only if $U = 0_{\mathcal{P}}$ or $1_{\mathcal{P}} - U$ contains a finite number of strong-soft points. One can easily show that O is a soft topology. To show that O is a soft T^1-structure. Let E^s and $\acute{E}^t$ be two distinct soft points s.t. $s, t \in \{x, y\}$ and $E, \acute{E} \subset \{1, 2, ...\}$. Let $i_s \widetilde{\in} E^s$ and $j_t \widetilde{\in} \acute{E}^t$. Define $U = 1_{\mathcal{P}} - \{j_t\}$ and $V = 1_{\mathcal{P}} - \{i_s\}$. Then U and V are open soft sets. Since E^s and $\acute{E}^t$ are distinct, we have $E^s \widetilde{\in} U$, $\acute{E}^t \widetilde{\in} V$, $E^s \widetilde{\notin} V$ and $\acute{E}^t \widetilde{\notin} U$. Thus O is a soft T^1-structure. Now, we will show that O is not a soft T^{00}-structure. Consider the two distinct soft points E^x and E^y s.t. $E = \mathcal{P}$ (note that $E^x = (x, \mathcal{P})$ and $E^y = (y, \mathcal{P})$). If U is an open soft set soft containing E^x, then $1_{\mathcal{U}} - U$ contains a finite number of strong-soft points. But E^y contains infinitely many strong-soft points, it follows: $E^y \notin U$ is false. Which implies that O is not a soft T^{00}-structure. Moreover, since $E^x = (x, \mathcal{P})$ and $E^y = (y, \mathcal{P})$, we have O is not a soft T_0^E structure. Thus we, also, conclude that $T^1 \nrightarrow T_0^E$.

Figure 2 summarize our previous results about different classes of SS-axioms. Some implications are well known and belongs to other authors.

Being soft T^1 does not imply that every soft point E^x is closed soft. The soft structure in Example 5.17 is soft T_1. But the soft point $\mathcal{P}^x = (x, \mathcal{P})$ is not a closed soft set.

Theorem 5.18. *$(\mathcal{U}, O, \mathcal{P})$ is a soft T^{01}-structure iff any soft point E^x is closed soft.*

Proof. **Sufficiency:** Take E^x and $\acute{E}^y$ as distinct soft points. Then $U = 1_{\mathcal{P}} - \acute{E}^y$ and $V = 1_{\mathcal{P}} - E^x$ are open soft sets with $E^x \widetilde{\in} U$, $\acute{E}^y \widetilde{\in} V$, $\acute{E}^y \notin U$ and $E^x \notin V$. Thus O is a soft T^{01}-structure.

Necessity: Let E^x be any soft point. To show that E^x is a closed soft subset of $\mathcal{U}$. For any $\varepsilon_y \widetilde{\in} 1_{\mathcal{U}} - E^x$ we have ε_y and E^x are distinct soft points. So there are open soft sets U and V s.t. $\varepsilon_y \widetilde{\in} U$, $E^x \widetilde{\in} V$, $\varepsilon_y \notin V$, $E^x \notin U$. Since $E^x \notin U$, we have $U \widetilde{\cap} E^x = 0_{\mathcal{U}}$. It follows that $1_{\mathcal{U}} - E^x$ is an open soft set, thus E^x is a closed soft set, and the proof is completed. □

Definition 5.19. We name $(\mathcal{U}, O, \mathcal{P})$:

1. A soft regular structure providing that for any soft point E^x and any closed soft set F s.t. $E^x \widetilde{\notin} F$ there are disjoint open soft sets U and V s.t. $E^x \widetilde{\in} U$ and $F \widetilde{\subseteq} V$.
2. A soft T^3-structure providing that it is soft regular and soft T^1.
3. A soft *regular*^{0} structure providing that for any soft point E^x and any closed soft set F s.t. $E^x \notin F$ there are disjoint open soft sets U and V s.t. $E^x \widetilde{\in} U$ and $F \widetilde{\subseteq} F$.
4. A soft T^{03}-structure providing that it is soft *regular*^{0} and soft T^{01}.
5. A soft normal structure providing that for any two disjoint closed soft sets F and G there are disjoint open soft sets U and V s.t. $F \widetilde{\subseteq} U$ and $G \widetilde{\subseteq} F$.
6. A soft T^4-structure providing that it is soft normal and soft T^1.
7. A soft T^{04}-structure providing that it is soft normal and soft T^{01}.

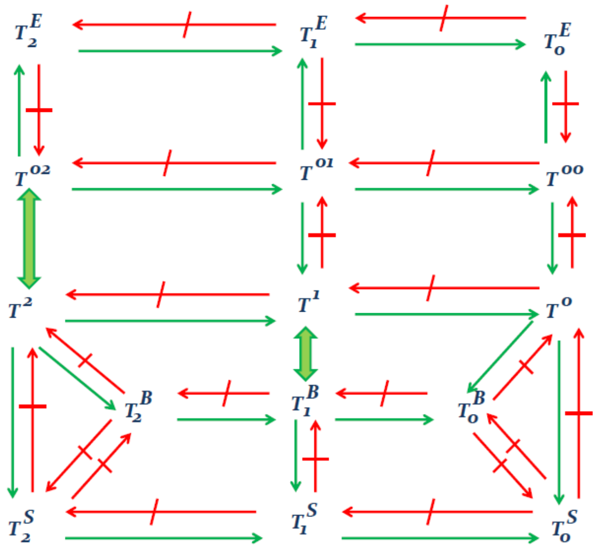


Figure 2: New classes of soft separation axioms and their relations with other existing classes

Theorem 5.20. 1. *Every soft regular structure is a regular* ^{0} *structure.*
2. *Every soft* T^{03}*-structure is a soft* T^{02}*-structure.*
3. *Every soft* T^{04}*-structure is a soft* T^{03}*-structure.*
4. *Every soft* T^{04}*-structure is a soft* T^4*-structure.*
5. *A structure is soft* T^4 *if and only if it is* T_4^B.

Proof. 1. Suppose that $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft regular structure. Take an arbitrary soft point E^x and a closed soft set F with $E^x \notin F$. Then $E^x \widetilde{\notin} F$. Since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is soft regular, there are disjoint open soft sets U and V s.t. $E^x \widetilde{\in} U$ and $F \widetilde{\in} V$. So $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is soft regular ^{0}.

2. Suppose that $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T^{03}-structure. Take arbitrary distinct soft points E^x and $\acute{E}^y$. Since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is soft T^{01}, $\acute{E}^y$ is a closed soft set (by Theorem 5.18) with $E^x \notin \acute{E}^y$. And since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T^{03}-structure, there are disjoint open soft sets U and V s.t. $E^x \widetilde{\in} U$ and $\acute{E}^y \widetilde{\subset} V$ (equivalently, $\acute{E}^y \widetilde{\in} V$). Thus $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is soft T^{02}.

3. Suppose that $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T^{04}-structure. Then $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T^{01}-structure. To show that $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft regular ^{0} structure. Let E^x be a soft point and F be a closed soft set with $E^x \notin F$. Since $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is soft T^{01}, E^x is a closed soft set. But $E^x \notin F$, so E^x and F are disjoint closed soft sets. As $(\mathcal{U}, \mathcal{O}, \mathcal{P})$ is a soft T^{04}-structure, there are disjoint open soft sets U and V s.t. $E^x \widetilde{\subset} U$ and $F \widetilde{\subset} V$, which completes the proof.

4. Since every soft T^{01}-structure is a soft T^1-structure.

5. By Theorem 5.12. □

The following example shows that regular $^0 \not\Longrightarrow$ regular.

Example 5.21. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{1, 2\}$. Set $O = \{0_{\mathcal{U}}, 1_{\mathcal{U}}, A, B\}$ s.t. $A = \{(1, \{x\}), (2, \{y\})\}$ and $B = \{(1, \{y\}), (2, \{x\})\}$. Then O is a soft topology. Since $A \widetilde{\cap} B = 0_{\mathcal{U}}$ and $A \widetilde{\cup} B = 1_{\mathcal{U}}$, we have A and B are the only none trivial open soft and closed soft sets. To show that O is a soft regular ^{0} structure. Let E^s be a soft point and F be a closed soft set with $E^s \notin F$. Since A and B are the only none trivial closed soft and open sets, it follows $F = A$ or $F = B$. If we set $U = 1_{\mathcal{U}} - F$ and $V = F$, then U and V are disjoint open soft sets with $E^s \widetilde{\in} U$ (since $E^s \notin F$ and $U = 1_{\mathcal{U}} - F$) and $F \widetilde{\subseteq} V$, hence O is soft regular ^{0}. O is not soft regular, since A is a closed soft set, $\mathcal{P}^x = (x, \mathcal{P}) \widetilde{\notin} A$ and there are no disjoint open soft sets U and V satisfying that $\mathcal{P}^x \widetilde{\in} U$ and $A \widetilde{\subseteq} V$.

Example 5.22. Let (X, τ_1) be a topological structure which is T_2 not T_3, and let (Y, τ_2) be a topological structure which is T_3 not T_4. Set $\mathcal{P} = \{1\}$ and define:

$$O_1 = \{(1, A); A \in \tau_1\}.$$

$$O_2 = \{(1, A); A \in \tau_2\}.$$

Then $(X, O_1, \mathcal{P})$ is a soft T^{02}-structure (T^2-structure) which is not T^{03} (T^3-structure), and $(Y, O_2, \mathcal{P})$ is a soft T^{03}-structure (T^3-structure) which is not T^{04} (T^4-structure).

Example 5.23. The structure $(X, O, \mathcal{P})$ in Example 3.1 is T_4^B, so it is a T^4-structure (by Theorem 5.20). But it is not T^{01}, since for the distinct soft points $\mathcal{P}^x$ and $\mathcal{P}^y$ there is no nonempty open soft set U with $\mathcal{P}^x \notin U$. Thus $(X, O, \mathcal{P})$ is not T^{04}.

Now, we will give a decomposition for soft T^3-structures.

Theorem 5.24. *Let $(\mathcal{U}, O, \mathcal{P})$ be a soft regular structure.*

1. *If F is a closed soft set, then there is $B \subset \mathcal{U}$ s.t.*

$$F = \bigcup_{x \in B} (x, \mathcal{P}).$$

2. *If U is an open soft set, then there is $A \subset \mathcal{U}$ s.t.*

$$F = \bigcup_{x \in A} (x, \mathcal{P}).$$

3. *The collection*

$$\tau = \{A \subset \mathcal{U}; U = \bigcup_{x \in A} (x, \mathcal{P}) \text{ is an open soft set}\}$$

is a topology on $\mathcal{U}$.

Proof. 1. Let F be a closed soft set. It suffices to show that for any $x \in \mathcal{U}$ if $x \notin F(\varepsilon_0)$ for some $\varepsilon_0 \in \mathcal{P}$, then $x \notin F(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$. Suppose that $x \notin F(\varepsilon_0)$ for some $\varepsilon_0 \in \mathcal{P}$, then $(x, \mathcal{P}) \widetilde{\notin} F$. Since O is soft regular, there is two disjoint open soft sets U and V s.t. $(x, \mathcal{P}) \widetilde{\in} U$ and $F \widetilde{\subset} V$. Since U and V are soft disjoint, we have $x \notin V(\varepsilon) \supset F(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$. Which completes the proof.

2. Obvious! Using (1) and the fact $1_{\mathcal{P}} - U$ is a closed soft set.

3. Straight forward! □

Theorem 5.25. *Let $(\mathcal{U}, O, \mathcal{P})$ be a soft T^3-structure. Then*

1. *$\mathcal{P}$ is a singleton.*
2. *$(\mathcal{U}, O, \mathcal{P})$ is a T_3 topological structure.*

Proof. 1. Suppose, by contrapositive, there are $a, b \in \mathcal{P}$ s.t. $a \neq b$. Consider the distinct strong soft points a_x and b_x. Since O is a soft T^1-structure, there are open soft sets U and V s.t. $a_x \widetilde{\in} U$, $a_x \widetilde{\notin} V$, $b_x \widetilde{\in} V$ and $b_x \widetilde{\notin} U$. But O is soft regular and $a_x \widetilde{\in} U$, so (from Theorem 5.24 part (2)) $x \in U(\varepsilon)$ for every $\varepsilon \in \mathcal{P}$. A contradiction with $b_x \widetilde{\notin} U$. Hence, $\mathcal{P}$ must be a singleton

2. From (1), $\mathcal{P}$ is singleton; suppose that $\mathcal{P} = \{\varepsilon\}$. Then for any soft set A we have $A = \{(\varepsilon, A(\varepsilon))\}$, so soft sets can be considered as points in the set $\{p\} \times 2^{\mathcal{U}}$. But $\{p\} \times 2^{\mathcal{U}} \cong 2^{\mathcal{U}}$ (here we used the symbol $\cong$ to refer equivalent sets). Thus soft sets are elements in the power set of $\mathcal{U}$. Which implies O (as a collection of subsets of $\mathcal{U}$) is a topology on $\mathcal{U}$. One can easily (using Theorem 5.24 part (2)) show that $(\mathcal{U}, O)$ is T^3. □

Corollary 5.26. 1. *Every soft T^3-structure is a soft T^2-structure.*
2. *Every soft T^3-structure is a soft T^{03}-structure.*

Proof. 1. Let $(\mathcal{U}, O, \mathcal{P})$ be a soft T^3-structure. Then $\mathcal{P} = \{a\}$ is a singleton and $(\mathcal{U}, \mathcal{T}^*)$ is a T_3-structure where $\mathcal{T}^* = \{U(a), U \in O\}$. To show that $(\mathcal{U}, O, \mathcal{P})$ is soft T^2. Let a_x and a_y be two distinct soft points (note that the only parameter in $\mathcal{P}$ is a). Then $x \neq y$. Since $(\mathcal{U}, \mathcal{T}^*)$ is a T_2-structure, there are disjoint open sets $U(a), V(a) \in O^*$ s.t. $x \in U(a)$ and $y \in V(a)$. It is clear that U and V are disjoint open soft sets with $a_x \widetilde{\in} U$ and $a_y \widetilde{\in} V$. That is $(\mathcal{U}, O, \mathcal{P})$ is soft T_2.

2. Let $(\mathcal{U}, O, \mathcal{P})$ be a soft T^3-structure, and $(\mathcal{U}, \mathcal{T}^*)$ as in part (1). From (1) we conclude $(\mathcal{U}, O, \mathcal{P})$ is a soft T^2-structure, so by (2) of Corollary 5.10) it is soft T^{02}. It remains to show that $(\mathcal{U}, O, \mathcal{P})$ is a soft regular ^{0} structure. Let F be a closed soft set and $a_x \widetilde{\notin} F$. Then $F(a)$ is a closed set in $(\mathcal{U}, \mathcal{T}^*)$ with $x \notin F(a)$. But $(\mathcal{U}, \mathcal{T}^*)$ is regular, so there are disjoint open sets $U(a)$ and $V(a)$ s.t. $x \in U(a)$ and $F(a) \subset V(a)$. It is clear that U and V are disjoint open soft sets with $a_x \widetilde{\in} U$ and $F \widetilde{\subset} V$. Which completes the proof. □

The following example shows that T^{03}-structure is not T^3-structure.

Example 5.27. Let $\mathcal{U} = \{x, y\}$ and $\mathcal{P} = \{a, b\}$. Define $O = \{A : A \text{ is a soft set}\}$. Then O is a soft topology. Since $\mathcal{P}$ is not singleton, O is not soft T^3 (see Theorem 5.25). And since each soft set is closed soft and open soft, O is soft T^{03}. Moreover, it is soft T^4.

By the preceding example and the succeeding example, we elucidate the independency of the structures of soft T^4 and soft T^3 of each other. That is, neither soft $T^4 \Rightarrow$ soft T^3 nor soft $T^3 \Rightarrow$ soft T^4.

Example 5.28. Let us consider a topological structure (X, τ) which is T_3 not T_4. Set $\mathcal{P} = \{\varepsilon\}$ and define:

$$O_\varepsilon = \{(\varepsilon, A); A \in \tau\}.$$

Then $(X, O_\varepsilon, \mathcal{P})$ is a soft T^3-structure which is not T^4.

6. Conclusion

It was founded the soft sets as a mathematical approach to dealing with uncertainty problems. Adequate parameterization capabilities are an advantage of soft sets that are lost by the previous approaches like fuzzy sets and rough sets. The theory of soft sets received attention from topologists, so they have studied the classical notions of topology in soft topology. Soft points are one of the most controversial concept in soft topological structures as it does not have a unified definition. And since soft separation axioms depend on soft points, we have different classes for soft separation axioms. We draw the attention of the readers to the main factors that should be taken into consideration when one studies soft separation axioms: 1) what is the form of soft points that are used? How the distinct soft points are defined? and which types of belonging and none belonging relations are applied?

We have aimed to achieve three goals. The first one was to solve an open problem "Is every soft T_i^B-structure soft T_i^E for $i = 2, 4$?" that was set up in [8]. We have clarified, with the aid of sophisticated counterexamples, that these soft structures are independent of each other in both cases of $i = 2, 4$. The second target was to build concrete examples illustrating the relationships between soft T_0^B and soft T_0^S

and investigate the condition under which we obtain soft T_0^B → soft T_0^S. Finally, we have derived a new definition of soft points from fuzzy sets, which includes two of existing definitions of soft points. We used the new definition of soft points to structure two new classes of soft separation axioms. At first glance, one expects the introduced two types of soft axioms to be equivalent with soft T^S and soft T^E existing in the published literature; however, after deep thought that takes into consideration the way of separating soft points inspired by the definition of fuzzy points, one can remark that the current types represent different categories of soft topology. In general, we have looked at the master properties of these classes and introduced a comparative study of the new classes with the old ones.

Last but not least, we hope the current contribution helps researchers construct an obvious view of the behaviors of separation axioms in the environment of soft topologies and how one exploits this environment to create further categories of soft structures as was done in the last section of this manuscript.

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