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Ramsey numbers of cycles versus multiple wheels

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Abstract. For given graphs G and H, the Ramsey number $R(G, H)$ is defined as the smallest positive integer N such that every red-blue edge-coloring of the complete graph K_N contains either a red copy of G or a blue copy of H. We denote by C_n the cycle on n vertices and by tW_{2m} the disjoint union of t copies of the wheel W_{2m}. We prove that for $m \geq 2$ and $n \geq 4tm$,

$$R(C_n, tW_{2m}) = 2n + t - 2.$$

This result generalizes previous findings by Surahmat, Baskoro, and Tomescu (Discrete Math., 2006), Chen, Cheng, Miao, and Ng (Appl. Math. Lett., 2009), Zhang, Broersma, and Chen (Graph Combin., 2015), as well as Sudarsana (Electron. J. Graph Theory Appl., 2021). Furthermore, the result confirms Sudarsana's conjecture for wheels of odd order.

1. Introduction

We use $[n]$ to denote the set $\{1, 2, \ldots, n\}$. The symbols $|G|$, $\delta(G)$, $\Delta(G)$, and $\chi(G)$ represent the number of vertices, minimum degree, maximum degree, and chromatic number of the graph G, respectively. The *chromatic surplus* of a graph G refers to the number of vertices in the smallest color class among all possible $\chi(G)$-colorings of G. The *connectivity* of a graph G refers to the minimum number of vertices that need to be removed to disconnect G. Given a subset $U \subseteq V(G)$, we write $G[U]$ for the subgraph of G induced by U. The graph obtained by deleting the vertex set U and all its incident edges from G is denoted by $G - U$; in the special case where $U = \{u\}$, we simply write $G - u$. When $U_1, U_2 \subseteq V(G)$ and $U_1 \cap U_2 = \emptyset$, we use $G[U_1, U_2]$ to denote the bipartite subgraph induced by all edges with one endpoint in U_1 and the other in U_2. The disjoint union of graphs G and H is denoted by $G \cup H$. A path with endpoints v_1 and v_2 is denoted by $v_1 P v_2$. We denote by C_n the cycle on n vertices and by W_ℓ the wheel on $\ell + 1$ vertices, which consists of the cycle C_ℓ together with an additional central vertex that is adjacent to every vertex of the cycle. Moreover, we use tW_ℓ to denote the disjoint union of t copies of W_ℓ. A graph of order n is said to be pancyclic if it contains a cycle of length k for every integer k with $3 \leq k \leq n$. For any other unexplained notation and terminology, we adhere to the textbook by Bondy and Murty [4].

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For two graphs G and H, the *Ramsey number* $R(G, H)$ is defined as the smallest positive integer n such that in any red-blue edge-coloring of the complete graph K_n, there either exists a red subgraph isomorphic to G or a blue subgraph isomorphic to H. In other words, in terms of graphs and their complements, the *Ramsey number* $R(G, H)$ is the smallest positive integer n such that for any graph F on n vertices, either F contains G as a subgraph or its complement $\overline{F}$ contains H as a subgraph. This paper focuses on the study of Ramsey numbers for cycles versus wheels, a line of research initiated by Burr and Erdős [7].

The study of Ramsey numbers for sparse graphs began to flourish in the 1970s. From the outset, cycles and wheels have been among the most frequently studied classes of sparse graphs. For results concerning the Ramsey numbers of cycles, see Bondy and Erdős [3], Rosta [16], Faudree and Schelp [12], among others. For results on the Ramsey numbers of wheels, see Faudree and McKay [11] (which disproved a conjecture of Erdős), Lidický and Pfender [14], Van Overberghe [22], and others.

The Ramsey numbers for cycles versus wheels were first studied by Burr and Erdős [7], where they proved that $R(C_3, W_\ell) = 2\ell + 1$ for $\ell \geq 5$. However, when C_3 is replaced by C_4, the complexity of the problem increases significantly. Zhang, Broersma, and Chen [25] proved that $R(C_4, W_\ell) = R(C_4, K_{1,\ell})$ for $\ell \geq 6$. The exact value of the latter is currently known only in certain special cases, and an upper bound problem in its general case includes a 100-dollar conjecture of Erdős [8].

For the Ramsey numbers of odd cycles versus large wheels, Zhang, Zhang, and Chen [27], Sanhueza-Matamala [17], and Alweiss [1] have obtained a series of results. For even cycles and even-order wheels, when $n < \ell < 3n/2$, Zhang, Broersma, and Chen [26] also determined the exact value of $R(C_n, W_\ell)$.

When the cycle is relatively large compared to the wheel, the parity of the wheel plays a crucial role in its Ramsey number. For convenience, we separately study even-order wheels W_{2m+1} and odd-order wheels W_{2m}.

For the wheel W_{2m+1}, it is easy to verify that $R(C_n, W_{2m+1}) \geq 3n - 2$. The question then arises: when does equality hold? Surahmat, Baskoro, and Tomescu [20] conjectured that for $n \geq 2m + 1 \geq 3$ and $(n, m) \neq (3, 1)$, the equality always holds. Two years later, the same group of authors [21] proved that this equality holds for $m \geq 2$ and $n > 5m - 2$. Shi [18] demonstrated that the equality holds for $n > 70$ or $n \geq 3m + 3$. Zhang, Chen, and Cheng [24] further extended the range of n, proving that the equality holds whenever $n \geq 20$. Finally, Chen, Cheng, Ng, and Zhang [10] completely resolved the Surahmat-Baskoro-Tomescu conjecture.

For the wheel W_{2m}, it is easy to verify that $R(C_n, W_{2m}) \geq 2n - 1$. The corresponding question is: when does equality hold? Surahmat, Baskoro, and Tomescu [21] conjectured that for $n \geq 2m \geq 4$ and $(n, m) \neq (4, 2)$, the equality always holds. Moreover, the same group of authors [20] had already proven that this equality holds for $n \geq 5m - 1$. Chen, Cheng, Miao, and Ng [9] extended the range of n to $n \geq 3m + 1$. Zhang, Broersma, and Chen [26] further extended the range to $n \geq 2m + 502$. Raeisi and Zaghian [15] independently extended the range of n to $n \geq 2m + 500$, requiring n to be even. Combining these results, the best known result to date states that for $m \geq 2$ and $n \geq 2m + \min\{m + 1, 502\}$, the equality holds.

Sudarsana [19] further generalized this problem by introducing multiple wheels. What happens if the five-vertex wheel W_4 is replaced by t pairwise disjoint copies of W_4? Sudarsana obtained the following theorem.

Theorem 1.1 (Sudarsana [19]). *$R(C_n, tW_4) = 2n + t - 2$ for $n \geq 15t^2 - 4t + 2$.*

Sudarsana [19] also proposed the following conjecture.

Conjecture 1.2. *For $t \geq 1$ and $m \geq 2$, there exists a number n_0 such that for $n \geq n_0$,*

$$R(C_n, tW_{2m}) = 2n + t - 2 \text{ and } R(C_n, tW_{2m+1}) = 3n + t - 3.$$

The main theorem of this paper confirms Conjecture 1.2 for odd-order wheels.

Theorem 1.3. *For $t \geq 1$ and $m \geq 2$, when $n \geq 4tm$, we have $R(C_n, tW_{2m}) = 2n + t - 2$.*

When $m = 2$, the above theorem states that as long as $n \geq 8t$, the equality holds. This result significantly improves Theorem 1.1. When $t = 1$, this result coincides with the classical cycle-wheel Ramsey number result. It is also worth noting that, unlike the proof of $R(C_n, W_{2m}) = 2n - 1$, our proof does not rely on the weakly pancyclic conditions established by Brandt [5] or Brandt, Faudree, and Goddard [6].

The structure of this paper is as follows: In Section 2, we present five useful lemmas, and in Section 3, we provide a complete proof of Theorem 1.3.

2. Useful lemmas

When $t = 1$, Theorem 1.3 can be derived from the following lemma.

Lemma 2.1 (Chen, Cheng, Miao, and Ng [9]). *$R(C_n, W_{2m}) = 2n - 1$ for $m \geq 2$ and $n \geq 3m + 1$.*

The following cycle exchange technique originates from the work of Bondy and Erdős on the Ramsey numbers of cycles versus complete graphs.

Lemma 2.2 (Bondy and Erdős [3]). *Suppose that a graph G contains a cycle of length ℓ but no cycle of length $\ell + 1$, and that its complement $\overline{G}$ does not contain K_r as a subgraph, where $r \geq 3$. Then any vertex in $V(G) \setminus V(C_\ell)$ is adjacent to at most $r - 2$ vertices in C_ℓ.*

Next, we present a pancyclic condition, a criterion for finding even cycles in bipartite graphs, and a panconnected condition.

Lemma 2.3 (Bondy [2]). *Let G be a graph of order n. If $\delta(G) \geq n/2$, then either G is pancyclic, or n is even and $G = K_{n/2,n/2}$.*

Lemma 2.4 (Jackson [13]). *Let $G = (X, Y)$ be a bipartite graph with bipartition classes X and Y such that $d(x) \geq k$ for all $x \in X$, where $|X| \geq 2$ and $2 \leq k \leq |Y| \leq 2k - 2$. Then G contains all cycles on $2m$ vertices for $2 \leq m \leq \min\{|X|, k\}$.*

Lemma 2.5 (Williamson [23]). *Every graph $G = (V, E)$ on n vertices with $\delta(G) \geq n/2 + 1$ has the following property. For every $v, w \in V$ and every k such that $2 \leq k \leq n - 1$, G contains a path of length k which starts at v and ends at w. In particular, G is pancyclic.*

3. Proof of the main theorem

For the lower bound, consider the graph $K_{n-1} \cup K_{n-1} \cup K_{t-1}$. Clearly, it does not contain C_n as a subgraph. Its complement is the complete tripartite graph $K_{n-1,n-1,t-1}$, whose chromatic surplus is $t - 1$. Since the chromatic surplus of tW_{2m} is t, it cannot be contained as a subgraph in $K_{n-1,n-1,t-1}$. Thus, we obtain

$$R(C_n, tW_{2m}) \geq 2n + t - 2.$$

Next, we prove the upper bound. When $t = 1$, the theorem follows from Lemma 2.1. We now consider the case $t \geq 2$.

Set $m \geq 2$ and $n \geq 4tm$, and let G be a graph of order $2n + t - 2$. Suppose, for the sake of contradiction, that G does not contain C_n as a subgraph, and its complement $\overline{G}$ does not contain tW_{2m} as a subgraph.

First, select as many pairwise disjoint copies of W_{2m} as possible from $\overline{G}$, and let x denote the number of such copies selected. By the assumption, we have $x \leq t - 1$. Now, remove all vertices of these x disjoint copies of W_{2m} from G, and denote the resulting graph by G'. By the maximality of x, $\overline{G'}$ does not contain W_{2m} as a subgraph, and we have

$$|G'| = |G| - x(2m + 1) \geq (2n + t - 2) - (t - 1)(2m + 1) = 2(n - tm + m) - 1.$$

By Lemma 2.1, we know that $R(C_{n-tm+m}, W_{2m}) = 2(n - tm + m) - 1$ for $n \geq tm + 2m + 1$. Since $\overline{G'}$ does not contain W_{2m} as a subgraph, it follows that G' contains C_{n-tm+m} as a subgraph. Let C_ℓ be a longest cycle in G' with length at most n. Then, we have $n - (t - 1)m \leq \ell \leq n - 1$.

Let U denote the set of vertices in G' that are not on C_ℓ. Then, we have $|U| \geq n - 2tm + 2m$. Since $\overline{G'}$ does not contain W_{2m} as a subgraph, it also does not contain K_{2m+1} as a subgraph. By Lemma 2.2, any vertex $u \in U$ has at most $2m - 1$ neighbors in the cycle C_ℓ. We now establish the following claims.

Claim 3.1. *The subgraph $\overline{G}[U]$ does not contain $K_{1,m}$ as a subgraph, and $G[U]$ contains a Hamiltonian cycle $C_{|U|}$.*

Proof. Suppose that $\overline{G}[U]$ contains $K_{1,m}$ as a subgraph. Let u_0 be the center of $K_{1,m}$, and let its leaves be $u_1, \dots, u_m$. In the graph $\overline{G}$, denote the set of neighbors of u_0 on the cycle C_ℓ by U_1. Since each vertex in U has at most $2m-1$ neighbors on C_ℓ, we have $|U_1| \geq |U| - (2m-1) \geq n - 2tm + 1$. Moreover, in the graph $\overline{G}$, for each $i \in [m]$, the number of neighbors of u_i in U_1 is at least $|U_1| - (2m-1)$. The sets $\{u_1, \dots, u_m\}$ and U_1 correspond to the sets X and Y in Lemma 2.4, respectively, with $|U_1| - (2m-1)$ corresponding to k in Lemma 2.4. Since $|U_1| \geq n - 2tm + 1$, it follows that $|U_1| \leq 2(|U_1| - (2m-1)) - 2$. By Lemma 2.4, there exists a cycle of length $2m$ in $\overline{G}$. This cycle with the vertex u_0 forms a copy of W_{2m} in $\overline{G}$, a contradiction. Hence, $\overline{G}[U]$ does not contain $K_{1,m}$ as a subgraph.

Since $\Delta(\overline{G}[U]) \leq m-1$, we have $\delta(G[U]) \geq |U| - m \geq |U|/2$. The last inequality holds because $|U| \geq n - 2tm + 2m$ and $n \geq 4tm$. By Lemma 2.3, $G[U]$ contains a Hamiltonian cycle $C_{|U|}$. □

Claim 3.2. *The connectivity of the graph G' is at most* 1.

Proof. On the contrary, let G' be a 2-connected graph. Then there exist two edges v_1v_2 and v_3v_4, where $v_1, v_3 \in V(C_\ell)$ and $v_2, v_4 \in U$. Given an orientation of the cycle C_ℓ, the vertices v_1 and v_3 divide C_ℓ into two paths: $v_1\overrightarrow{C}v_3$ and $v_1\overleftarrow{C}v_3$. One of these paths must have length at least $\lceil \ell/2 \rceil$, meaning it contains at least $\lceil \ell/2 \rceil + 1$ vertices.

Now, select a shortest path v_1Pv_3 in $G[V(C_\ell)]$ from v_1 to v_3 with length at least $\lceil \ell/2 \rceil$, and let this path contain x vertices.

If $x \leq n-3$, we only need to show that there exists a path in $G[U]$ from v_2 to v_4 containing exactly $n-x$ vertices. Such a path, together with v_1Pv_3 and the edges v_1v_2 and v_3v_4, would form a cycle of length n in G, contradicting our assumption.

To find a path in $G[U]$ from v_2 to v_4 with $n-x$ vertices, we first prove that $|U| \geq n-x$. Suppose, for contradiction, that $|U| \leq n-x-1$. Note that in $G[V(C_\ell)]$, the number of vertices not on v_1Pv_3 is at most $\lfloor \ell/2 \rfloor - 1$. Thus,

$$\begin{aligned}
|G'| \leq & x + (\lfloor \ell/2 \rfloor - 1) + |U| \\
\leq & n + \lfloor \ell/2 \rfloor - 2 \\
\leq & n + (n-1)/2 - 2 \\
< & 2(n - tm + m) - 1 \\
= & |G'|.
\end{aligned}$$

This contradiction confirms that $|U| \geq n-x$.

Next, we prove that $\delta(G[U]) \geq |U|/2 + 1$. By Claim 3.1, we have $\overline{G}[U] \leq m-1$. Hence,

$$\delta(G[U]) \geq |U| - 1 - (m-1) \geq |U|/2 + 1.$$

The last inequality follows from $|U| \geq n - 2tm + 2m$.

By Lemma 2.5, there exists a path in $G[U]$ from vertex v_2 to vertex v_4 containing exactly $n-x$ vertices. This completes the proof for the case $x \leq n-3$.

Now let $x \geq n-2$ and $z_1z_2\cdots z_x$ be such a path, where $v_1 = z_1$ and $v_3 = z_x$. Set $Z_1 = \{z_3, z_4, \dots, z_{m+2}\}$ and $Z_2 = \{z_{m+4}, z_{m+5}, \dots, z_{2m+3}\}$.

We claim that the subgraph induced by z_1, Z_1, and Z_2 in $\overline{G}$ contains $K_{1,m,m}$ as a subgraph. To establish this, we need to show that for all $1 \leq i < i+1 < j \leq 2m+3$, the edge z_iz_j is not in $E(G)$. Indeed, if $z_iz_j \in E(G)$, then the path $z_1 \cdots z_iz_jz_x$ would be a shorter path between v_1 and v_3 of length at least $(x-1)-(j-i-1)$. But

$$\begin{aligned}
& (x-1) - (j-i-1) \\
\geq & (n-2-1) - (2m+3-1-1) \\
\geq & n/2 \\
\geq & \lceil \ell/2 \rceil .
\end{aligned}$$

The second inequality follows from $n \geq 4tm \geq 4m + 8$. In this way, we obtain a shorter path of length at least $\lceil \ell/2 \rceil$, contradicting the minimality of x. This contradiction confirms that the subgraph induced by z_1, Z_1, and Z_2 in $\overline{G}$ contains $K_{1,m,m}$ as a subgraph.

Since $K_{1,m,m}$ contains W_{2m} as a subgraph, we have a contradiction. Thus, the case $x \geq n-2$ cannot occur. □

Claim 3.3. *There exists a vertex w in G', such that $G' - w$ has exactly two connected components.*

Proof. First, we prove that there exists a vertex w in G', such that $G' - w$ has at least two connected components. By Claim 3.2, the graph G' is either disconnected or has a cut vertex. If the latter holds, then the cut vertex of G' is the desired vertex w. According to Claim 3.1, there are no isolated vertices in G'. If G' itself is disconnected, then any vertex in G' can serve as the required vertex w.

Next, we prove that for any vertex w, $G' - w$ has at most two connected components. By Claim 3.1, every vertex of G' lies either on the cycle C_ℓ or on the cycle $C_{|U|}$. This implies that removing any single vertex does not create more than two connected components. □

Let X and Y denote the vertex sets of the components of $G' - w$. We assert the following.

Claim 3.4. *$\delta(G[X]) \geq |X| - m + 1$, $\delta(G[Y]) \geq |Y| - m + 1$, $n - 2tm + 2m - 1 \leq |X| \leq n - 1$, and $n - 2tm + 2m - 1 \leq |Y| \leq n - 1$.*

Proof. According to Claim 3.1, each vertex of G' is either on the cycle C_ℓ or on the cycle $C_{|U|}$. Therefore, the number of vertices in each connected component of $G' - w$ is at least

$$\min\{\ell - 1, |U| - 1\} \geq n - 2tm + 2m - 1.$$

Next, we prove that $\delta(G[X]) \geq |X| - m + 1$. Suppose not, then there exists a star $K_{1,m}$ in $\overline{G}[X]$. By selecting m arbitrary vertices from Y and combining them with this $K_{1,m}$, we obtain a complete tripartite graph $K_{1,m,m}$ in $\overline{G}$, which contains W_{2m} as a subgraph, leading to a contradiction. Hence, we conclude that $\delta(G[X]) \geq |X| - m + 1$. Similarly, we can prove that $\delta(G[Y]) \geq |Y| - m + 1$.

Finally, we prove that $|X| \leq n - 1$. Suppose not, then $|X| \geq n$. It follows that

$$\delta(G[X]) \geq |X| - m + 1 > |X|/2.$$

By Lemma 2.3, $G[X]$ contains a cycle C_n as a subgraph, which is a contradiction. Therefore, we conclude that $|X| \leq n - 1$. Similarly, we can prove that $|Y| \leq n - 1$. □

Claim 3.5. *The vertex w has at least $|X| - m + 1$ neighbors in X, or at least $|Y| - m + 1$ neighbors in Y.*

Proof. Suppose the claim is false. Then the vertex w has at least m non-neighbors in both X and Y. Select m non-neighbors of w from X and Y, denoted as X_0 and Y_0, respectively. Consequently, the set consisting of w, X_0, and Y_0 forms a complete tripartite graph $K_{1,m,m}$ in $\overline{G}$, which contains W_{2m} as a subgraph. This contradiction establishes the claim. □

Recall that the graph $\overline{G} - V(G')$ contains x vertex-disjoint copies of W_{2m+1}. For each of these W_{2m+1}, we assert the following.

Claim 3.6. *In W_{2m+1}, all but at most one vertex have at least $|X| - 2m + 1$ neighbors in X, or at least $|Y| - 2m + 1$ neighbors in Y.*

Proof. We proceed by contradiction. Suppose that there exists a W_{2m+1}, say H, in which two vertices w_1 and w_2 each have at most $|X| - 2m$ neighbors in X and at most $|Y| - 2m$ neighbors in Y.

Under this assumption, we can select a set X_1 of m non-neighbors of w_1 in X and a set X_2 of m non-neighbors of w_2 in X, ensuring that $X_1 \cap X_2 = \emptyset$. Similarly, we can choose a set Y_1 of m non-neighbors of w_1 in Y and a set Y_2 of m non-neighbors of w_2 in Y, satisfying $Y_1 \cap Y_2 = \emptyset$.

Since X and Y are the two connected components of $G' - w$, the bipartite subgraph $\overline{G}[X, Y]$ is a complete bipartite graph. Consequently, the sets $\{w_1\} \cup X_1 \cup Y_1$ and $\{w_2\} \cup X_2 \cup Y_2$ each contain a copy of W_{2m} as a subgraph in $\overline{G}$.

Now, by removing H from the x vertex-disjoint copies of W_{2m} and adding the two newly constructed copies of W_{2m}, we obtain $x + 1$ vertex-disjoint copies of W_{2m} in $\overline{G}$, contradicting the maximality of x. This completes the proof of the claim. □

We classify the vertices outside $X \cup Y$ as follows: A vertex is assigned to X' if it has at least $|X| - 2m + 1$ neighbors in X, and to Y' if it has at least $|Y| - 2m + 1$ neighbors in Y. By Claims 3.5 and 3.6, at most x vertices do not belong to $X \cup Y \cup X' \cup Y'$. In other words, the number of vertices in $X \cup Y \cup X' \cup Y'$ is at least $2n + t - 2 - x \geq 2n - 1$. By the pigeonhole principle, either $|X| + |X'| \geq n$ or $|Y| + |Y'| \geq n$. Without loss of generality, assume the former holds. Extracting $n - |X|$ vertices from X' and still denoting the resulting set as X', we proceed with the proof.

Next, we show that $G[X \cup X']$ contains a Hamiltonian cycle, which is C_n. By Claims 3.4, 3.5, and 3.6, we have

$$\delta(G[X \cup X']) \geq |X| - 2m + 1 \geq (n - 2tm + 2m - 1) - 2m + 1 = n - 2tm \geq n/2.$$

By Lemma 2.3, the theorem follows.

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Data Availability Statement

No data was used or generated in this research.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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