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Graphic contraction and perimetric contractions

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Abstract. We discuss the relation of graphic contraction and perimetric contractions: mappings contracting perimeters of triangles, generalized Kannan-type mappings, generalized Chatterjea-type mappings, and generalized ĆRR-type mappings. Generalized Kannan-type and ĆRR-type mappings are graphic contractions, while mappings contracting perimeters of triangles and generalized Chatterjea-type mappings mappings are graphic contractions under properly introduced remetrization that preserves completeness.

1. Introduction

The notion of a mapping contracting perimeters of triangles was introduced by E. Petrov [15] in 2023. and further studied in [3, 10, 14, 16, 17, 27] among others. This concept represents a generalization of a Banach contraction and differs from it due to the lack of uniqueness of a fixed point in the general case. Hence, it belongs to the class of weakly Picard operators introduced by I. A. Rus [28] in 1993, which presents an extension of the class of Picard operators. Numerous authors continued the research on the topic of weakly Picard operators [1, 8, 13, 18, 30, 31]. A graphic contraction [23] is a weakly Picard operator that is not necessarily a Picard operator. The saturated principle of graphic contraction [32] claims the existence of a fixed point of a graphic contraction under some additional assumptions, such as orbital continuity. There are numerous recent results related to graphic contraction [7, 19–22, 29]. In this paper, we intend to establish some correlations between different modifications of mappings contracting perimeters of triangles and graphic contractions.

In the sequel, we present some basic definitions and main results regarding the mentioned topics.

Research on perimetric contractions starts in [15], where a mapping contracting perimeters of triangles was introduced.

Definition 1.1. *[15] A mapping $f : X \mapsto X$ on a metric space (X, d) is a mapping contracting perimeters of triangles if there exists some $q \in [0, 1)$ such that for all mutually distinct points $x, y, z \in X$ we have*

$$d(fx, fy) + d(fy, fz) + d(fz, fx) \leq q\left(d(x, y) + d(y, z) + d(z, x)\right). \tag{1}$$

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The main result of [15] is the existence of a fixed point of a mapping contracting perimeters of triangles under the additional condition that the observed mapping has no periodic points of a prime period two.

Theorem 1.2. *[15] Let (X, d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ satisfy the following two conditions:*

(i) $f^2x = x$ implies $fx = x$ for all $x \in X$;

(ii) f is a mapping contracting perimeters of triangles.

Then f has a fixed point, and the number of fixed points is at most two.

Recall that $x \in X$ is a periodic point of a mapping $f : X \mapsto X$ if there exists a natural number $k \in \mathbb{N}$ such that $f^k x = x$. The smallest number k fulfilling the observed equality is a prime period of x related to the mapping f. Hence, a mapping has no periodic points of a prime period two if and only if $f^2x = x$ must imply $fx = x$ as originally stated in [15].
The further research driven by the idea of mappings contracting perimeters of triangles was related to the Kannan-type and Chatterjea-type mappings in a setting of a complete metric space.
Kannan [9] modified the original Banach contractive condition in 1960. and obtained an existence and uniqueness result for a class of mappings that does not require a continuity assumption. The class of Kannan contractions also characterizes the completeness of a metric space, as can be seen in [33], which is not the case for the class of Banach contractions [6].

Theorem 1.3. *[9] If (X, d) is a complete metric space and $f : X \mapsto X$ a mapping fulfilling*

$$d(fx, fy) \leq q\,(d(x, fx) + d(y, fy)) \tag{2}$$

for all $x, y \in X$ and some $q \in \left[0, \frac{1}{2}\right)$, then f has a unique fixed point in X and the sequence $(f^n x)$ converges to the fixed point of mapping f for any initial point $x \in X$.

The concept of generalized Kannan-type mappings, inspired by the results of [15], was introduced in [16].

Definition 1.4. *[16] A mapping $f : X \mapsto X$ is a generalized Kannan-type mapping on a metric space (X, d) if there exists some $q \in \left[0, \frac{2}{3}\right)$ such that the inequality*

$$d(fx, fy) + d(fy, fz) + d(fz, fx) \leq q\,(d(x, fx) + d(y, fy) + d(z, fz)) \tag{3}$$

holds for all pairwise distinct $x, y, z \in X$.

Kannan contractions are generalized Kannan-type mappings only if $q \in \left[0, \frac{1}{3}\right)$ in (2). Reverse hold only if the generalized Kannan-type mapping is continuous and X has no isolated points. However, the presented example in [16] testifies that even though those two classes coincide on the subset of a class of continuous mappings, there are discontinuous generalized Kannan-type mappings. A mapping contracting perimeters of triangles on a set with more than three points for $q \in \left[0, \frac{1}{4}\right)$ is a generalized Kannan-type mapping.
Existence of a fixed point for a generalized Kannan-type mapping on a complete metric space is obtained under the same presumption related to the periodic points of a prime period two.

Theorem 1.5. *[16] Let (X, d) for $|X| \geq 3$ be a complete metric space and let a mapping $f : X \mapsto X$ satisfy the following conditions:*

(i) f has no periodic points of a prime period two;

(ii) f is a generalized Kannan-type mapping.

Then f has a fixed point, and the number of fixed points is at most two.

Chatterjea presented a modification of a Kannan contractive condition in [4] and proved the existence and uniqueness of a fixed point of a Chatterjea contraction on a complete metric space. Chatterjea and Kannan contractions are independent.

Theorem 1.6. *[4] If (X, d) is a complete metric space and $f : X \mapsto X$ is a mapping such that*

$$d(fx, fy) \leq q\,(d(x, fy) + d(y, fx))$$

for some $q \in [0, \frac{1}{2})$ and for all $x, y \in X$, then a mapping f has a unique fixed point $x^* \in X$ and the sequence $(f^n x)$ converges to the fixed point of mapping f for any initial point $x \in X$.

In [14], a generalized Chatterjea-type mapping was presented in the sense of [15].

Definition 1.7. *[14] A mapping $f : X \mapsto X$ is a generalized Chatterjea-type mapping on a metric space (X, d) if there exists some $q \in \left[0, \frac{1}{2}\right)$ such that the inequality*

$$\begin{aligned} d(fx, fy) + d(fy, fz) + d(fz, fx) \leq q\,(&d(x, fy) + d(x, fz) + d(y, fx) \\ &+ d(y, fz) + d(z, fx) + d(z, fy)) \end{aligned} \tag{4}$$

holds for all pairwise distinct $x, y, z \in X$.

In this case, any Chatterjea contraction is a generalized Chatterjea-type mapping, while the converse does not hold, as validated by Example 2.3 of [14]. The generalized Chatterjea-type mappings on a complete metric space have at least one and at most two fixed points, assuming that the mapping has no periodic points of a prime period two.

Theorem 1.8. *[14] Let (X, d) for $|X| \geq 3$ be a complete metric space and let a mapping $f : X \mapsto X$ satisfy the following conditions:*

(i) f has no periodic points of a prime period two;

(ii) f is a generalized Chatterjea-type mapping.

Then f has a fixed point, and the number of fixed points is at most two.

This idea is extended in a different direction by making some type of Ćirić-Reich-Rus generalized mappings. The contractive conditions related to mappings contracting perimeters of triangles and generalized Kannan mappings are combined in [3].

Definition 1.9. *[3] A mapping $f : X \mapsto X$ is a generalized ĆRR-type mapping on a metric space (X, d) if there exists some $\alpha, \beta \geq 0$ such that $2\alpha + \frac{3}{2}\beta < 1$ for which the inequality*

$$\begin{aligned} d(fx, fy) + d(fy, fz) + d(fz, fx) \leq\; &\alpha\,(d(x, y) + d(y, z) + d(z, x)) \\ &+ \beta\,(d(x, fx) + d(y, fy) + d(z, fz)) \end{aligned} \tag{5}$$

holds for all pairwise distinct $x, y, z \in X$.

The main result of [3] is the following:

Theorem 1.10. *[3] Let (X, d) for $|X| \geq 3$ be a complete metric space and let a mapping $f : X \mapsto X$ satisfy the following conditions:*

(i) f has no periodic points of a prime period two;

(ii) f is a generalized ĆRR-type mapping.

Then f has a fixed point, and the number of fixed points is at most two.

Remark 1.11. *Observe that the concepts of perimetric contractions can be observed in the setting of a G-metric space proposed by Mustafa and Sims [12]. Concretely, for a mapping contracting perimeters of triangles* $f : X \mapsto X$ *on a complete metric space* (X, d)*, the contractive condition* (1) *can be transformed into a Banach-type contractive condition*

$$G(fx, fy, fz) \leq qG(x, y, z)$$

whenever $x, y, z \in X$ *are three mutually distinct points and* $G : X \times X \times X \mapsto [0, \infty)$ *defined by* $G(x, y, z) = d(x, y) + d(y, z) + d(z, y)$ *for* $x, y, z \in X$. (X, G) *is a complete G-metric space. However, the existence of a fixed point of a mapping contracting perimeters of triangles does not follow directly from the fixed point result because of the mentioned restriction of* x, y*, and* z *being pairwise distinct. Still, we believe that the same technique can be applied even in the case of the restriction. Similar considerations can be done for generalized Kannan-type, generalized Chatterjea-type, and generalized ĆRR-type contractions in the context of an adequate G-metric space.*

The class of weakly Picard operators (WPOs) was introduced in [28], and all presented perimetric contractions are types of weakly Picard operators.

Definition 1.12. *[28] A mapping* $f : X \mapsto X$ *is a weakly Picard operator (WPO) if for any* $x \in X$ *the sequence* $(f^n x)$ *converges to a fixed point of a mapping* f.

The class of weakly Picard operators contains the class of Picard operators.

Definition 1.13. *A mapping* $f : X \mapsto X$ *is a Picard operator (PO) if for any* $x \in X$ *the sequence* $(f^n x)$ *converges to the unique fixed point of a mapping* f.

Moreover, there are weakly Picard operators that are not Picard operators.

Example 1.14. *Banach contraction [2], Kannan contraction [9], and Chatterjea contraction [4] are Picard operators as well as weakly Picard operators. However, the mapping* $f : [0, 1] \mapsto [0, 1]$ *defined by*

$$fx = \begin{cases} 0, & x \in [0, 1) \\ 1, & x = 1 \end{cases}$$

is a weakly Picard operator on a complete metric space $([0, 1], d)$ *assuming that* $d : X \times X \mapsto [0, \infty)$ *is an Euclidean metric on* X *defined by*

$$d(x, y) = |x - y|, \tag{6}$$

for all $x, y \in X$*, but it is not a Picard operator.*

A mapping contracting perimeters of triangles is always a weakly Picard operator, as established in the proof of Theorem 1.2, but not necessarily a Picard operator.

Example 1.15. *[15] Let* $X = \{a, b, c\}$ *be a set equipped with the discrete metric* $d : X \times X \mapsto [0, \infty)$ *defined by*

$$d(x, y) = \begin{cases} 1, & x \neq y \\ 0, & x = y \end{cases}$$

for $x, y \in X$ *and a mapping* $f : X \mapsto X$ *defined by* $fa = fb = a$ *and* $fc = c$*. Then* f *is a mapping contracting perimeters of triangles, which is a weakly Picard operator, but not a Picard operator.*

As expected, not every weakly Picard operator is a mapping contracting perimeters of triangles, as can be seen in the Example 1.14.
The orbit of a mapping $f : X \mapsto X$ at the point $x \in X$ is a set $O(x) = \{f^n x \mid n \in \mathbb{N}_0\}$ where $f^0 x = x$ assuming that f^0 is an identity mapping on X. The notion of orbital continuity was introduced by Ćirić in [5] as the self-mapping being continuous on the orbit of each point of an underlying metric space.

Definition 1.16. *[5] A mapping $f : X \mapsto X$ is orbitally continuous if* $\lim_{n\to\infty} f^{m_n}x$ *exists, then* $f(\lim_{n\to\infty} f^{m_n}x) = \lim_{n\to\infty} f(f^{m_n}x)$ *for any $(m_n) \subseteq \mathbb{N}$ and $x \in X$.*

A graphic contraction is an example of a weakly Picard operator that was introduced in [23]. In [32], the fixed point results for a graphic contraction of [23] were modified by the assumption of orbital continuity.

Definition 1.17. *[23] A mapping $f : X \mapsto X$ is a graphic contraction on a metric space (X, d) if the inequality*

$$d(fx, f^2x) \leq qd(x, fx) \tag{7}$$

holds for some $q \in [0, 1)$ and all $x \in X$.

For a weakly Picard operator $f : X \mapsto X$ we may define $f^\infty : X \mapsto X$ as

$$f^\infty(x) = \lim_{n\to\infty} f^n x$$

for all $x \in X$. For a fixed point x^* of a mapping f let $X_{x^*} = \{x \in X \mid \lim_{n\to\infty} f^n x = x^*\}$.
The saturated principle of a graphic contraction presented below in Theorem 1.18 claims the existence of a fixed point of an orbitally continuous graphic contraction on a complete metric space.

Theorem 1.18. *[32] Let (X, d) be a complete metric space and $f : X \mapsto X$ a mapping. If f is an orbitally continuous graphic contraction on X for some $q \in [0, 1)$, then*

(i) f is a weakly Picard operator on X;

(ii) $d(x, f^\infty x) \leq \frac{1}{1-q} d(x, fx)$ for all $x \in X$;

(iii) If $(x_n) \subseteq X_{x^*}$ is a sequence such that $\lim_{n\to\infty} d(x_n, fx_n) = 0$, *then* $\lim_{n\to\infty} x_n = x^*$;

(iv) If $q < \frac{1}{3}$ and $(x_n) \subseteq X_{x^*}$ is a sequence such that $\lim_{n\to\infty} d(x_{n+1}, fx_n) = 0$, *then* $\lim_{n\to\infty} x_n = x^*$.

2. Main results

We will discuss the relation of mappings contracting perimeters of triangles and graphic contraction in two different aspects. We will show for what values of contractive constant a mapping contracting perimeters of triangles is always a graphic contraction, and also provide a remetrization approach that will prove that any mapping contracting perimeters of triangles may be observed as a graphic contraction in a remetrized metric space.

If $q \in [0, \frac{1}{2})$ in (1), then a mapping contracting perimeters of triangles without periodic points of a prime period two is a graphic contraction.

Theorem 2.1. *If $f : X \mapsto X$ is a mapping contracting perimeters of triangles on a complete metric space (X, d) such that $|X| \geq 3$ for $q \in \left[0, \frac{1}{2}\right)$ and the mapping f has no periodic points of a prime period two, then it is a continuous graphic contraction on (X, d).*

Proof. Let $x \in X$ be arbitrary. If x is a fixed point of the mapping f, then (7) evidently holds. Otherwise, $fx \neq x$ and as f has no periodic points of prime period two, we additionally obtain $f^2x \neq x$. If $fx = f^2x$, again (7) easily follows, while otherwise (1) may be applied on these three points. Hence,

$$d(fx, f^2x) + d(f^2x, f^3x) + d(f^3x, fx) \leq q\left(d(x, fx) + d(fx, f^2x) + d(f^2x, x)\right)$$

and

$$\begin{aligned} d(fx, f^2x) &\le \frac{1}{2}\Big(d(fx, f^2x) + d(f^2x, f^3x) + d(f^3x, fx)\Big) \\ &\le \frac{q}{2}\Big(d(x, fx) + d(fx, f^2x) + d(f^2x, x)\Big) \\ &\le \frac{q}{1-q} d(x, fx). \end{aligned}$$

Since $\frac{q}{1-q} \in [0, 1)$ for $q \in \left[0, \frac{1}{2}\right)$, we conclude that the mapping f is a graphic contraction on a metric space (X, d) for $q < \frac{1}{2}$.
The mapping contracting perimeters of triangles is continuous as noted in [15]. Indeed, if $(x_n) \subseteq X$ is a non-stationary sequence converging to $x \in X$ with respect to the metric d, then, without loss of generality, we may assume that $\{x_n, x_m, x\}$ are three distinct points whenever $n \neq m$. Consequently, (1) holds and

$$\begin{aligned} 2d(fx, fx_n) &\le d(fx, fx_n) + d(fx_n, fx_m) + d(fx_m, fx) \\ &\le q\left(d(x, x_n) + d(x_n, x_m) + d(x_m, x)\right) \end{aligned}$$

whenever $n \neq m$. As $n, m \to \infty$, we conclude that (fx_n) is a convergent sequence with the limit fx, further implying that f is a continuous mapping as being a sequentially continuous mapping on a metric space. □

Remark 2.2. *Since a metric space is Hausdorff, any continuous mapping has a closed graph.*

Hence, for $q \in [0, \frac{1}{2})$ the main result of [15] is a direct corollary of Theorem 1.18.

Remark 2.3. *Graphic contractions do not have periodic points of prime period two. Indeed, if $fx \neq x$ and $f^2x = x$ for some self-mapping f of a metric space (X, d), then*

$$d(fx, x) = d(fx, f^2x) \le qd(x, fx)$$

which is impossible.

It can be discussed on the types of weakly Picard operators that are not Picard operators and that do have periodic points of a prime period two. As noted, a mapping contracting perimeters of triangles for $q \in [0, \frac{1}{2})$ is a graphic contraction with a closed graph, but the reverse does not hold.

Example 2.4. *Let $X = \bigcup_{n=1}^{\infty} X_n$ where $X_n = \left\{n, n - \frac{1}{m} \mid m \in \mathbb{N} \setminus \{1\}\right\}$ for $n \in \mathbb{N}$ and let a metric $d : X \times X \mapsto [0, \infty)$ be determined in* (6) *for all $x, y \in X$. A metric space (X, d) is complete.*
Define a mapping $f : X \mapsto X$ such that for $x \in X_n$ we have $fx = n$ if $x \in \{n, n - \frac{1}{m} \mid m \in \mathbb{N} \setminus \{1\}\}$ for all $n \in \mathbb{N}$. Since $\{n, n - \frac{1}{m} \mid m \in \mathbb{N} \setminus \{1\}\} \subseteq (n-1, n]$ for $n \in \mathbb{N}$, a mapping is well-defined.
Since $fx = f^2x$ for all $x \in X$, f is a graphic contraction on X and, additionally, it is a continuous mapping. Nevertheless, for any three mutually distinct natural numbers n, m and k ($|\{n, m, k\}|$=3), we observe that

$$d(fn, fm) + d(fm, fk) + d(fk, fn) = d(n, m) + d(m, k) + d(k, n)$$

and f is not a mapping contracting perimeters of triangles. The same conclusion can be derived from the cardinality of a set of fixed points of a mapping f.

Note that a mapping contracting perimeters of triangles satisfies more restrictive conditions than (1) of a Banach contraction, $d(fx, fy) \le qd(x, y)$, if x or y are accumulation points. Hence, the discrepancy between the concept of graphic contraction and the mapping contracting perimeters of triangles lies in the set of isolated points. Indeed, if $x \in X'$ and $y \in X$ are arbitrary and $(x_n) \subseteq X$ is a non-stationary sequence converging to x, then

$$d(fx, fy) + d(fy, fx_n) + d(fx_n, fy) \le q\left(d(x, y) + d(y, x_n) + d(x_n, y)\right)$$

implies, as $n \to \infty$, that $d(fx, fy) \le qd(x, y)$.
There exist mappings contracting perimeters of triangles with a unique fixed point that are not graphic contractions.

Example 2.5. *Let $X = \{x_n \mid n \in \mathbb{N}_0\}$ where $n \neq m$ implies $x_n \neq x_m$ be equipped with the metric $d : X \times X \mapsto [0, \infty)$ such that*

$$d(x_n, x_m) = \begin{cases} 0, & n = m \\ \frac{1}{2^{\left[\frac{n-1}{2}\right]}}, & m = n+1, n > 0 \\ \frac{1}{2^{\left[\frac{m-1}{2}\right]}}, & n = m+1, m > 0 \\ \sum_{i=n}^{m-1} d(x_i, x_{i+1}), & m > n+1 \\ \sum_{i=m}^{n-1} d(x_i, x_{i+1}), & n > m+1 \\ 4 - \sum_{i=1}^{n} d(x_i, x_{i+1}), & m = 0, n > 0 \\ 4 - \sum_{i=1}^{m} d(x_i, x_{i+1}), & n = 0, m > 0 \end{cases}$$

Observe a mapping $f : X \mapsto X$ such that

$$fx_n = \begin{cases} x_0, & n = 0 \\ x_{n+1}, & n > 0 \end{cases}$$

As observed in [15], (X, d) is a complete metric space and f is a mapping contracting perimeters of triangles for $q = \frac{7}{8}$, and it has a unique fixed point x_0.
However, it is not a graphic contraction since $d(fx_1, f^2x_1) = d(x_2, x_3) = 1$ and $d(x_1, fx_1) = d(x_1, x_2) = 1$.

We will present a remetrization under which any mapping contracting perimeters of triangles may be observed as a graphic contraction.

Theorem 2.6. *If $f : X \mapsto X$ is a mapping on a complete metric space (X, d), and a mapping $d^* : X \times X \mapsto [0, \infty)$ is defined by*

$$d^*(x, y) = \begin{cases} d(x, fx) + d(y, fy) + d(x, fy) + d(y, fx), & x \neq y \\ 0, & x = y \end{cases} \tag{8}$$

then (X, d^*) is a complete metric space.

Proof. The mapping $d^* : X \times X \mapsto [0, \infty)$ defined by (8) is a well-defined mapping since $d : X \times X \mapsto [0, \infty)$. If $x = y$, then $d^*(x, y) = 0$ by (8). If $d^*(x, y) = 0$ and $x \neq y$, then

$$d(x, fx) + d(y, fy) + d(x, fy) + d(y, fx) = 0$$

implies $x = y$, which is a contradiction. Thus, $d^*(x, y) = 0$ if and only if $x = y$.
The mapping d^* is symmetric due to its definition and the symmetry of a metric d, so $d^*(x, y) = d^*(y, x)$ for all $x, y \in X$.
Let $x, y, z \in X$ be arbitrary, then the triangle inequality easily follows, and as a conclusion, (X, d^*) is a metric space.
Assume that $(x_n) \subseteq X$ is a Cauchy sequence in a metric space (X, d^*). Then, for any $n, m \in \mathbb{N}$ such that $m > n$ we have

$$d(x_n, x_m) \leq d(x_n, fx_n) + d(fx_n, x_m) \leq d^*(x_n, x_m),$$

so (x_n) is a Cauchy sequence in a metric space (X, d). Let $x^* = \lim_{n\to\infty} x_n$, then $d(x_n, fx_n) \leq d^*(x_n, x_m)$ leads to $lim_{n\to\infty} fx_n = fx^* = x^*$. Accordingly,

$$d^*(x_n, x^*) = d(x_n, fx_n) + d(x^*, fx^*) + d(x_n, fx^*) + d(x^*, fx_n).$$

Thus, the sequence (x_n) converges to x^* with respect to the metric d^*. Therefore, (X, d^*) is a complete metric space. □

Additionally, a mapping contracting perimeters of triangles without periodic points of a prime period two in (X, d) will be a graphic contraction in (X, d^*).

Theorem 2.7. *If $f : X \mapsto X$ is a mapping contracting perimeters of triangles without periodic points of a prime period two on a complete metric space (X, d) such that $|X| \geq 3$, and a mapping $d^* : X \times X \mapsto [0, \infty)$ defined by* (8)*, then f is an orbitally continuous graphic contraction on a complete metric space (X, d^*).*

Proof. Theorem 2.6 claims the completeness of an induced metric space (X, d^*) determined by (8). For an arbitrary $x \in X$, if $fx = x$ or $f^2x = fx$, the inequality (7) is evidently true. Otherwise, x, fx and f^2x are three distinct points. Observe

$$d^*(fx, f^2x) = d(fx, f^2x) + d(f^2x, f^3x) + d(fx, f^3x)$$
$$d^*(x, fx) = d(x, fx) + d(fx, f^2x) + d(x, f^2x)$$

which further implies that $d^*(fx, f^2x) \leq qd^*(x, fx)$ holds for all $x \in X$.
To prove orbital continuity of f with respect to d^*, suppose that $(f^{m_n}x)$ converges to some y in (X, d^*) for some $x, y \in X$ and a sequence $(m_n) \subseteq \mathbb{N}$. Since

$$d^*(f^{m_n}x, y) = d(f^{m_n}x, f^{m_n+1}x) + d(y, fy) + d(f^{m_n}x, fy) + d(y, f^{m_n+1}x),$$

it follows that $fy = y$ and f is an orbitally continuous on (X, d^*). □

Now, we can claim that the main result of [15] can be derived from Theorem 1.18 except for the part of the maximal number of fixed points that is easily derived from the condition (1).

Corollary 2.8. *Let (X, d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ satisfy the following two conditions:*

(i) $f^2x = x$ implies $fx = x$ for all $x \in X$;

(ii) f is a mapping contracting perimeters of triangles.

Then f has a fixed point, and the number of fixed points is at most two.

Proof. Theorems 2.6 and 2.7 claim that f is an orbitally continuous graphic contraction on a complete metric space (X, d^*), so the existence of a fixed point, as well as the convergence of iterative sequence (f^nx) to a fixed point for any $x \in X$, directly follow from Theorem 1.18. Recall that the contractive condition (1) implies that the number of fixed points is at most two due to the fact that for $fx = x$, $fy = y$, and $fz = z$ for three mutually distinct points, we deduce

$$d(x, y) + d(y, z) + d(z, y) \leq q\,(d(x, y) + d(y, z) + d(z, y))$$

which leads to the contradiction. □

Continuous generalized Kannan contraction on a metric space without isolated points is a Kannan contraction. However, talking about Kannan contraction, we intend to avoid the continuity presumption. Hence, it is an open question whether a generalized Kannan contraction can be observed as a Kannan contraction, assuming it is discontinuous. Examples 2.5 and 2.6 of [16] testify to the independence of these two classes of mappings. The main result of [16] is the existence of a fixed point of the generalized Kannan contraction on a complete metric space, assuming that the mapping does not have periodic points of a prime period two. We will prove that, under these presumptions, a generalized Kannan-type mapping is an orbitally continuous graphic contraction. Moreover, the number of fixed points is at most two, which follows from (3).

Theorem 2.9. *If $f : X \mapsto X$ is a generalized Kannan-type mapping in the sense of* (3) *on a complete metric space (X, d) with $|X| \geq 3$ such that it has no periodic points of prime period two, then the mapping f is an orbitally continuous graphic contraction on X.*

Proof. If $x = fx$ or $fx = f^2x$, then the inequality (7) trivially holds. Otherwise, suppose that (x, fx) and (fx, f^2x) are pairs of distinct points, and taking into account that f has no periodic points of prime period 2, x and f^2x are also mutually distinct. Therefore, (3) is applicable on $\{x, fx, f^2x\}$, so

$$d(fx, f^2x) + d(f^2x, f^3x) + d(f^3x, fx) \leq q\left(d(x, fx) + d(fx, f^2x) + d(f^2x, f^3x)\right)$$

further implies

$$\begin{aligned}(1-q)d(fx, f^2x) &\leq qd(x, fx) - (1-q)d(f^2x, f^3x) - d(f^3x, fx)\\ &\leq qd(x, fx) - (1-q)d(fx, f^2x).\end{aligned}$$

Hence,

$$d(fx, f^2x) \leq \frac{q}{2(1-q)}d(x, fx)$$

and f is graphic $\frac{q}{2(1-q)}$-contraction since $\frac{q}{2(1-q)} \in [0, 1)$ for $q \in [0, \frac{2}{3})$.
It remains to prove that the mapping f fulfilling (7) is orbitally continuous. Assume that $\lim_{n\to\infty} f^{m_n}x = y$ for some $x, y \in X$ and $(m_n) \subseteq \mathbb{N}$. Taking into account the proven fact that f is also a graphic contraction, by a principle of mathematical induction, easily follows that

$$d(f^{m_n}x, f^{m_n+1}x) \leq \left(\frac{q}{2(1-q)}\right)^{m_n} d(x, fx).$$

As $n \to \infty$, we deduce that $\lim_{n\to\infty} d(f^{m_n}x, f^{m_n+1}x) = 0$ and, moreover, $\lim_{n\to\infty} f^{m_n+1}x = y$. If $(f^{m_n}x)$ is a stationary sequence, then $f^{m_n}x = y$ and $f^{m_n+1}x = fy$ starting from some $n \geq n_0$, but having in mind that the sequence $(f^{m_n+1}x)$ converges to y, we get $fy = y$. If that is not the case, $f^{m_n}x$, $f^{m_{n+1}}x$ and y are three mutually distinct points for infinitely many $n \in \mathbb{N}$. Thus,

$$\begin{aligned}&d(f^{m_n+1}x, f^{m_{n+1}+1}x) + d(f^{m_{n+1}+1}x, fy) + d(fy, f^{m_n+1}x)\\ &\leq q\left(d(f^{m_n}x, f^{m_n+1}x) + d(f^{m_{n+1}}x, f^{m_{n+1}+1}x) + d(y, fy)\right)\end{aligned}$$

leads to the conclusion that $fy = y$ since the inequality

$$2d(y, fy) \leq qd(y, fy)$$

holds after letting $n \to \infty$.
Hence, a generalized Kannan-type mapping without periodic points of prime period two is an orbitally continuous graphic contraction. □

This result will imply that the main result of [16] given in the Introduction as Theorem 1.5 is a direct corollary of the Saturated principle of graphic contraction, with the exception that the part that the maximal number of fixed points is two must be proved additionally, which is a trivial consequence of the contractive condition.

Corollary 2.10. *Let (X, d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ be a generalized Kannan-type mapping without periodic points of a prime period two. The mapping f has a fixed point, and the number of fixed points is at most two.*

Proof. Theorem 2.9 asserts that a generalized Kannan-type mapping without periodic points of a prime period two is an orbitally complete graphic contraction on a complete metric space, so Theorem 1.18 asserts existence of a fixed point as well as the convergence of the iterative sequence (f^nx) to a fixed point of a mapping for arbitrary initial point $x \in X$.

As mentioned, uniqueness is easily obtained from (3). Suppose, contrary to what we intend to prove, that $fx = x$, $fy = y$, and $fz = z$ hold for pairwise distinct points $x, y, z \in X$. Then,

$$\begin{aligned} d(x,y) + d(y,z) + d(z,x) &= d(fx,fy) + d(fy,fz) + d(fz,fx) \\ &\leq q\,(d(x,fx) + d(y,fy) + d(z,fz)) \end{aligned}$$

leads to the contradiction.
Therefore, a generalized Kannan-type mapping without periodic points of a prime period two on a complete metric space has one or two fixed points. □

Another type of perimetric contractions are generalized Chatterjea-type mappings [14]. Chatterjea contractions are generalized Chatterjea-type mappings, which was not the case for generalized Kannan-type mappings and Kannan contractions. Examples 2.2 and 2.3 of [14] present generalized Chatterjea-type mappings that are not Kannan or Chatterjea mappings, and also not mappings contracting perimeteres of triangles or generalized Kannan-type mappings.

Theorem 2.11. *If $f : X \mapsto X$ is a generalized Chatterjea-type mapping in a sense of* (4) *for a $q \in [0, \frac{1}{3})$ on a complete metric space (X, d) with $|X| \geq 3$ such that it has no periodic points of prime period two, then the mapping f is an orbitally continuous graphic contraction on X.*

Proof. If $x = fx$ or $fx = f^2x$, the inequality (7) trivially holds. Otherwise, as f has no periodic points of prime period two, x and f^2x are also distinct. Thus, we may apply (4) on $\{x, fx, f^2x\}$, leading to

$$\begin{aligned} d(fx,f^2x) + d(f^2x,f^3x) + d(f^3x,fx) \leq q\Big(&d(x,f^2x) + d(x,f^3x) + d(fx,fx) \\ &+ d(fx,f^3x) + d(f^2x,fx) + d(f^2x,f^2x)\Big). \end{aligned}$$

Consequently,

$$(1-q)d(fx,f^2x) \leq 2qd(x,fx) - (1-q)d(f^2x,f^3x) - (1-2q)d(f^3x,fx)$$

and

$$d(fx,f^2x) \leq \frac{q}{1-2q}d(x,fx).$$

Thus, f is a graphic $\frac{q}{1-2q}$-contraction since $\frac{q}{1-2q} \in [0,1)$ for $q \in [0, \frac{1}{3})$.
It remains to prove that the mapping f fulfilling (4) for $q < \frac{1}{3}$ is orbitally continuous. Assume that $\lim_{n\to\infty} f^{m_n}x = y$ for some $x \in X$. Taking into account the proven fact that f is also a graphic contraction, by a principle of mathematical induction, it follows that

$$d(f^{m_n}x, f^{m_n+1}x) \leq \left(\frac{q}{1-2q}\right)^{m_n} d(x,fx).$$

As $n \to \infty$, we deduce that $\lim_{n\to\infty} d(f^{m_n}x, f^{m_n+1}x) = 0$ and moreover $\lim_{n\to\infty} f^{m_n+1}x = y$. If $(f^{m_n}x)$ is a stationary sequence, then $f^{m_n}x = y$ and $f^{m_n+1}x = fy$ starting from some $n \geq n_0$, but having in mind that the sequence $(f^{m_n+1}x)$ converges to y, we get $fy = y$. If that is not the case, $f^{m_n}x$, $f^{m_{n+1}}x$ and y are three mutually distinct points for infinitely many $n \in \mathbb{N}$. Thus,

$$\begin{aligned} &d(f^{m_n+1}x, f^{m_{n+1}+1}x) + d(f^{m_{n+1}+1}x, fy) + d(fy, f^{m_n+1}x) \\ &\leq q\Big(d(f^{m_n}x, f^{m_{n+1}+1}x) + d(f^{m_n}x, fy) + d(f^{m_{n+1}}x, f^{m_n+1}x) \\ &+ d(f^{m_{n+1}}x, fy) + d(y, f^{m_n+1}x) + d(y, f^{m_{n+1}+1}x)\Big) \end{aligned}$$

leads to the conclusion that $fy = y$ since the inequality

$$2d(y, fy) \leq 2qd(y, fy)$$

holds after letting $n \to \infty$. Hence, $fy = y$.
Hence, a generalized Chatterjea-type mapping without periodic points of a prime period two is an orbitally continuous graphic contraction. □

This result will imply that for $q \in [0, \frac{1}{3})$, the main result of [14] given in the Introduction as Theorem 1.8 is a direct corollary of the Saturated principle of graphic contraction, except for the upper bound of the number of fixed points.

Corollary 2.12. *Let (X, d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ satisfy the following two conditions:*

(i) f has no periodic points of a prime period two;

(ii) f is a generalized Chatterjea-type mapping for $q \in [0, \frac{1}{3})$.

Then f has a fixed point, and the number of fixed points is at most two.

Proof. Theorem 2.11 asserts that assumptions (i) and (ii) imply that f is an orbitally complete graphic contraction on a complete metric space, so Theorem 1.18 gives existence of a fixed point as well as the convergence of the iterative sequence $(f^n x)$ to a fixed point of a mapping for arbitrary initial point $x \in X$.
The maximal number of fixed points is easily obtained from (3). Suppose, contrary to what we intend to prove, that $fx = x$, $fy = y$, and $fz = z$ hold for pairwise distinct points $x, y, z \in X$. Then,

$$\begin{aligned} d(x, y) + d(y, z) + d(z, x) &\leq d(fx, fy) + d(fy, fz) + d(fz, fx) \\ &\leq q\left(d(x, fy) + d(x, fz) + d(y, fx) + d(y, fz)\right. \\ &\left.+ d(z, fx) + d(z, fy)\right) \\ &= 2q\left(d(x, y) + d(y, z) + d(z, x)\right) \end{aligned}$$

leads to the contradiction.
Therefore, a generalized Chatterjea-type mapping for $q \in [0, \frac{1}{3})$ without periodic points of a prime period two on a complete metric space has one or two fixed points. □

Similarly to the mapping contracting perimeters of triangles, we will prove that a generalized Chatterjea-type mapping without periodic points of a prime period two on a complete metric space (X, d) will be a graphic contraction in (X, d^*).

Theorem 2.13. *If $f : X \mapsto X$ is a generalized Chatterjea-type mapping without periodic points of a prime period two on a complete metric space (X, d) such that $|X| \geq 3$, and a mapping $d^* : X \times X \mapsto [0, \infty)$ is defined by (8), then f is an orbitally continuous graphic contraction on a complete metric space (X, d^*).*

Proof. According to Theorem 2.6, the induced metric space (X, d^*) is complete, where the metric is determined by (8). Let $x \in X$ be arbitrary. If $fx = x$ or $f^2x = fx$, the inequality (7) holds, while for distinct x, fx and f^2x we obtain

$$\begin{aligned} d^*(fx, f^2x) &= d(fx, f^2x) + d(f^2x, f^3x) + d(fx, f^3x) \\ d^*(x, fx) &= d(x, fx) + d(fx, f^2x) + d(x, f^2x). \end{aligned}$$

The contractive condition (3) implies

$$\begin{aligned} d(fx, f^2x) + d(f^2x, f^3x) + d(fx, f^3x) &\leq q\Big(d(x, f^2x) + d(x, f^3x) \\ &+ d(fx, f^3x) + d(f^2x, fx)\Big) \end{aligned}$$

and

$$\begin{aligned}&(1-q)\left(d(fx,f^2x)+d(f^2x,f^3x)+d(fx,f^3x)\right)\\ &\leq q\left(d(x,f^2x)+d(x,f^3x)-d(fx,f^3x)+d(fx,f^3x)-d(f^2x,f^3x)\right)\\ &\leq q\left(d(x,f^2x)+d(x,fx)+d(fx,f^2x)\right).\end{aligned}$$

Consequently, $d^*(fx,f^2x) \leq \frac{q}{1-q}d^*(x,fx)$ holds for all $x \in X$ and $\frac{q}{1-q} \in [0,1)$ for $q \in [0,\frac{1}{2})$.
To prove orbital continuity of f with respect to d^*, suppose that $(f^{m_n}x)$ converges to some y in (X,d^*) for some $x,y \in X$ and a sequence $(m_n) \subseteq X$. Since

$$d^*(f^{m_n}x,y) = d(f^{m_n}x,f^{m_n+1}x)+d(y,fy)+d(f^{m_n}x,fy)+d(y,f^{m_n+1}x),$$

it follows that $fy=y$ and f is an orbitally continuous on (X,d^*). □

Thus, the main result of [14] can be derived from Theorem 1.18 except for the part of the maximal number of fixed points that is easily derived from the condition (4).

Corollary 2.14. *Let (X,d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ satisfy the following two conditions:*

(i) $f^2x = x$ implies $fx = x$ for all $x \in X$;

(ii) f is a generalized Chatterjea-type mapping.

Then f has a fixed point, and the number of fixed points is at most two.

Proof. Theorems 2.6 and 2.13 imply that f is an orbitally continuous graphic contraction on a complete metric space (X,d^*). Therefore, from Theorem 1.18, f has a fixed point. Recall that the contractive condition (4) implies that the number of fixed points is at most two due to the fact that for $fx = x$, $fy = y$, and $fz = z$ for three mutually distinct points, we deduce

$$d(x,y)+d(y,z)+d(z,y) \leq 2q\,(d(x,y)+d(y,z)+d(z,y))$$

further implying the contradiction. Thus, f has at least one and at most two fixed points in X. □

Based on the results of Ćirić, Reich and Rus and inspired by the perimetric contractive condition, the generalized Ćirić-Reich-Rus type mapping was introduced.

Theorem 2.15. *If $f : X \mapsto X$ is a generalized ĆRR-type mapping in a sense of (5) on a complete metric space (X,d) with $|X| \geq 3$ such that it has no periodic points of prime period two, then the mapping f is an orbitally continuous graphic contraction on X.*

Proof. Assume that (X,d) is a complete metric space and $f : X \mapsto X$ is a generalized ĆRR-type mapping on X. Let $x \in X$ be arbitrary. Then if $fx = x$ or $f^2x = fx$, f has a fixed point in X. Otherwise, $fx \neq x$ additionally implies that $f^2x \neq x$ and x, fx, f^2x are mutually distinct points, so (5) may be applied for $y = fx$ and $z = f^2x$. Accordingly,

$$\begin{aligned}d(fx,f^2x)+d(f^2x,f^3x)+d(f^3x,fx) &\leq \alpha\left(d(x,fx)+d(fx,f^2x)+d(f^2x,x)\right)\\ &\quad+\beta\left(d(x,fx)+d(fx,f^2x)+d(f^2x,f^3x)\right)\end{aligned}$$

implies

$$\begin{aligned}(1-\alpha-\beta)d(fx,f^2x) &\leq (\alpha+\beta)d(x,fx)+\alpha d(f^2x,x)\\ &\quad-(1-\beta)d(f^2x,f^3x)-d(f^3x,fx)\\ &\leq (2\alpha+\beta)d(x,fx)+\alpha d(fx,f^2x)-(1-\beta)d(fx,f^2x).\end{aligned}$$

Consequently,

$$d(fx, f^2x) \leq \frac{2\alpha + \beta}{2 - 2\alpha - 2\beta} d(x, fx)$$

and as $\frac{2\alpha+\beta}{2-2\alpha-2\beta} < 1$ due to $2\alpha + \frac{3}{2}\beta < 1$, f is a graphic contraction on X.
To prove orbital continuity of f suppose that $(f^{m_n}x)$ converges in X for some $x \in X$ and $(m_n) \subseteq \mathbb{N}$ and denote the limit point with y. Then, from the fact that f is a graphic contraction, by the principle of mathematical induction, we obtain

$$d(f^{m_n}x, f^{m_n+1}x) \leq \left(\frac{2\alpha + \beta}{2 - 2\alpha - 2\beta}\right)^{m_n} d(x, fx)$$

and by letting $n \to \infty$, it follows that $\lim_{n\to\infty} f^{m_n+1}x = y$. We assume that $(f^{m_n}x)$ is a non-stationary sequence since otherwise $y = fy$ easily follows, so we can observe, for infinitely many $n \in \mathbb{N}$, three mutually distinct points $f^{m_n}x$, $f^{m_{n+1}}x$ and y. The inequality (5) implies

$$\begin{aligned}
&d(f^{m_n+1}x, f^{m_{n+1}+1}x) + d(f^{m_{n+1}+1}x, fy) + d(fy, f^{m_n+1}x)\\
&\leq \alpha \left(d(f^{m_n}x, f^{m_{n+1}}x) + d(f^{m_{n+1}}x, y) + d(y, f^{m_n}x)\right)\\
&+ \beta \Big(d(f^{m_n}x, f^{m_n+1}x) + d(f^{m_{n+1}}x, f^{m_{n+1}+1}x) + d(y, fy)\Big)
\end{aligned}$$

and by letting $n \to \infty$, we acquire

$$2d(y, fy) \leq \beta d(y, fy).$$

Hence, $fy = y$ and f is an orbitally continuous graphic contraction. □

Consequently, the main result of [3] presented in Theorem 1.10 of the Introduction follows from the Saturated graphic contraction principle while the upper bound of the number of fixed points must be derived directly from the contractive condition.

Corollary 2.16. *Let (X, d) for $|X| \geq 3$ be a complete metric space and let the mapping $f : X \mapsto X$ satisfy the following two conditions:*

(i) f has no periodic points of a prime period two;

(ii) f is a generalized ĆRR-type mapping.

Then f has a fixed point, and the number of fixed points is at most two.

Proof. Theorem 2.15 claims that a generalized ĆRR-type mapping without periodic points of prime period two is an orbitally continuous graphic contraction. The underlying metric space is supposed to be complete, so from Theorem 1.18 it follows that f has a fixed point in X and that the iterative sequence $(f^n x)$ converges to the fixed point for arbitrary $x \in X$.
To claim that the number of fixed points is at most two, suppose that $fx = x$, $fy = y$, and $fz = z$ while $x, y, z \in X$ are three mutually distinct points. Then, from (5) follows

$$\begin{aligned}
d(x, y) + d(y, z) + d(z, x) &= d(fx, fy) + d(fy, fz) + d(fz, fx)\\
&\leq \alpha \left(d(x, y) + d(y, z) + d(z, x)\right)\\
&+ \beta \left(d(x, fx) + d(y, fy) + d(z, fz)\right)\\
&= \alpha \left(d(x, y) + d(y, z) + d(z, x)\right)
\end{aligned}$$

which is possible if and only if $x = y = z$.

Note that if some of the sequences $(f^{m_n}x)$ is stationary, so it is $(f^{m_n+1}x)$ and $fy = y$.
Thus, the number of fixed points of a generalized ĆRR-type mapping without periodic points of prime period two on a complete metric space is one or two. □

3. Conclusion

In this paper, we present a detailed analysis of the relation of four types of perimetric contractions-mapping contracting perimeters of triangles, generalized Kannan-type mappings, generalized Chattrjea-type mappings, and generalized ĆRR-type mappings. It is proven that generalized Kannan and ĆRR-type mappings are graphic contractions, while additional constraints for contractive constants are required in the case of mappings contracting perimeters of triangles and generalized Chatterjea-type mappings. However, we propose a remetrization under which completeness is preserved, and both mappings contracting perimeters of triangles and generalized Chatterjea-type mappings are orbitally continuous graphic contractions on the induced metric space. Consequently, we conclude that the existence of a fixed point for these perimetric contractions can be derived from results concerning graphic contractions, specifically from the Saturated graphic contraction principle.
Several questions remain open, such as whether a generalized Kannan-type contractive condition can be modified to indeed generalize the Kannan contraction, how the relationship among perimetric contractions might change under a remetrization, and whether these mappings can be classified under one category of weakly Picard operators.

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